

Gear Trains

Linearity Series

Instructor's Guide

CONTENTS

INTRO

COURSEWORK

RESOURCES

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DEVELOPED BY THE TEACHING AND LEARNING LABORATORY AT MIT
FOR THE SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN



Introduction

When to Use this Video

- In a mechanical design course, after learning about gears.
- Prior knowledge: be able to identify and describe the forces and torques that act on a system.

Learning Objectives

After watching this video students will be able to:

- Model the forces and torques in a gear train as a system of linear equations.
- Combine these linear equations into a matrix equation.

Motivation

- Students typically have difficulty using linear algebra to model the force and torque balance in a gear train.
- This video provides a reference for modeling this problem using linear algebra, and provides a jumping off point for using the model to understand gear train design elements.

Student Experience

It is highly recommended that the video is paused when prompted so that students are able to attempt the activities on their own and then check their solutions against the video.

During the video, students will:

- Explain how a matrix equation is modeling the force and torque balance on one rigid body gear consisting of two gears molded together.
- Work out an analogous matrix for another gear body whose free body diagram is given.
- Write a linear equation that describes the torque balance due to the gear attached to the motor.
- Write a linear equation that describes the torque balance due at the output gear.

Key Information

Duration: 16:55

Narrator: Prof. Dan Frey

Materials Needed:

- Paper
- Pencil

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Video Highlights

This table outlines a collection of activities and important ideas from the video.

| Time | Feature | Comments |
|-------|---|--|
| 0:00 | Video clip of servomotors | The use and function of servomotors is introduced. |
| 1:13 | Prerequisites and Learning Objectives | What students should know before watching the video, and what they will learn in the video. |
| 1:35 | Chapter 1: Servomotor Design | Output torque in the idealized, friction-less case is computed. |
| 3:38 | Output torque measured experimentally | Servomotor is front-loaded with overhanging weights to find an estimate for torque with friction. |
| 4:53 | Chapter 2: Modeling Forces and Torques | |
| 6:20 | Forces and torques on Gear Body 2 modeled in detail | |
| 8:32 | Activity | Students verify that matrix equation describes a balance of forces and torques on the gear body. |
| 9:15 | Gear Body 3 modeled | |
| 9:40 | Linking equation described | An equation to model the connection between Gear Bodies 2 and 3 is developed to connect two matrix equations into 1 larger matrix equation. |
| 12:02 | Activity | Students model Gear Body 1 with matrix equation. |
| 12:38 | Activity | Students model the pinion and output gears. |
| 13:45 | Inspect model and solutions | Full matrix is examined to determine if it is correct by checking the signs of solutions. |
| 14:10 | Activity | Students are asked to determine why one component of a normal force was pointing opposite the direction predicted. |
| 14:53 | Chapter 3: Apply model to explore design | Values for efficiency and friction are introduced to compute the efficiency of the gear design. This computed value is compared to the rough experimental estimate to suggest that loading the servomotors with torque is not a good idea. |
| 16:10 | To Review | Video content is summarized. |

Video Summary

This video leads students through using linear algebra to model a gear train. This is achieved by modeling each rigid gear body as a system of 3 equations, the input and output gears as 1 linear equation each, and then modeling the connection between gears as another linear equation derived from Newton's second law. The resulting matrix is explored for consistency and design elements.

Course Materials

Pre-Video Materials

When appropriate, this guide is accompanied by additional materials to aid in the delivery of some of the following activities and discussions.

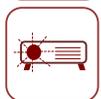
The first two questions check student background in understanding solutions to linear equations. The last question checks student conceptual understanding of gears and gear trains.



1. Overdetermined System (Appendix A1)



Determine conditions on the vector entry b such that the following matrix equation has a solution.



$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ b \end{bmatrix}$$



2. Rank and Solutions (Appendix A2)



Determine conditions on the matrix entry a and solution entry b so that the following matrix equation has

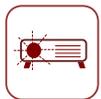


- (a) one solution,
- (b) many solutions, and
- (c) no solutions.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ b \end{bmatrix}$$



3. Gears (Appendix A3–A7)



Explain the following for each of the gear configurations in Appendix A3.

- (a) Does the force of the output increase, decrease, or stay the same as the input?
- (b) Does the torque of the output increase, decrease, or stay the same as the input?
- (c) Is the direction of rotation of the output the same as or the reverse of the input gear rotation?

