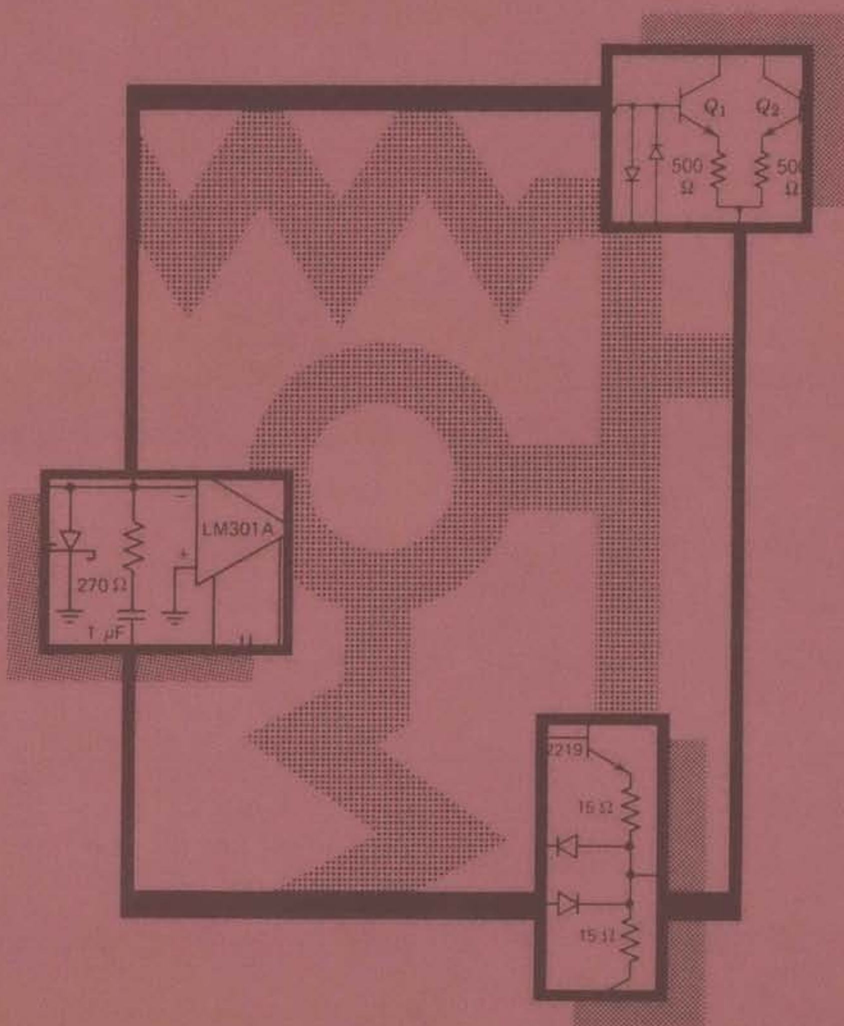
VIDEO COURSE MANUAL

Solutions to Problems



S O L U T I O N S

Introduction and Basic Concepts 1

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 1.1

Following the example of Section 1.2.3 of the textbook, with $N = 3$, we use the connection shown in Figure S1.1:

We then apply Equation 1.20 to write

$$V_o = -\frac{R_f}{R_{i1}} V_{i1} - \frac{R_f}{R_{i2}} V_{i2} - \frac{R_f}{R_{i3}} V_{i3} \quad \textbf{(S1.1)}$$

The desired gain expression is:

$$V_o = -V_{i1} - 2V_{i2} - 3V_{i3} \quad \textbf{(S1.2)}$$

Thus, $R_{i1} = R_f$, $R_{i2} = \frac{1}{2}R_f$, and $R_{i3} = \frac{1}{3}R_f$. By choosing $R_f = 60$ kΩ, we satisfy these conditions with $R_{i1} = 60$ kΩ, $R_{i2} = 30$ kΩ, and $R_{i3} = 20$ kΩ.

To derive the block diagram, we follow the method of Section 2.4 of the textbook to write a pair of equations in V_a, V_o, V_{i1}, V_{i2}, and V_{i3}.

