Oscillators (Intentional) 18

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 18.1 (P12.1)

From Equation 12.1, with $\omega = \frac{1}{RC}$,

$$V_a\left(\frac{j}{RC}\right) = \frac{j}{j^2 + 3j + 1} = \frac{1}{3} \quad \text{(S18.1)}$$

Thus, at $\omega = \frac{1}{RC}$, the feedback path to the noninverting terminal has the same transfer function as the feedback path to the inverting terminal. Thus, the voltages at both terminals are equal.

The modified topology will not function as an oscillator because in this case, the resistive positive feedback makes the op-amp connection unstable.

Solution 18.2 (P12.2)

To make the example specific let the parallel leg resistance increase by 5% to 1.05 R. Then,

$$\frac{V_a(s)}{V_o(s)} = \frac{1.05\,RCs}{1.05\,R^2C^2s^2 + 3.1\,RCs + 1} \quad \text{(S18.2)}$$

Now, let the closed-loop gain of the noninverting connection equal k. (In Section 12.1.1, $k = 3$.) Then, the characteristic equation is:

$$\begin{aligned} 1 - L(s) &= 1 - \frac{1.05\,kRCs}{1.05\,R^2C^2s^2 + 3.1\,RCs + 1} \\ &= \frac{1.05\,R^2C^2s^2 + (3.1 - 1.05k)\,RCs + 1}{1.05\,R^2C^2s^2 + 3.1\,RCs + 1} \end{aligned} \quad \text{(S18.3)}$$

This has imaginary zeros at $s = \frac{\pm j}{\sqrt{1.05}RC}$ when $k = 2.95$. Thus, the component values in the resistive network must satisfy $\frac{R_1 + R_2}{R_2} = 2.95$ where R_1 connects the amplifier output to the inverting input, and R_2 is connected between the inverting input and ground. This is satisfied by $R_1 = 1.95\,R_2$, a 2.5% change in R_1. (In Section 12.1.1, $R_1 = 2R_2$.)

Solution 18.3 (P12.4)

Consider the double integrator of Figure 11.12 with the $R/2$ valued resistor replaced by a resistance of $\frac{R}{2}(1 + \Delta)$, where Δ is the fractional change in resistance. Then, Equation 11.21 becomes

$$I_f(s) = \frac{(1 + \Delta)RC^2s^2}{2((1 + \Delta)RCs + 1)} V_o(s) \qquad \textbf{(S18.4)}$$

Then, using Equation 11.20, and applying the constraint $I_f = -I_i$ yields

$$\frac{V_o(s)}{V_i(s)} = -\frac{(1 + \Delta)RCs + 1}{(RCs + 1)(1 + \Delta)(RCs)^2} \qquad \textbf{(S18.5)}$$

Note that if $\Delta = 0$ this reduces to the original Equation 11.22, as expected.

Then, with the output of the double integrator connected back to its input, the loop transmission $L(s)$ is given by Equation S18.5. The characteristic equation then is

$$1 - L(s) = 1 + \frac{(1 + \Delta)RCs + 1}{(RCs + 1)(1 + \Delta)(RCs)^2}$$
$$= \frac{R^3C^3(1 + \Delta)s^3 + R^2C^2(1 + \Delta)s^2 + RC(1 + \Delta)s + 1}{(RCs + 1)(1 + \Delta)(RCs)^2} \qquad \textbf{(S18.6)}$$