Post-Video Materials



1. To determine the motor's output requires a bit of measurement and calculation. The brushed DC motor in this servo has a torque constant, according to our measurements, of 0.0052 Newton Meters per Amp. We used a 4.8V Nickel Metal Hydride battery, and given the winding resistance of the motor (0.87 Ohms), and the discharge capacity of that battery, we find it delivers 4.0 Amps near "stall" when the output shaft is stationary or moving slowly.



- Compute the torque at the DC motor.
- Under ideal conditions, how much torque might this 326:1 compound gear produce?



2. In modeling this problem, we assumed that the friction along the shaft was uniform. In reality, it is more complicated. Typically, the friction at one side of the bushing is larger than at the other side, which creates additional torque.



- Discuss with a partner how this information would change the model we developed in the video.
- Try to write an equation to balance the torques and forces on a gear with non-uniform friction along the shaft.



3. Gear Train Configuration (Appendix A8)



Another interesting design aspect is that the gear train reverts back to the axle about which the pinion turns. This has two major impacts. One is that the bushing needs to be wider to fit over the cap on the pinion.

- Describe the direction of the separation forces on this gear.
- How does this effect the normal force at the shaft for this gear?
- How does this effect the tangential pressure forces?
- What would happen to these forces if this gear was rotating in the opposite direction?
- Do you think this placement and direction of rotation was a good design decision?

Additional Resources

Going Further

Applying linear algebraic methods conceptually and numerically to solve force and torque balance equations is a powerful way to decompose complex linear systems, and gain sophisticated understanding of the design principles involved.

The application of tensors to the study of mechanics is in many ways a natural extension of this type of discrete, linear balance problem. Tensors are used much as matrices in this video, to study the forces and torques applied to continuous, deformable objects.

References

The following article goes into more detail about the friction losses at the shaft and at the meshing of gear teeth and may be of interest.

- Kuria, J. and Kihui, J., (2011). Prediction of Overall Efficiency in Multistage Gear Trains, *International Journal of Aerospace and Mechanical Engineering*, 5(3), 171-177.

The following text books provide references for the material in this video.

- Strang, G., (1998). *Introduction to Linear Algebra*. Wellesley, MA: Wellesley-Cambridge Press.
- Shigley, J., Mischke, C., and Budynas, R., (2003). *Mechanical Engineering Design*. Boston, MA: McGraw-Hill.

Determine conditions on the vector b such that the matrix equation below has a solution.

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ b \end{bmatrix}$$

Determine conditions on the matrix entry a and the solution entry b such that the matrix equation below has:

- a) one solution,
- b) infinitely many solutions, and
- c) no solutions

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ b \end{bmatrix}$$

For the gear configuration below:

- a) Does the force of the output increase, decrease, or stay the same as the input?
- b) Does the torque of the output increase, decrease, or stay the same as the input?
- c) Is the direction of rotation of the output the same as or the reverse of the input gear rotation?