We have, by superposition:

$$V_a = \frac{R_f \| R_{i2} \| R_{i3}}{R_{i1} + R_f \| R_{i2} \| R_{i3}} V_{i1} + \frac{R_f \| R_{i1} \| R_{i3}}{R_{i2} + R_f \| R_{i1} \| R_{i3}} V_{i2} + \frac{R_f \| R_{i1} \| R_{i2}}{R_{i3} + R_f \| R_{i1} \| R_{i2}} V_{i3} + \frac{R_{i1} \| R_{i2} \| R_{i3}}{R_f + R_{i1} \| R_{i2} \| R_{i3}} V_o \quad \textbf{(S1.3)}$$

and

$$V_o = -aV_a \quad \textbf{(S1.4)}$$

Substitution of numerical values into Equation S1.3 yields

$$-V_a = -\tfrac{1}{7}V_{i1} - \tfrac{2}{7}V_{i2} - \tfrac{3}{7}V_{i3} - \tfrac{1}{7}V_o \quad \textbf{(S1.5)}$$

This, combined with Equation S1.4, yields the block diagram in Figure S1.2:

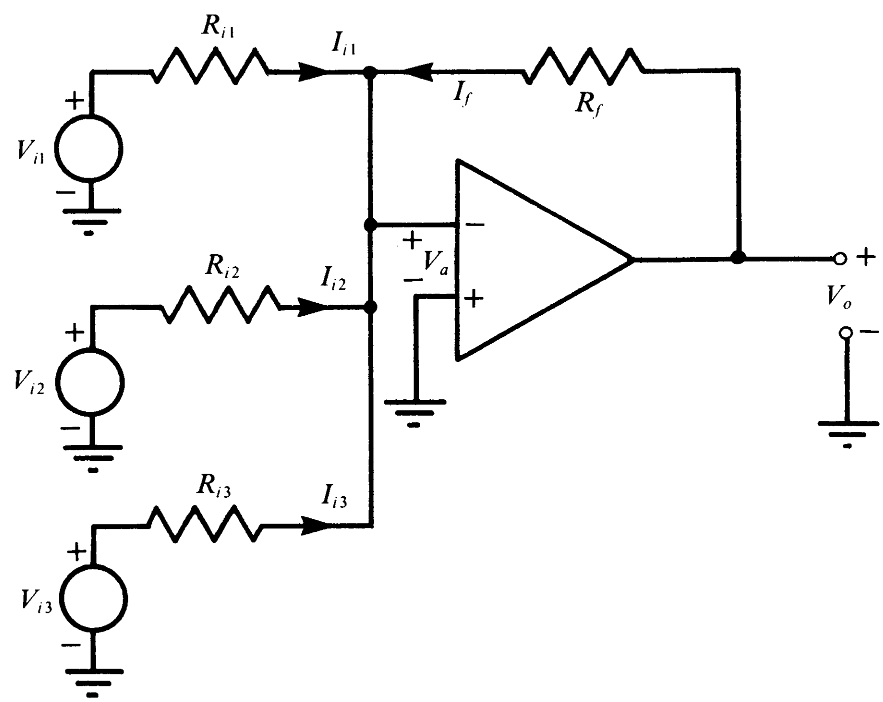


Figure S1.1 Three-input inverting amplifier.

Thus, for the specified accuracy, we require that the closed-loop gain of the output loop V_o/V' equal seven within 0.01%. That is,

$$\frac{V_o}{V'} = \frac{a}{1 + a \times 1/7} = 7 \times \frac{a}{7 + a} \geq 6.9993 \qquad \textbf{(S1.6)}$$

Thus, the minimum a is such that:

$$\frac{a}{7 + a} = (1 - 10^{-4}) \qquad \textbf{(S1.7)}$$

which is satisfied by: $a \geq 69{,}993$. That is, the loop-transmission magnitude must be greater than about 10,000.

