Describing Functions 15

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 15.1 (P6.6)

This problem is most readily solved by recognizing that the given nonlinearity can be represented by combining two elements as shown in Figure S15.1.

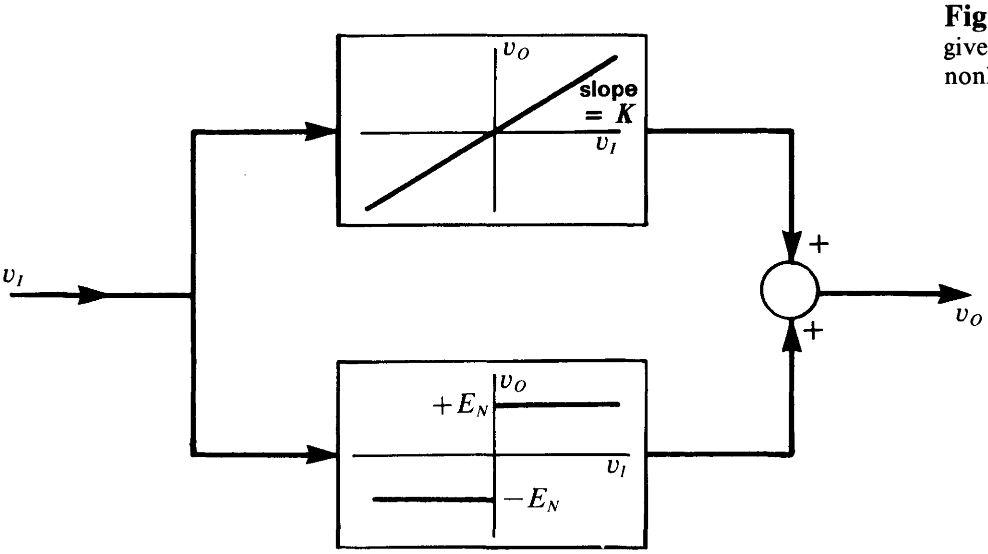

Figure S15.1 Decomposition of given nonlinearity into two nonlinearities.

The describing function for the upper element of Figure S15.1 is simply its linear gain K with zero phase. The describing function for the lower element of Figure S15.1 is tabulated in Table 6.1 of the textbook as $\frac{4E_N}{\pi E} \measuredangle 0°$. Then, the overall describing function is the sum of the two individual describing functions, and is given by

$$G_D(E) = \left(K + \frac{4E_N}{\pi E}\right) \measuredangle 0° \tag{S15.1}$$

Solution 15.2 (P6.7)

Following the analysis of Chapter 6 in the textbook, stable oscillations may exist where $a(j\omega) = \frac{-1}{G_D(E)}$ when the system is in the form shown in Figure 6.9. The nonlinear oscillator of Figure 6.26 is in the appropriate form, with $a(s) = \frac{10}{(s+1)(0.1s+1)(0.01s+1)}$. For the given nonlinearity, $G_D(E) = \frac{4}{\pi E} \angle 0°$, as tabulated in Table 6.1. Thus, oscillations of frequency ω can exist where

$$\frac{10}{(j\omega+1)(0.1j\omega+1)(0.01j\omega+1)} = -\frac{\pi E}{4} \qquad \textbf{(S15.2)}$$

Notice that the phase of the right-hand side of Equation S15.2 is $-180°$ for all ω.

A rough sketch of the Bode plot of $a(s)$ shows that the phase of the left-hand side is $-180°$ at about the geometric mean of the breakpoints at 10 and 100 rad/sec, which is $\omega \simeq 32$ rad/sec. An exact solution indicates that equality in S15.2 occurs for $\omega = 33$ rad/sec. At this frequency, the magnitude of $a(s)$ is 8.3×10^{-2}. Thus to satisfy S15.2, we must have $8.3 \times 10^{-2} = \frac{\pi E}{4}$, which is solved by $E = 0.11$ volts. This is the amplitude of the signal into the nonlinearity. (Note that the peak-to-peak value of the signal into the nonlinearity is $2E$ or 0.22 volts.)

The above analysis can also be carried out graphically in the gain-phase plane. Such a graphical analysis will appear very similar to the plot in Figure 6.13 of the textbook. This analysis will also verify that the oscillation at 33 rad/sec is stable, because following the discussion in Section 6.3.2, increasing E moves the point on the $-\frac{1}{G_D(E)}$ curve upwards, and thus to the left of the $a(j\omega)$ curve.

We have seen that the signal v_A is a 33 rad/sec sinusoid with an amplitude of 0.11 volts. Thus, v_B is a square wave in phase with v_A and with an amplitude of 1 volt (2 volts peak to peak). Now, consider the level of the third harmonic at the output of the nonlinearity. By the usual Fourier series calculations we find that the amplitude of the third harmonic is ⅓ that of the fundamental. Thus, because the fundamental of v_B has an amplitude of $4/\pi = 1.27$ volts, the third harmonic of v_B has an amplitude of 0.42 volts. Of course, this third harmonic is at a frequency of $3 \times 33 \simeq 100$ rad/sec, and is thus attenuated by a factor of about 0.007 by the third-order transfer function that filters v_B. Thus, the amplitude of the third harmonic in v_A is 0.42 volts $\times$ 0.007 = 0.0029 volts. The ratio in v_A of the amplitude of the third harmonic to the fundamental, then, is given by $\dfrac{0.0029 \text{ volts}}{0.11 \text{ volts}} = 0.027$.

RES.6-010 Electronic Feedback Systems
Spring 2013