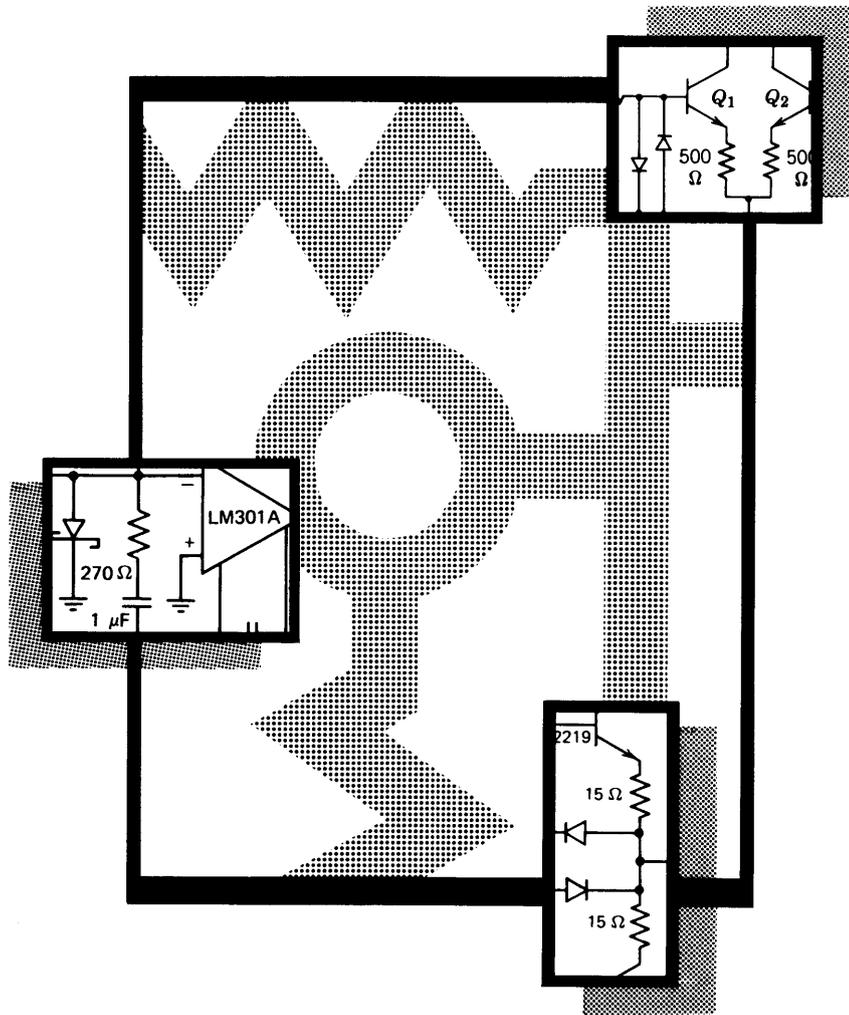


# Linearized Analysis of Nonlinear Systems

# 14



**Blackboard 14.1**

Linearization	$v_0 = F(v_{I1}, v_{I2}, \dots, v_{IN})$
Total Variable: $v_0, v_{I1}$	$v_0 = V_0 + v_0'$
Operating Point: $V_0, V_{I1}$	$F(V_{I1}, V_{I2}, \dots, V_{IN})$
Incremental: $v_0, v_{I1}$	$+ \left. \frac{\partial v_0}{\partial v_{I1}} \right _{op pt} v_{i1} +$
Frequency Domain: $V_0, V_{i1}$	$\left. \frac{\partial v_0}{\partial v_{I2}} \right _{op pt} v_{i2} + \dots$
	$+ \left. \frac{\partial v_0}{\partial v_{IN}} \right _{op pt} v_{iN} + \text{higher order terms}$