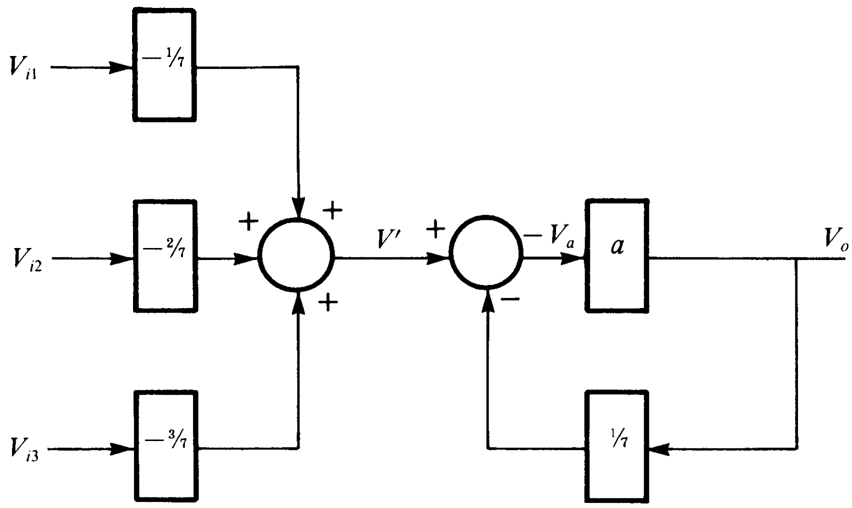


Figure S1.2 Block diagram for three-input inverting amplifier.

Solution 1.2 (P1.6)

We note that the connection of Figure 1.7*a* is simply a special case of Figure 1.7*b* with $V_{i1} = V_{i2}$. Thus, we first solve for the input–output relationship for Figure 1.7*b*. By superposition:

$$V_o = V_{i1} \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 10\text{ k}\Omega} \times \frac{10\text{ k}\Omega + 10\text{ k}\Omega}{10\text{ k}\Omega} - V_{i2} \times \frac{10\text{ k}\Omega}{10\text{ k}\Omega} \tag{S1.8}$$

using results derived in Section 1.2.2 of the text for inverting and noninverting connections. This reduces to:

$$V_o = V_{i1} - V_{i2} \tag{S1.9}$$

Thus, this connection is a unity-gain differential amplifier. (How would you design for gains greater or less than unity?) To consider Figure 1.7*a*, we set $V_{i1} = V_{i2} = V_i$, to find:

$$V_o = 0, \text{ for all } V_i \tag{S1.10}$$

This shows the useful property that differential amplifiers reject common-mode input signals.

Solution 1.3 (P2.1)

With reference to Figure 2.20, we write

$$V_o = a_2 V_a \text{ or, equivalently, } V_a = \frac{V_o}{a_2} \qquad \textbf{(S1.11)}$$

$$V_a = a_1 V_e \text{ or, equivalently, } V_e = \frac{V_o}{a_1 a_2} \qquad \textbf{(S1.12)}$$

$$V_e = V_i - V_f = V_i - f_1 V_a - f_2 V_o \qquad \textbf{(S1.13)}$$

Now, we eliminate V_a and V_e by substituting S1.11 and S1.12 into S1.13:

$$\frac{V_o}{a_1 a_2} = V_i - \frac{f_1 V_o}{a_2} - f_2 V_o$$

Collecting terms yields:

$$V_i = \left(\frac{1}{a_1 a_2} + \frac{f_1}{a_2} + f_2\right) V_o$$

or,

$$V_i = \frac{1 + a_1 f_1 + a_1 a_2 f_2}{a_1 a_2} V_o$$

Therefore,

$$\frac{V_o}{V_i} = \frac{a_1 a_2}{1 + a_1 a_2 (f_2 + f_1/a_2)} \qquad \textbf{(S1.14)}$$

which is the desired input–output relationship.

We solve the second part by a block-diagram manipulation method. This is often easier than working through the algebra as we have done in the first half of this problem. A valid manipulation on Figure 2.20 yields the block diagram shown in Figure S1.3, which reduces further to that of Figure S1.4, which is the desired reduced form. We see that by using the relation $A = \frac{a}{1 + af}$, Equation S1.14 can be derived by inspection of this reduced block diagram.

