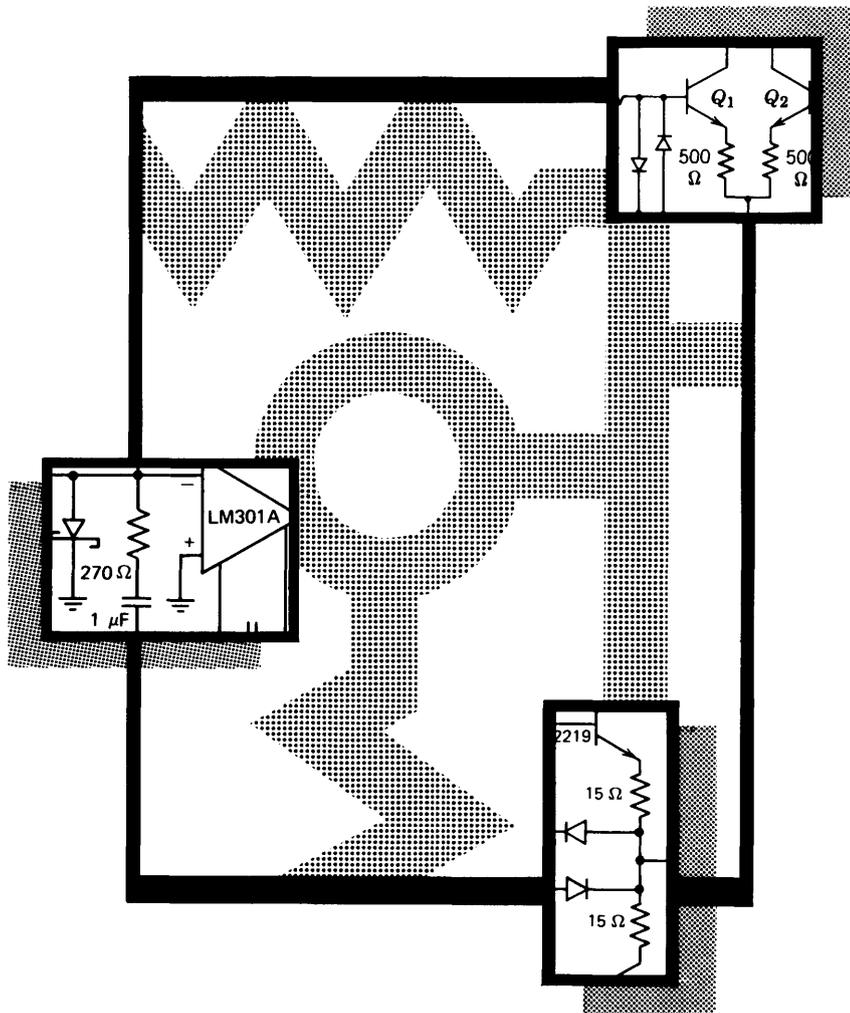


# Root Locus

# 5



**Blackboard 5.1**

**Root Locus**

C.E. = 1 - L.T. = 1 + a(s)f(s)  
 poles of A(s) @ 1 + a(s)f(s) = 0  
 $a(s)f(s) = a_0 f_0 g(s)$   
 $1 + a_0 f_0 g(s) = 0$   
 $a_0 f_0 g(s) = -1$   
 $|a_0 f_0 g(s)| = 1$   
 $\angle g(s) = (2n+1)180^\circ$

s-plane  
 $\sigma_a = \frac{\sum p_i - \sum z_i}{n - m}$   
 $\theta = \theta_1 - \theta_2 - \theta_3 - \theta_4$   
 $|g(s)| = \frac{1}{a_0 f_0}$

Rules:  
 1. # of branches, starting + ending points.  
 2. Real Axis  
 3. Breakaway or entry @  $\frac{da(s)}{ds} = 0$   
 4. If  $P \geq Z + 2$ , average distance of closed-loop poles from the imaginary axis is constant

5-1

**Blackboard 5.2**

5. Asymptotes for large  $a_0 f_0$   
 $\sum \theta - P\theta = (2n+1)180^\circ$   
 $\theta = \frac{(2n+1)180^\circ}{P-Z}$   
 Intersect @  
 $\frac{\sum \text{Re poles} - \sum \text{Re zeros}}{P-Z}$

6.  $\theta_p = 180^\circ + \sum \gamma - \sum \beta$   
 $\theta_z = 180^\circ + \sum \beta - \sum \gamma$

7. Ignore remote poles and zeros at points near the origin

8.  $a_0 f_0 = \left| \frac{1}{g(s_p)} \right|$

$a(s) = \frac{a_0}{c_a s + 1}$ ,  $f = f_0$   
 $A(s) = \frac{1}{f_0} \frac{1}{\frac{c_a s}{a_0 f_0} + 1 + 1}$