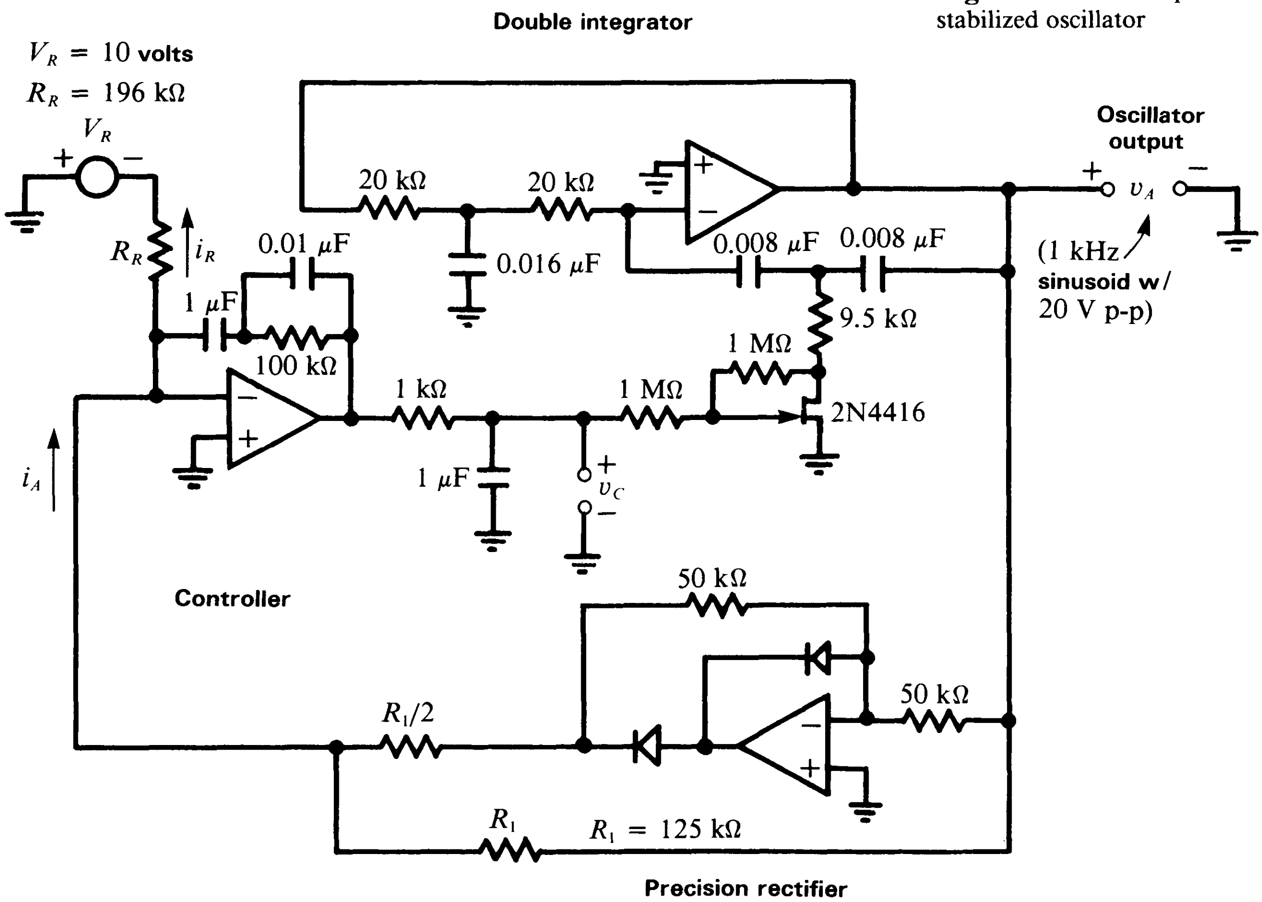
This is identical with Equation 12.9, and thus for small Δ, Equation 12.10 applies. Therefore, the performance of the oscillator is dominated by a complex-conjugate root pair with $\omega_n \simeq \frac{1}{RC}$, and $\zeta \simeq \Delta/4$. The closed-loop poles can be made to lie in either the left half or the right half of the s plane according to the sign of Δ. Thus, by adjusting Δ, the envelope of the sinusoidal output may be made exponentially increasing ($\Delta < 0$) or exponentially decreasing ($\Delta > 0$). By this mechanism the amplitude can be controlled.

Now, because Equation 12.10 applies, following the analysis on pp. 490–91 of the textbook, the linearized transfer function relating envelope amplitude to Δ is

$$\frac{E_a(s)}{\Delta(s)} = -\frac{E_A}{4\,RCs} \qquad \textbf{(S18.7)}$$

as given by Equation 12.17. Note that we are letting $v_A(t)$ be equal to the double integrator output voltage $v_o(t)$ in order to conform to the notation of Section 12.1.4.

Figure S18.1 Amplitude stabilized oscillator



Now, as shown in Figure S18.1, we use the same FET circuit as in Figure 12.4, including the 9.5 kΩ resistor, so that nominally $R/2$ = 10 kΩ. (Thus, R = 20 kΩ.) Then, all the equations describing the FET in Section 12.1.4 apply. Specifically, from Equation 12.23

$$\left.\frac{\delta\Delta}{\delta v_C}\right|_{v_C=-4\text{V}} = -0.0125\ \text{V}^{-1} \tag{S18.8}$$

Now, to set the operating frequency to 1 kHz, we require that

$$\omega_n = \frac{1}{RC} = 2\pi \times 10^3 \tag{S18.9}$$

Because we have already determined that R = 20 kΩ, this is solved by C ≃ 0.008 μF. Thus, the components of the double-integrator loop are chosen as shown in Figure S18.1.