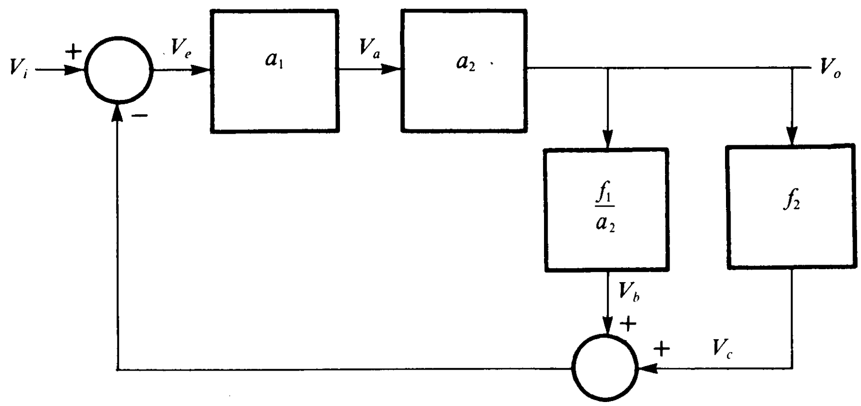


Figure S1.3 Manipulated block diagram.

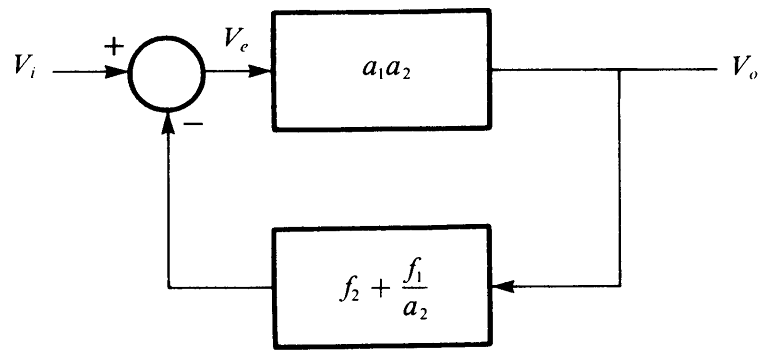


Figure S1.4 Reduced block diagram.

Solution 1.4 (P2.11)

This problem is most readily solved by removing the load and analyzing the output impedance of the op-amp connection, which we shall call R_{amp}. The output impedance with the load connected is then simply R_{amp} in parallel with R_L.

To solve for R_{amp}, we analyze the circuit of Figure S1.5:

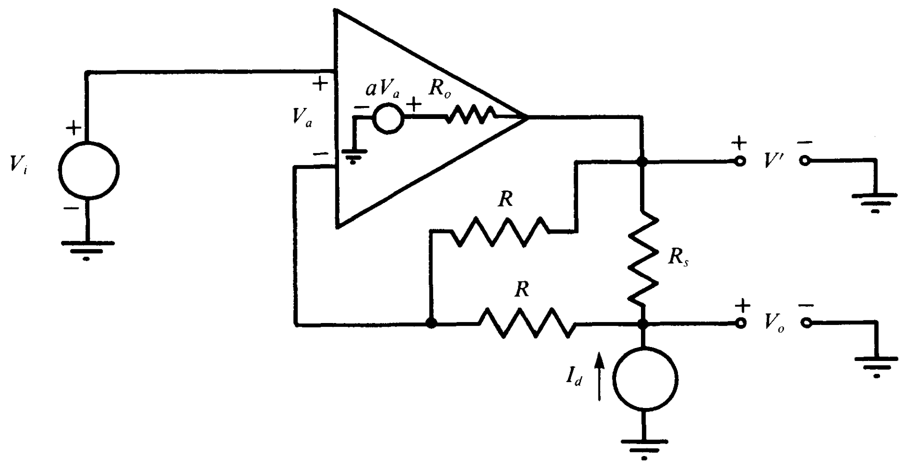


Figure S1.5 Operational amplifier connection with load removed.

Because $R \gg R_s$, we assume that all of I_d flows through the series connection of R_o and R_s. Thus

$$V_o = aV_a + I_d(R_o + R_s) \qquad \textbf{(S1.15)}$$

Now, we require an expression for V_a:

$$V_a = V_i - \frac{(V' + V_o)}{2} \qquad \textbf{(S1.16)}$$

where V' is as defined in Figure S1.5. But,

$$V' \simeq V_o - I_dR_s \qquad \textbf{(S1.17)}$$

Therefore:

$$V_a = V_i - V_o + \frac{I_dR_s}{2} \qquad \textbf{(S1.18)}$$

Equations S1.15 and S1.18 together define the block diagram shown in Figure S1.6:

