

Moments of Distributions

Probability and Statistics Series

Instructor's Guide

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DEVELOPED BY THE TEACHING AND LEARNING LABORATORY AT MIT
FOR THE SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN



Introduction

When to Use this Video

- In EPDE 101 or ESD 102, at home or in recitation.
- Prior knowledge: integration, some familiarity with probability

Learning Objectives

After watching this video students will be able to:

- Define moments of distributions.
- Compute moments and understand what they mean.

Motivation

Death, taxes, and uncertainty are part of life. We represent uncertainty using probability distributions. However, distributions, especially in many dimensions, are complicated beasts. Moments of distributions allow us to represent important features of probability distributions using just a few numbers.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Inspect an expression for variance to convince them that the resultant value will always be non-negative.
- Compute the variance and sketch it as a function of p for a simple discrete distribution.
- Compute the first and second moments and the variance for the uniform distribution.

Key Information

Duration: 15:03

Narrator: Prof. Sanjoy Mahajan

Materials Needed:

- paper
- pencil

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Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:00	A question is posed - why does driving to the airport seem to take more time than driving home from the airport?	This question will be answered at the end of the video.
1:06	Prerequisite knowledge and Learning objectives	
2:43	The first moment of a distribution is defined.	
3:40	The second moment of a distribution is defined.	
4:35	Variance is defined.	
5:19	Standard deviation is defined.	
5:58	Student activity	Students are asked to pause the video and inspect an expression for variance to convince themselves that the resultant value will always be non-negative.
8:30	Student activity	Students are asked to pause the video and compute the variance and sketch it as a function of p for a simple discrete distribution.
11:08	Student activity	Students are asked to pause the video and compute the first and second moments and the variance for the uniform distribution.
12:37	The question posed at the beginning of the video is answered.	

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Video Summary

This video introduces students to the moments of a distribution and how they can be used to characterize the shape of a distribution. Students apply what they learned to examples of discrete and continuous distributions.

Course Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.



1. Discussion question

Which kind of subway system would you prefer: one system that gets you from home to work in 30 minutes, give or take 2 minutes; or another system that needs needs 30 minutes, give or take 10 minutes?



2. Integration practice

Find the area under the parabola $y=x^2$ from $x=0$ to $x=1$.



3. Probability review

You flip a (fair) coin three times. What are the probabilities of getting 0 heads, 1 head, 2 heads, and 3 heads?

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Post-Video Materials



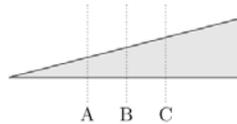
1. Dice moments: Make a table of probabilities for the possible sums of two dice. Use the table to compute the first moment of the distribution (the expected sum). How does this moment compare to the first moment of the distribution for one die roll?



2. Moment of a continuous distribution: A strange random-number generator produces, like most random-number generators, a real number between 0 and 1; however, unlike most random-number generators, its probability of producing a number x is proportional to x . Sketch the probability density $p(x)$, making sure that $p(x)$ is normalized. Then find its first and second moments and the variance.



3. Moment of inertia: A uniform triangular piece of metal is to be spun about a vertical axis. Where should you place the axis in order to minimize the moment of inertia about the axis?



4. Connections: Explain the connection between the preceding two problems.

Additional Resources

References

The following papers discuss common difficulties students may have in understanding basic statistical concepts.

- Batanero, C., Godino, J. D., Vallecillos, A., Green, D., & Holmes, P. (1994). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematical Education in Science and Technology*, 25(4), 527-547.
- delMas, R., Garfield, J., Ooms, A., Chance, B. (2007). Assessing students' conceptual understanding after a first course in statistics. *Statistics Education Research Journal*, 6(2), 28-58.
- delMas, R., & Liu, Y. (2005). Exploring students' conceptions of the standard deviation. *Statistics Education Research Journal*, 4(1), 55-82. Retrieved July 29, 2014 at [http://www.stat.auckland.ac.nz/~iase/serj/SERJ4\(1\)_delMas_Liu.pdf](http://www.stat.auckland.ac.nz/~iase/serj/SERJ4(1)_delMas_Liu.pdf)

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