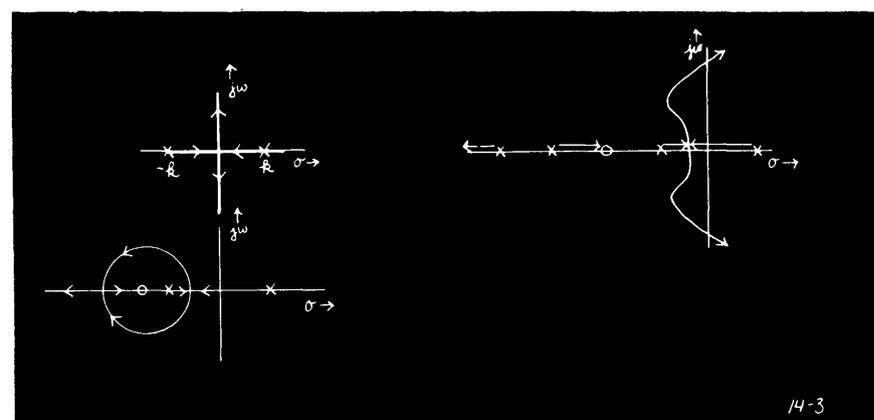
14-1

**Blackboard 14.2**

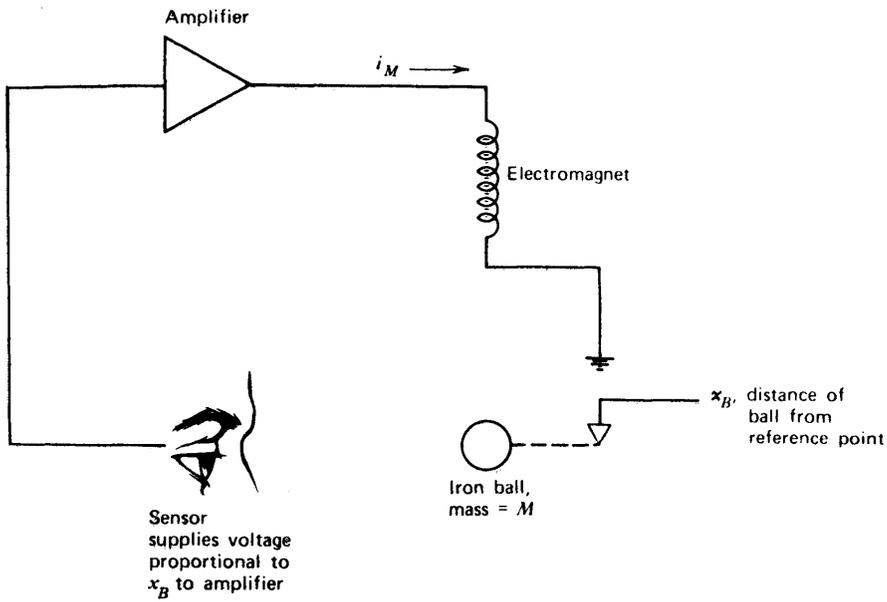
$f_M = \frac{C I_M^2}{X_B^2}$	$\frac{M X_B^2}{2C I_M^2} \frac{d^2 x_f}{dt^2} = -\frac{X_B}{I_M} x_m + x_f$	$I_m(s) \rightarrow \left[ \frac{X_B}{I_M} \right] \rightarrow \left[ \frac{1}{1 - \frac{s^2}{k}} \right] \rightarrow X_f(s)$
$f_M = \frac{C I_M^2}{X_B^2} + \frac{2C I_M}{X_B^2} x_m - \frac{2C I_M^2}{X_B^2} x_f + \dots$	$\frac{s}{k} X_f(s) = -\frac{X_B}{I_M} I_m(s) + X_f(s)$	$I_m(s) \rightarrow \left[ \frac{X_B}{I_M} \right] \rightarrow \left[ \frac{1}{1 - \frac{s^2}{k}} \right] \rightarrow X_f(s)$
$M a = F:$	$X_f(s) = \frac{s}{k} X_f(s) + \frac{X_B}{I_M} I_m(s)$	$I_m(s) \rightarrow \left[ \frac{-X_B/I_M}{(\frac{s}{k} + 1)(\frac{s}{k} - 1)} \right] \rightarrow X_f(s)$
$\frac{M d^2 x_B}{dt^2} = M g - \frac{C I_M^2}{X_B^2} - \frac{2C I_M}{X_B^2} x_m + \frac{2C I_M^2}{X_B^2} x_f$		