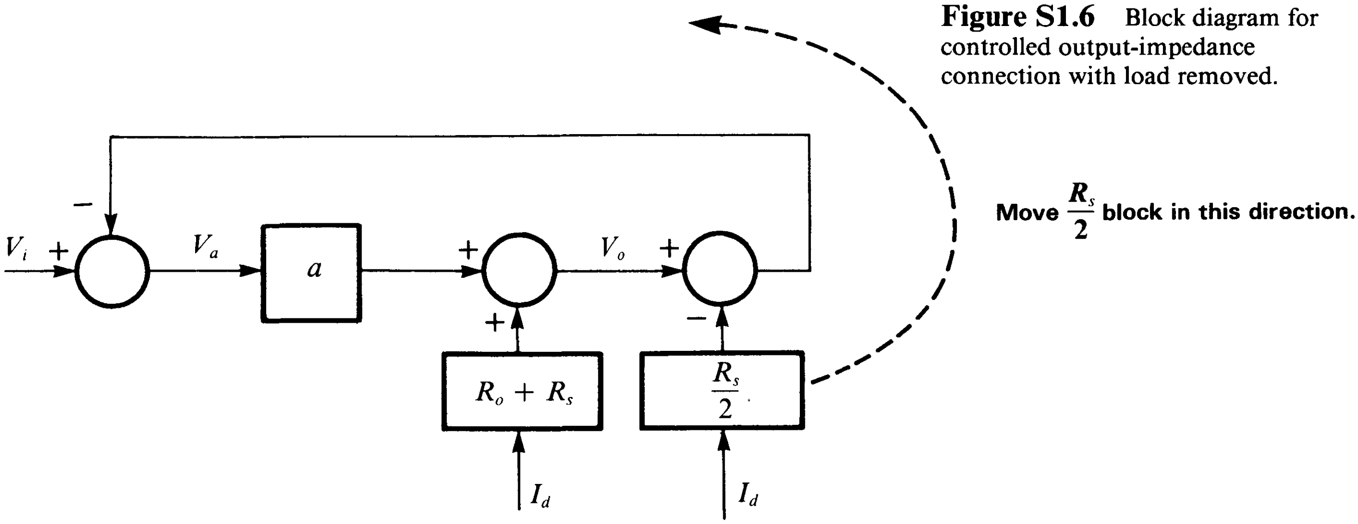


Figure S1.6 Block diagram for controlled output-impedance connection with load removed.

To calculate output-resistance R_{amp}, we need the ratio $\frac{V_o}{I_d}$. Note that V_i does not affect this ratio (as a consequence of linearity); thus, we set $V_i = 0$. We then manipulate the block diagram by propagating the $\frac{R_s}{2}$ block forward around the loop, as indicated in Figure S1.6, to arrive at the reduced block diagram of Figure S1.7:

With V'' defined as in Figure S1.7, we have $V_o = \frac{1}{1 + a} V''$, thus

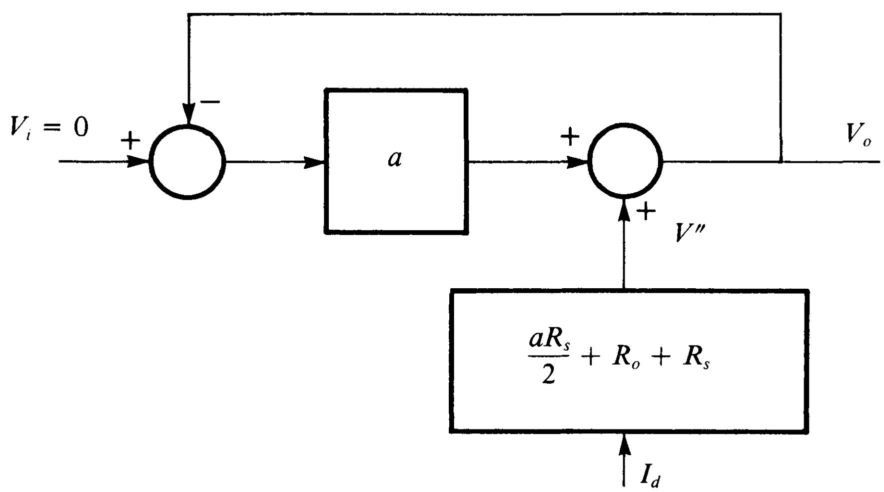


Figure S1.7 Reduced block diagram.