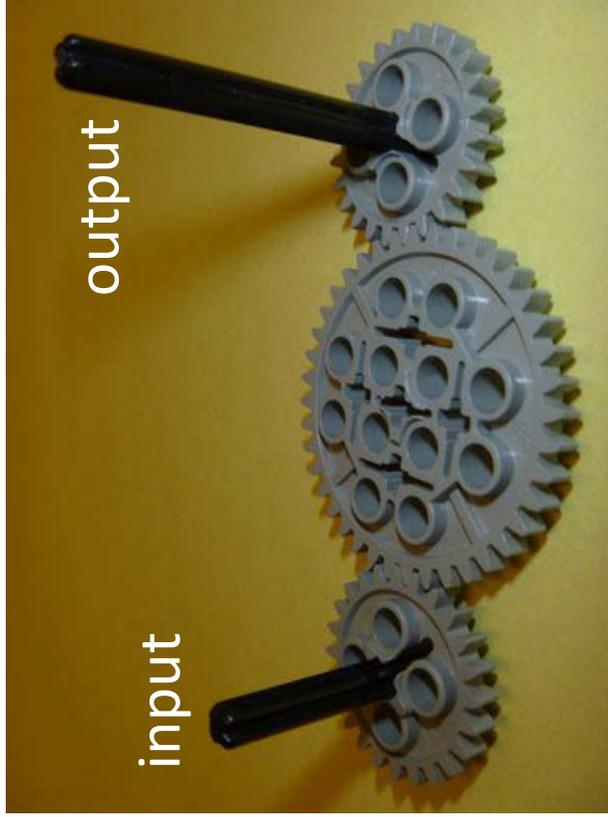
output



input

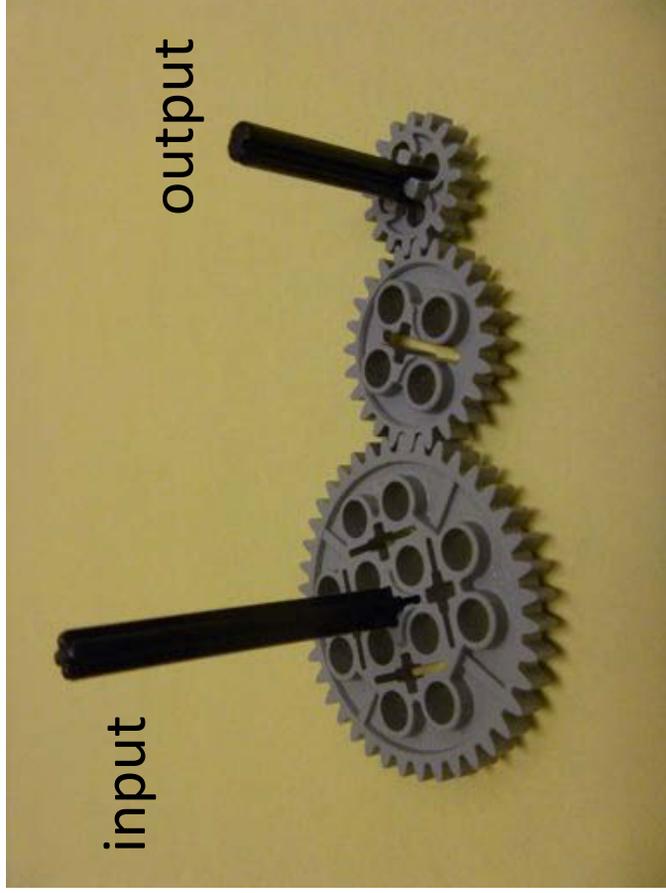
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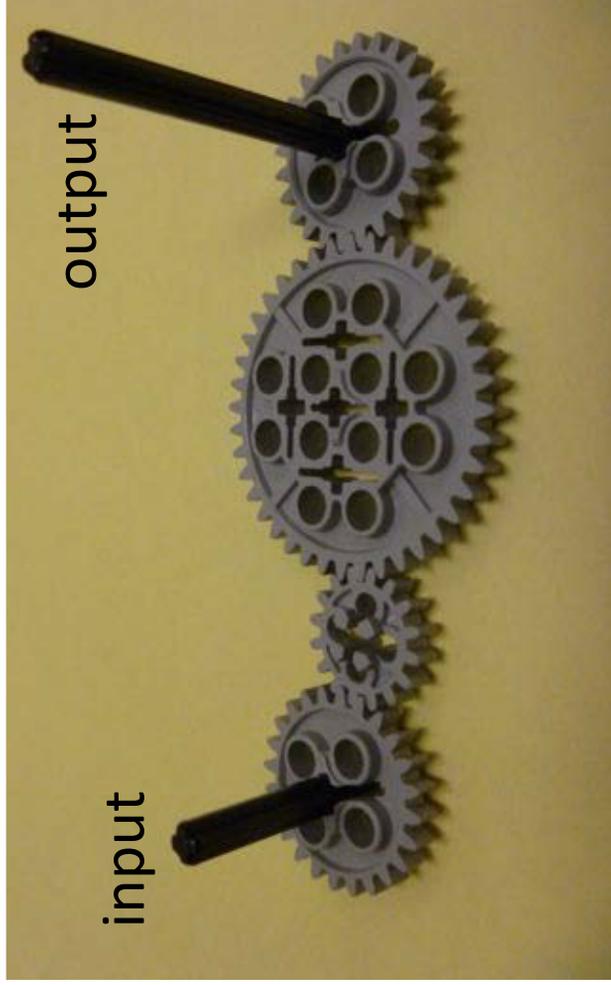
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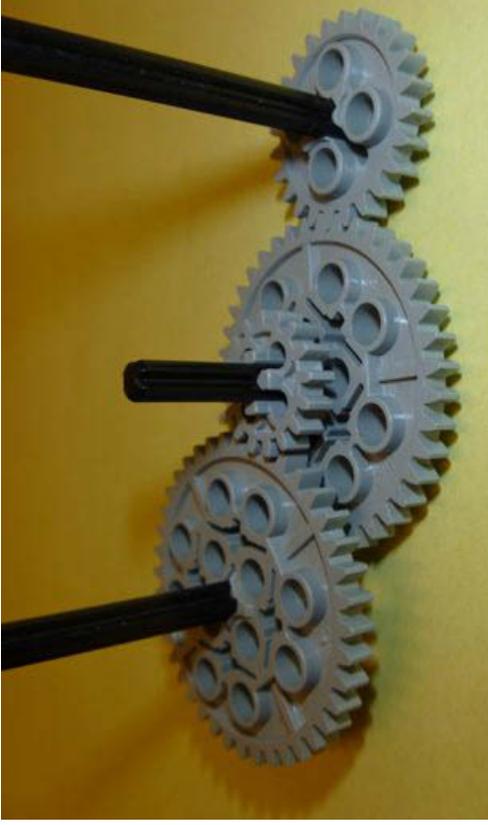


