

# Rigid Body Kinematics

## Representations Series

### *Instructor's Guide*

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# Introduction

## When to Use this Video

- In SOPH 301, in recitation, after Lecture 1, Eigenvalues and Eigenvectors.
- Prior knowledge: eigenvalues and eigenvectors; the standard basis of a real, 3-dimensional, vector space; and orthogonal matrices.

## Learning Objectives

After watching this video students will be able to:

- Identify rotation matrices.
- Decompose the motion of the wobbly, thrown disk into translational and rotational components.
- Write the rotational motion of the disk as a product of rotation matrices.

## Motivation

- This video aims to help students connect notions of eigenvalues and eigenvectors to geometric transformations of vector spaces, in particular, the rotation.
- After connecting concepts in linear algebra to geometric transformation, we show the power of linear operators in describing the motion of rigid bodies by introducing time dependent rotation matrices and solving a real world kinematics problem.

## Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Identify the eigenvalues and eigenvectors of rotation matrices.
- Explain what complex eigenvalues mean in terms of a geometric transformation of a vector space.
- Determine whether all orthonormal matrices describe rotational transformations.
- Write matrices describing rotational components of a real-world, rigid body motion.
- Find an expression for a complex rigid body, rotational, time-dependent motion in terms of combinations of simple rotations about standard basis vectors.

## Key Information

*Duration:* 17:26

*Narrator:* Prof. Dan Frey

*Materials Needed:*

- Paper
- Pencil

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## Video Highlights

This table outlines a collection of activities and important ideas from the video.

Time	Feature	Comments
0:05	Frisbee/Disk tosses	The motion that will be described in pieces throughout the video is shown here.
0:50	Prerequisites and Learning Objectives	
1:24	Chapter 1: Kinematics	Review of rigid body kinematics
2:14	Chapter 2: Three-Dimensional Rotation Matrices	Linear algebra of rotations, including eigenvalues and eigenvector analysis.  There are 5 activities for students to pause the video and find eigenvalues and vectors for various rotation matrices.
6:40	Rotation matrices - classified	Rotation matrices are classified as orthogonal matrices with determinant 1.
7:25	Chapter 3: Time Dependent Rotations	
8:45	Activity	How do you make a rotation matrix time dependent?
9:45	Rotation rate	How to compute a constant rotation rate.
10:05	Wobble described	Wobble described as a rotation of a tilt axis, and the wobble rotation rate is described.
11:15	Initial condition of disk described	
11:35	Motion decomposed into rotations about principle axes	
12:07	Activity	Students are asked to write a matrix that removes the tilt from the initial position of the disk.
13:45	Activity	Students are asked to determine how to combine the 4 matrices together to describe the rotational motion of the disk.
14:14		Process of decomposition is checked to see that that the combination of rotations does in fact describe the position of the disk for all times.
15:15	Translation added into equation	
16:30	To Review	Video content is summarized.

### Video Summary

This video leads students through describing the motion of all points on a wobbly disk as a function of time. Properties of time independent rotation matrices are explored. The problem of the wobbly disk is scaffolded through a sequence of increasingly more complicated motions.

# SOPH 301 Materials

## Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.



1. Have students identify examples that show that combinations of rotations are not commutative. These examples can be geometric descriptions, or matrix descriptions.



2. If the determinant of real matrix is 1, and one eigenvalue is complex, what does that tell you about the other eigenvalues?



3. Consider a real matrix that has eigenvalues equal to 2, 3 and  $1/2$  corresponding to eigenvectors  $e_1$ ,  $e_2$ , and  $e_3$  respectively.



(a) Draw a picture that describes what happens to a unit cube after the matrix is applied to the vector space.

(b) How would this picture change if the eigenvector  $e_3$  is replaced by an eigenvector that is a linear combination of the vector  $e_3$  and  $e_2$ , but has the same eigenvalue?

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## Post-Video Materials



1. Prove the following using linear algebra.

(a) A matrix which does not change the lengths of vectors must be an orthogonal matrix.



(b) A matrix which is orthogonal has determinant equal to plus or minus 1.

(c) Explain why rotation matrices must have determinant plus 1.



2. In the video, we chose initial positions of the marked point and the tilt axis to make the matrices the identity matrix at time zero. Determine how you must change the matrices if there is a change in the initial position so that:



(a) The marked point is in a different location initially.

(b) The tilt axis is in the different location initially.

(c) The angular velocities of the tilt and the marked point are equal.



3. In this video, we decomposed the rotations of a complex system into a sequence of rotations about the  $x$ ,  $y$ , and  $z$  axes. Show that you can describe any rotation about an arbitrary axis in terms of a sequence of rotations about principle axes.



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## Additional Resources

### Going Further

Inspired by a plate thrown up in the air in the cafeteria, Richard Feynman used dynamics to find a relationship between the angular velocity of the plate compared to the angular velocity of the wobble. In a course such as EPDM 301, it would be appropriate to build off of this example and solve this dynamics problem.

### References

This book inspired the example considered in this video. It highlights the playful and exploratory nature of scientific discovery, and may be motivational for students.

- Feynman, R. P. (1985). *Surely you're joking Mr. Feynman!*. New York, NY: W. W. Norton & Company, Inc.

The following text is common for use in undergraduate linear algebra courses, and has many excellent matrix algebra and linear algebra problems and questions.

- Strang, G. (1998). *Introduction to Linear Algebra*. Wellesley, MA: Wellesley-Cambridge Press.

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