$$V_o = I_d\left(\frac{aR_s}{2} + R_o + R_s\right)\left(\frac{1}{1+a}\right) \quad \textbf{(S1.19)}$$

and

$$R_{\text{amp}} = \frac{V_o}{I_d} = \frac{\left(\frac{a}{2}+1\right)R_s + R_o}{1+a} \quad \textbf{(S1.20)}$$

If we define R_{out} as the output impedance with the load connected, we have:

$$R_{\text{out}} = R_{\text{amp}} \| R_L = \frac{\left(\frac{a}{2}+1\right)R_s + R_o}{1+a} \| R_L \quad \textbf{(S1.21)}$$

In the limit of large a, we have

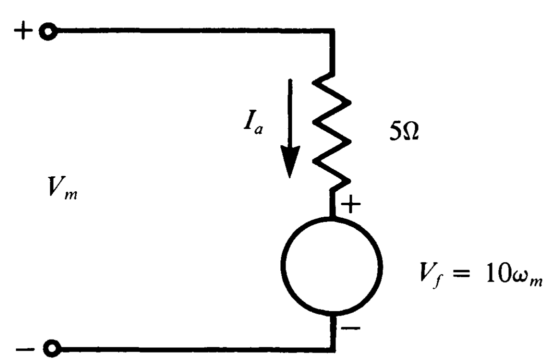
$$R_{\text{out}} = \lim_{a\to\infty}\left[\frac{\left(\frac{a}{2}+1\right)R_s + R_o}{1+a} \| R_L\right] = \frac{R_s}{2} \| R_L \quad \textbf{(S1.22)}$$

Solution 1.5 (P2.9)

Let's start by modeling the physical system of motor and antenna (the "plant" in control terminology), which has V_m as its input and θ_o as its output.

The motor is modeled as shown in Figure S1.8:

Figure S1.8 Motor model.



where ω_m is the motor rotational speed in radians per second and V_f represents the motor's back e.m.f. voltage.

Now, motor torque T_m is related to I_a by:

$$T_m = 10 \times I_a \quad \textbf{(S1.23)}$$

From Figure S1.8, we write:

$$I_a = \frac{1}{5\Omega}(V_m - 10\omega_m) \quad \textbf{(S1.24)}$$

Thus, the first part of the block diagram appears as shown in Figure S1.9.

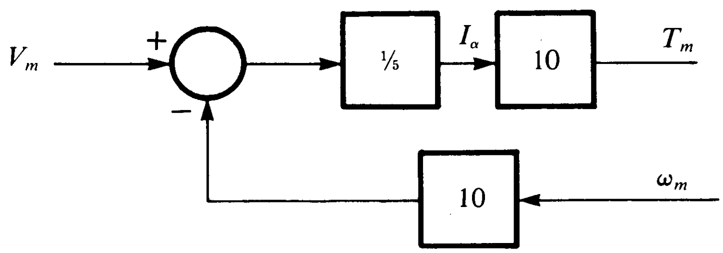


Figure S1.9 Partial block diagram for Problem 1.5 (P2.9).

Now, we require a relation between T_m and ω_m. The rotational equivalent of Newton's law $F = ma$ is $T = I\alpha$ where I is the rotational inertia and α is the angular acceleration. Applying this relationship with $\alpha_m = \frac{d\omega_m}{dt}$ and $I_m = 2$ kg-m we have