For the gear configuration below:

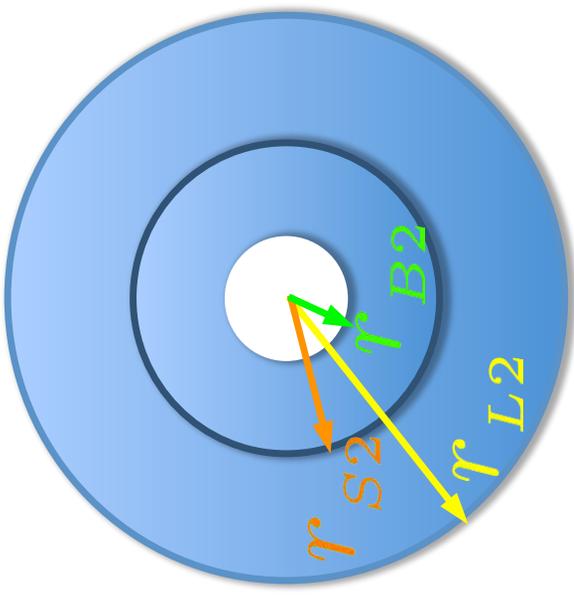
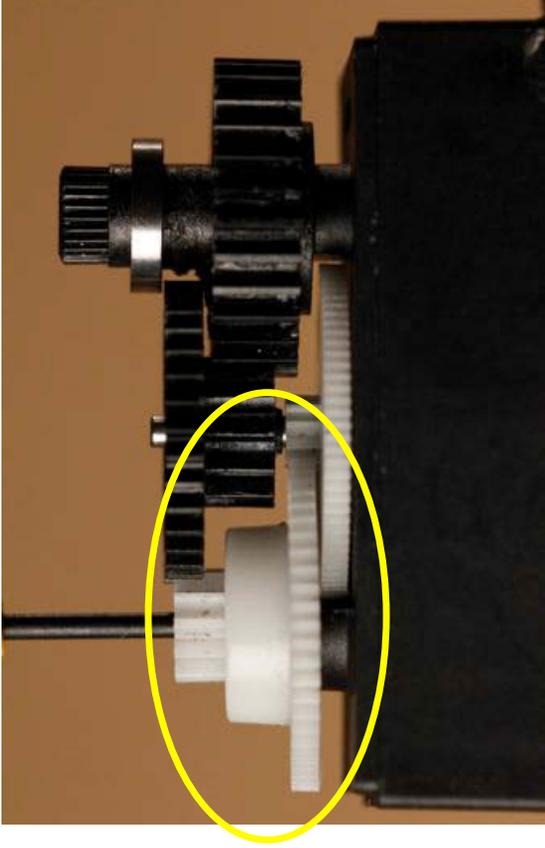
- a) Does the force of the output increase, decrease, or stay the same as the input?
- b) Does the torque of the output increase, decrease, or stay the same as the input?
- c) Is the direction of rotation of the output the same as or the reverse of the input gear rotation?

input

output



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- Describe the direction of the separation forces on this gear.
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