14-2

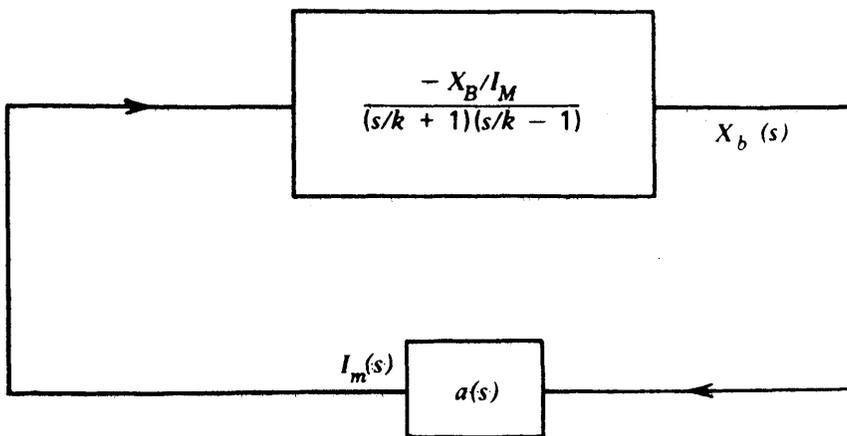
**Blackboard 14.3**



Viewgraph 14.1

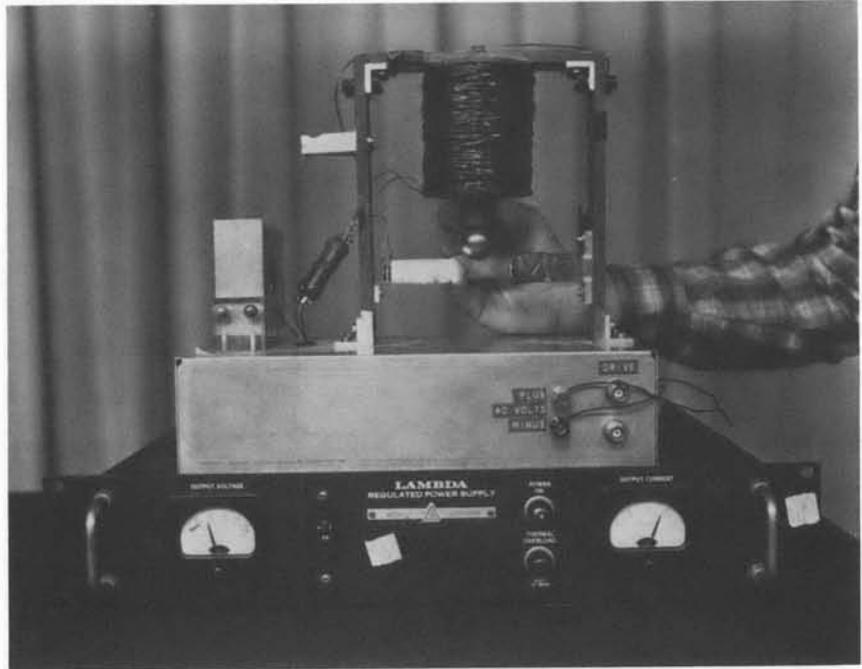


Viewgraph 14.2



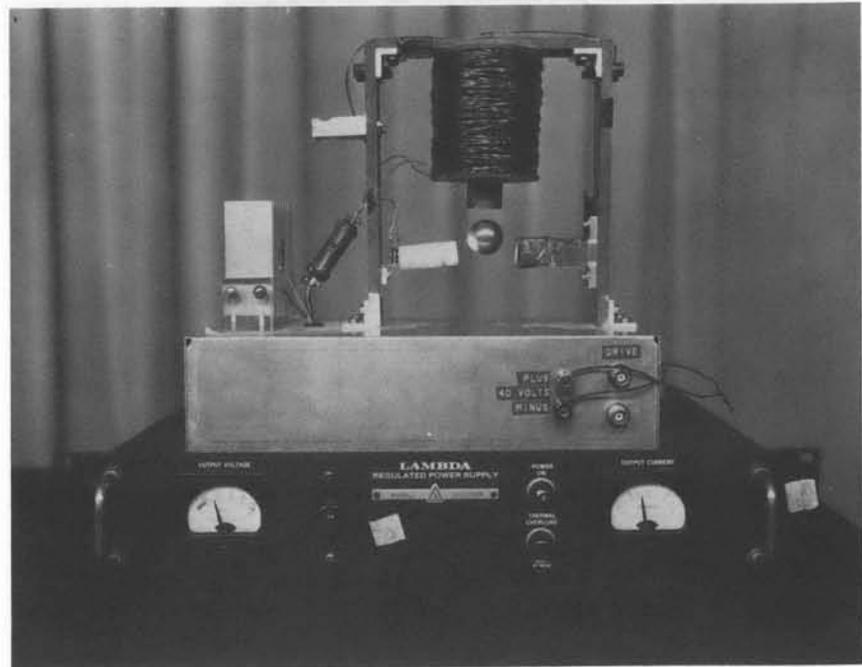
**Demonstration Photograph**

**14.1** Trying to get the magnetic-suspension system started



**Demonstration Photograph**

**14.2** The magnetic-suspension system in operation



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The analytical techniques introduced up to this point in the course make liberal use of superposition. Unfortunately, superposition does not generally apply in nonlinear systems, and consequently we need to develop new methods of analysis for systems where nonlinearity influences performance.

**Comments**

One possible method is to linearize the system equations about an operating point, recognizing that the linearized equations can be used to predict behavior over an appropriately restricted region around the operating point. This technique is used to determine compensation for a magnetic suspension system. The linearized equations of motion of this system have a loop-transmission pole in the right-half plane, reflecting the inherent instability that exists when the magnetic field strength is fixed.

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Textbook: Chapter 6 through Section 6.2.

**Reading**

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**Problems**

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**Problem 14.1 (P6.1)**

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**Problem 14.2 (P6.2)**

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**Problem 14.3 (P6.3)**

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RES.6-010 Electronic Feedback Systems  
Spring 2013

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