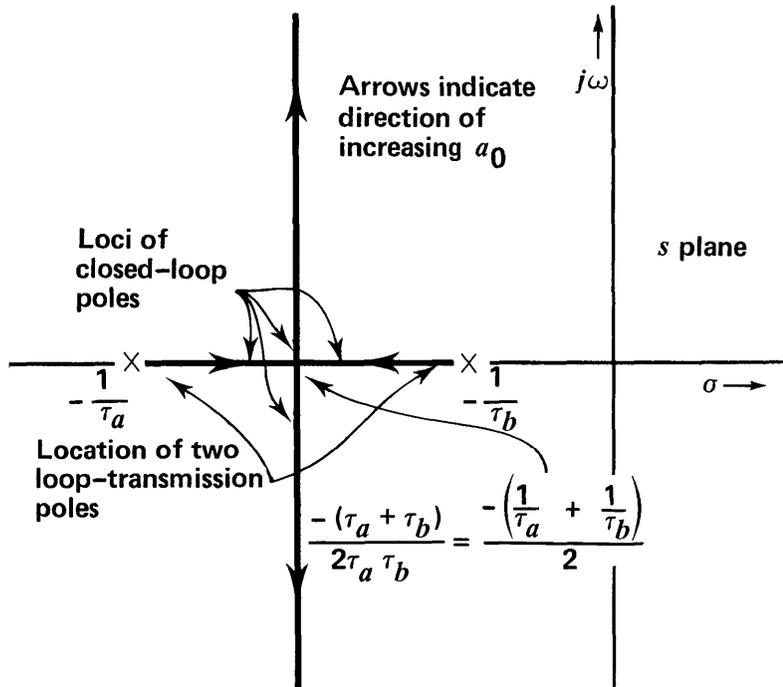
5-2

**Blackboard 5.3**

$a(s)f(s) = \frac{a_0 f_0}{(s+1)(0.5s+1)(0.1s+1)}$   
 unstable for  $a_0 f_0 > 19.8$   
 $\zeta = 0.5$  for  $a_0 f_0 = 2.2$

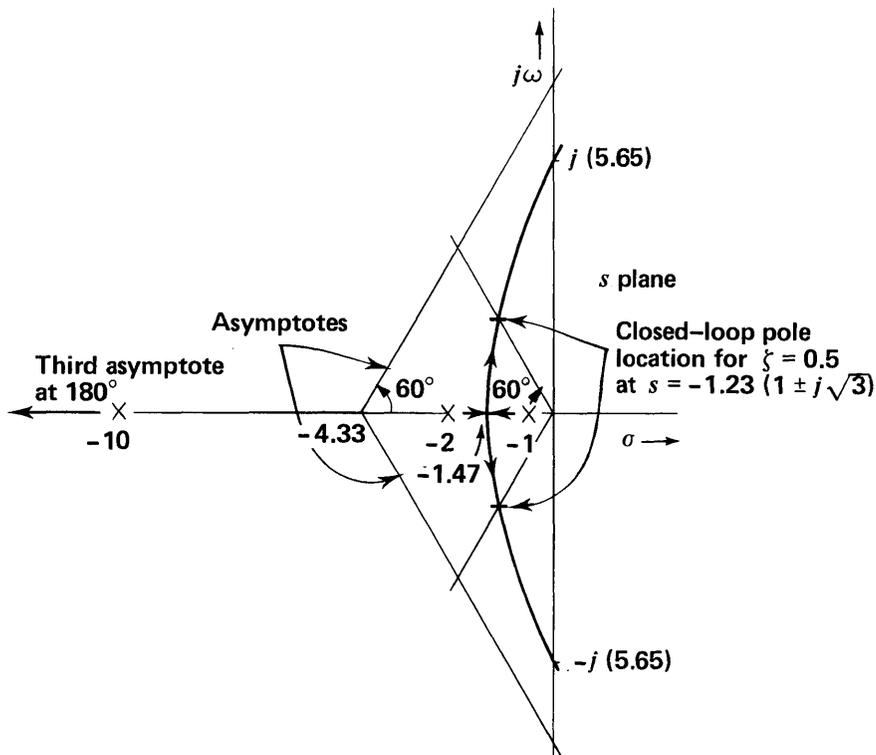
5-3

Viewgraph 5.1



Root-locus diagram for second order system.

Viewgraph 5.2



Root-locus diagram for third-order system

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**Comments**

In Lecture 4 we introduced the concept of a root-locus diagram by directly factoring the characteristic equation of the two-pole system used for illustration. This method is tedious for higher-order systems.

The material in this lecture shows how the fact that the  $a(s)f(s)$  product must equal  $-1$  at a closed-loop pole location can be exploited to determine rapidly important features of the root-locus diagram. We also see that simple numerical methods can provide certain quantitative results when required.

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**Corrections**

Note that there is a mistake in the videotape on blackboard 5-1 where it states root-locus Rule 4. The blackboard says that the average distance from the real axis is constant. This is identically satisfied for all physically realizable systems. The corrected blackboard in the Video Course Manual states that the average distance from the *imaginary* axis is constant under the conditions of Rule 4. See page 123 of the textbook for clarification.

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**Reading**

Textbook: Sections 4.3.1 and 4.3.2.

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**Problems**

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**Problem 5.1 (P4.5)**

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**Problem 5.2 (P4.7)**

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