$$\frac{d\omega_m}{dt} = \frac{T_m}{2} \quad \textbf{(S1.25)}$$

Thus, the transfer function between T_m and ω_m is

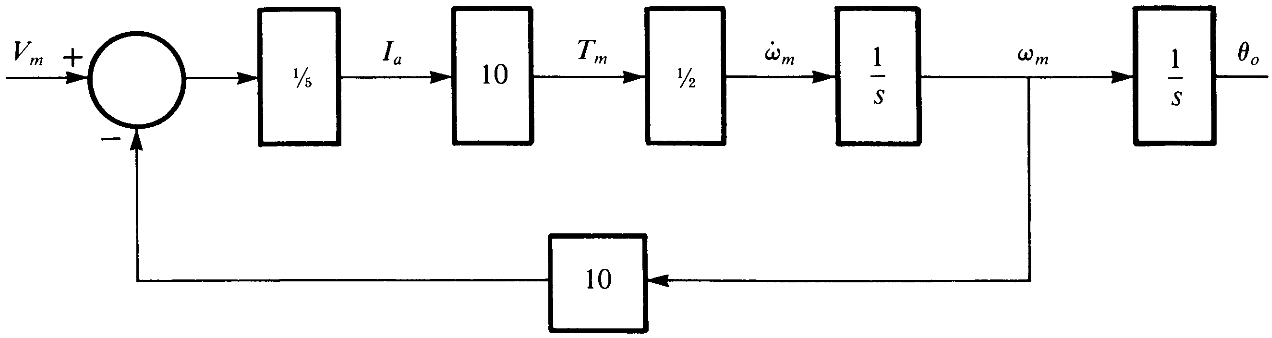
$$\omega_m = \frac{T_m}{2} \times \frac{1}{s} \quad \textbf{(S1.26)}$$

Angular position is the integral of angular velocity. Therefore:

$$\theta_o = \frac{1}{s} \times \omega_m \quad \textbf{(S1.27)}$$

Now, we can apply relations S1.26 and S1.27 to draw the complete block diagram for the motor and antenna:

Figure S1.10 Motor and antenna block diagram.

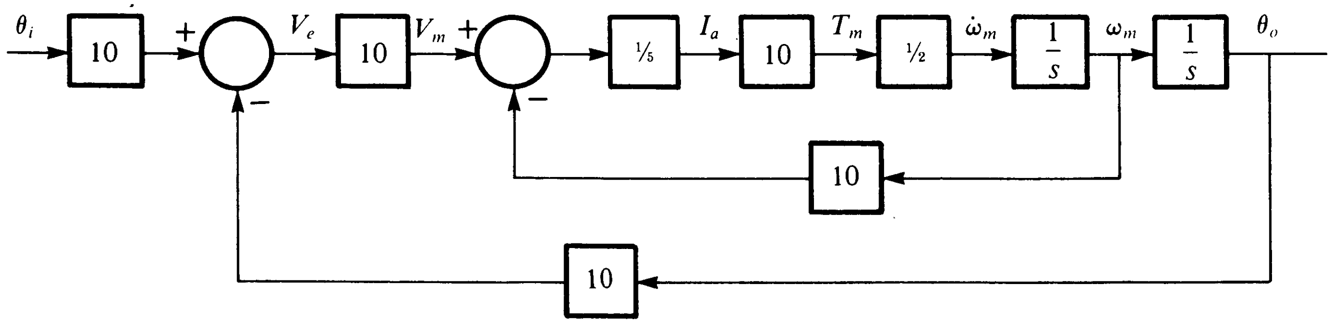


Lastly, we add the differential amplifier. With the error signal defined as in Figure 2.27, and an amplifier gain of 10, we have

$$V_m = 10[10(\theta_i - \theta_o)] \qquad \textbf{(S1.28)}$$

This relationship allows us to draw the complete block diagram as shown in Figure S1.11.

Figure S1.11 Complete block diagram for antenna-rotator system.



This may be greatly simplified. The transfer function of the inner loop is:

$$\frac{a}{1 + af} = \frac{\frac{1}{s}}{1 + \frac{10}{s}} = \frac{1}{s + 10} \qquad \textbf{(S1.29)}$$

Then, the reduced block diagram is as shown in Figure S1.12:

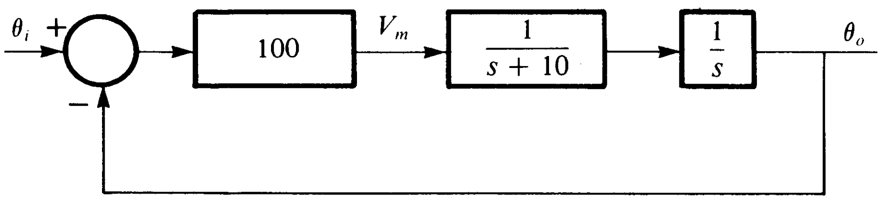


Figure S1.12 Reduced antenna-rotator block diagram.