Also, for 20 volts peak-to-peak output, $E_A = 10$. With these values, Equation S18.7 becomes

$$\frac{E_a(s)}{\Delta(s)} = -\frac{1.57 \times 10^4}{s} \tag{S18.10}$$

Combining this with Equation S18.8 yields the linearized incremental relation between $E_a(s)$ and $V_c(s)$.

$$\frac{E_a(s)}{V_c(s)} = \frac{196}{s} \tag{S18.11}$$

Now, we turn our attention to the amplitude-measuring circuit and the controller circuit. As shown in Figure S18.1, we use a precision full-wave rectifier to provide an amplitude measurement. The controller amplifier provides a ground potential current-summing node, so only one additional amplifier is required to realize the precision rectifier. This is the same connection as is used in the precision phase shifter of Figure 12.32. The precision rectifier is discussed in more detail in Section 11.5.1.

The controller circuit has the same topology as in Figure 12.6. Note, however, that the R_1 and $R_1/2$ valued resistors of the precision rectifier replace the 312.5 kΩ input resistor of Figure 12.6. Because the loop in Figure 12.4 is very similar to the loop in Figure S18.1, we can use the same controller. That is, because they are oscillating at similar frequencies we can set crossover for both loops at 100 rad/sec. Further, the incremental relations between $V_c(s)$ and $E_a(s)$ (Equations 12.24 and S18.11) differ only in the gain constant. Thus, by simply scaling the gain constant of $a(s)$ (i.e., by varying the controller input resistor), the two loops can be made to have identical amplitude-control loop transmissions.

The only remaining subtlety is in determining the incremental relation between $E_a(s)$ and $V_c(s)$. With ω_c set to 100 rad/sec, the controller $a(s)$ effectively filters out all signal components at the oscillator frequency (1 kHz) and above. Thus, we can effectively ignore these harmonics, and focus on the propagation of the amplitude signal around the loop. That is, the amplitude-control loop is really feeding back on the amplitude parameter $e_A(t)$, and not on the detailed waveform $v_A(t)$.

Because it is full-wave rectified, the current into the controller summing junction $i_A(t)$ is always greater than or equal to zero and is given by $\frac{1}{R_1}\left|e_A(t)\sin\omega t\right|$. For slowly varying $e_A(t)$, the low frequency portion of this signal is given by finding the d-c Fourier component under the assumption that $e_A(t)$ is fixed. That is, the low-frequency current $i_{ALF}(t)$ into the summing junction is

$$i_{ALF}(t) \simeq \frac{\omega}{R_1\pi}\int_0^{\pi/\omega_0} e_A(t)\sin\omega t\,dt \simeq \frac{2}{R_1\pi}e_A(t) \qquad \textbf{(S18.12)}$$

Now, the transfer function from this low-frequency current to voltage v_c is given by

$$\frac{V_c(s)}{I_{alf}(s)} = -\frac{(0.1s + 1)}{10^{-6}s(10^{-3}s + 1)^2} \qquad \textbf{(S18.13)}$$

Then, combining Equations S18.11, S18.12, and S18.13 yields the amplitude-control loop transmission

$$L(s) = -\left(\frac{196}{s}\right)\left(\frac{2}{R_1\pi}\right)\left[\frac{0.1s + 1}{10^{-6}s(10^{-3}s + 1)^2}\right] = \frac{-1.25 \times 10^8(0.1s + 1)}{R_1 s^2(10^{-3}s + 1)^2} \qquad \textbf{(S18.14)}$$

Equating this with the negative of Equation 12.26, so the loop transmissions are identical, yields $R_1 = 125\ \text{k}\Omega$. The system crossover frequency is 100 rad/sec and the phase margin exceeds 70°.

One consequence of the chosen topology is that the amplitude reference signal is negative. That is, when the loop is in equilibrium, the average current drawn by the reference $\left(i_R = \dfrac{V_R}{R_R}\right)$ will be equal to the average current i_{ALF} supplied by the precision rectifier. We choose R_R so that the steady-state amplitude of $v_A(t)$ will be equal to V_R. That is, equating i_{ALF} with i_R, and setting $V_R = e_A$ gives

$$\frac{2}{R_1\pi}e_A = \frac{V_R}{R_R} = \frac{e_A}{R_R} \qquad \textbf{(S18.15)}$$

Given $R_1 = 125\ \text{k}\Omega$, this is solved by $R_R = 196\ \text{k}\Omega$. Then, for an output amplitude of 10 volts, $V_R = 10$ volts.

RES.6-010 Electronic Feedback Systems
Spring 2013