By inspection of Figure S1.12, the transfer function between θ_o and θ_i is given by:

$$\frac{\theta_o}{\theta_i} = \frac{\frac{100}{s(s+10)}}{1 + \frac{100}{s(s+10)}} = \frac{100}{s^2 + 10s + 100} = \frac{1}{\frac{s^2}{100} + \frac{s}{10} + 1} \qquad \textbf{(S1.30)}$$

To consider a wind disturbance, we sum a disturbance torque T_d at the point labeled T_m in Figure S1.11 to get Figure S1.13:

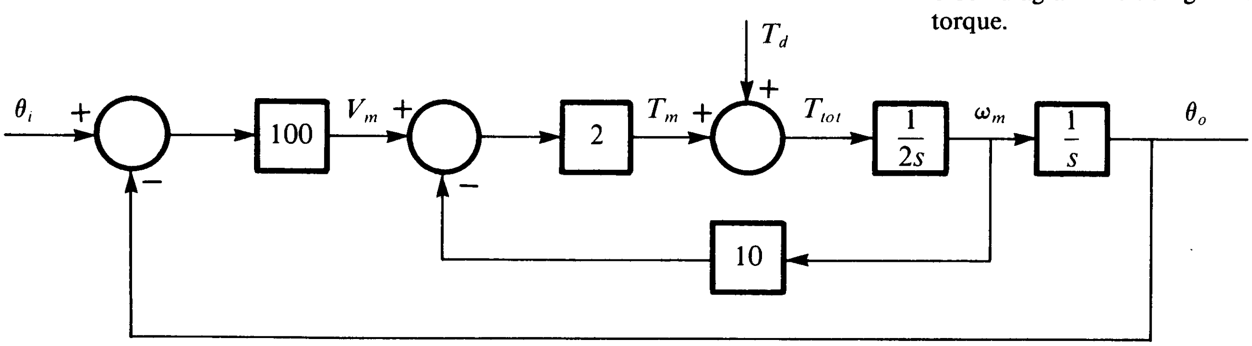


Figure S1.13 Antenna-rotator block diagram including disturbance torque.

At this point, we save further calculations by using a diagram manipulation to move the T_d input to the same summing junction as θ_i, yielding Figure S1.14:

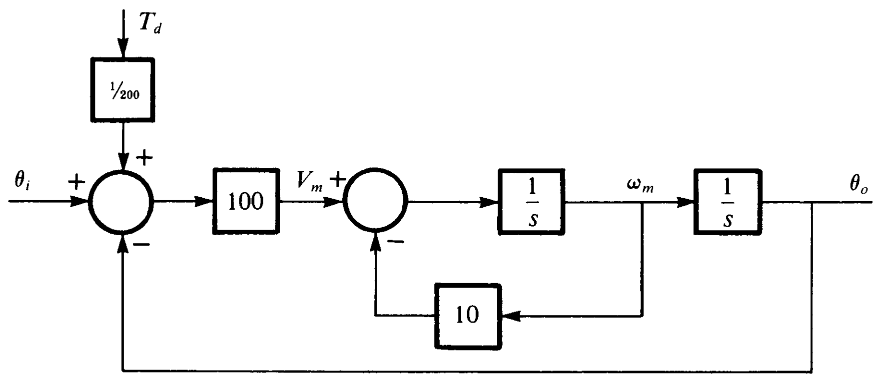


Figure S1.14 Reduced block diagram.

Thus, we see that

$$\frac{\theta_o}{T_d} = \frac{1}{200}\frac{\theta_o}{\theta_i} = \frac{\frac{1}{200}}{\frac{s^2}{100} + \frac{s}{10} + 1} \tag{S1.31}$$

For a constant input of $T_d = 1$ N-m, we evaluate the transfer function at $s = 0$ to find $\theta_o = \frac{1}{200}$ radians.

In closing, there are two useful points to notice. First, both $\frac{\theta_o}{\theta_i}$ and $\frac{\theta_o}{T_d}$ have the same transfer function denominator. This is the term $1 + af$, which is the system characteristic equation and does not depend on where an input drives the system. Secondly, the input T_d is attenuated relative to θ_i by a factor of 200, which is simply the forward path gain that precedes the torque disturbance.