

**OPERATIONAL  
AMPLIFIERS:**  
Theory and Practice

**JAMES K. ROBERGE**

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**To Nancy**



# PREFACE

The operational amplifier is responsible for a dramatic and continuing revolution in our approach to analog system design. The availability of high performance, inexpensive devices influences the entire spectrum of circuits and systems, ranging from simple, mass-produced circuits to highly sophisticated equipment designed for complex data collection or processing operations. At one end of this spectrum, modern operational amplifiers have lowered cost and improved performance; at the other end, they allow us to design and implement systems that were previously too complex for consideration.

An appreciation of the importance of this component, gained primarily through research rather than academic experience, prompted me in 1969 to start a course at M.I.T. focusing on the operational amplifier. Initially the course, structured as part of an elective sequence in active devices, concentrated on the circuit techniques needed to realize operational amplifiers and on the application of these versatile elements.

As the course evolved, it became apparent that the operational amplifier had a value beyond that of a circuit component; it was also an excellent instructional vehicle. This device supplied a reason for studying a collection of analytic and design techniques that were necessary for a thorough understanding of operational amplifiers and were also important to the general area of active-circuit design. For example, if we study direct-coupled amplifiers in detail, with proper attention given to transistor-parameter variation with temperature, to loading, and to passive-component peculiarities, we can improve our approach to the design of a large class of circuits dependent on these concepts and also better appreciate operational amplifiers. Similarly, the use of an active load to increase dramatically the voltage gain of a stage is a design technique that has widespread applicability. The

integrated-circuit fabrication and design methods responsible for the economical realization of modern operational amplifiers are the same as those used for other linear integrated circuits and also influence the design of many modern discrete-component circuits.

Chapters 7 to 10 reflect the dual role of the operational-amplifier circuit. The presentation is in greater detail than necessary if our only objective is to understand how an operational amplifier functions. However, the depth of the presentation encourages the transfer of this information to other circuit-design problems.

A course based on circuit-design techniques and some applications material was taught for two years. During this period, it became clear that in order to provide the background necessary for the optimum use of operational amplifiers in challenging applications, it was necessary to teach material on classical feedback concepts. These concepts explain the evolution of the topology used for modern amplifiers, suggest configurations that should be used to obtain specific closed-loop transfer functions, and indicate the way to improve the dynamics of operational-amplifier connections.

The linear-system theory course that has become an important part of most engineering educational programs, while providing valuable background, usually does not develop the necessary facility with techniques for the analysis and synthesis of feedback systems. When courses are offered in feedback, they normally use servomechanisms for their examples. Although this material can be transferred to a circuits context, the initial assimilation of these ideas is simplified when instruction is specifically tailored to the intended field of application.

Chapters 2 to 6 and Chapter 13 present the techniques necessary to model, analyze, and design electronic feedback systems. As with the circuit-related material, the detail is greater than the minimum necessary for a background in the design of connections that use operational amplifiers. This detail is justifiable because I use the operational amplifier as a vehicle for presenting concepts valuable for the general area of electronic circuit and system design.

The material included here has been used as the basis for two rather different versions of the M.I.T. course mentioned earlier. One of these concentrates on circuits and applications, using material from Chapters 7 to 10. Some application material is included in the examples in these chapters, and further applications from Chapters 11 and 12 are included as time permits. Some of the elementary feedback concepts necessary to appreciate modern operational-amplifier topologies are also discussed in this version.

The second variation uses the feedback material in Chapters 2 to 6 and Chapter 13 as its central theme. A brief discussion of the topology used

for modern operational amplifiers, such as that presented in portions of Chapters 8 and 10, is included in this option. The applications introduced as examples of feedback connections are augmented with topics selected from Chapters 11 and 12.

A laboratory has been included as an integral part of both options. In the circuits variation, students investigate specific circuits such as direct-coupled amplifiers and high-gain stages, and conclude their laboratory experience by designing, building, and testing a simple operational amplifier. In the feedback version, connections of operational amplifiers are used to verify the behavior of linear and nonlinear feedback systems, to compare time-domain and frequency-domain performance indices, and to investigate stability.

We have found it helpful to have ready access to some kind of computational facilities, particularly when teaching the feedback material. The programs made available to the students reduce the manual effort required to draw the various plots and to factor polynomials when exact singularity locations are important.

Both versions of the course have been taught at least twice from notes essentially identical to the book. The student population consisted primarily of juniors and seniors, with occasional graduate students. The necessary background includes an appreciation of active-circuit concepts such as that provided in *Electronic Principles* by P. E. Gray and C. L. Searle (Wiley, New York, 1969), Chapters 1 to 14. An abbreviated circuits preparation is acceptable for the feedback version of the course. Although a detailed linear-systems background stressing formal operational calculus and related topics is not essential, familiarity with concepts such as pole-zero diagrams and elementary relationships between the time and the frequency domain is necessary.

Some of the more advanced applications in Chapters 11 and 12 have been included in a graduate course in analog and analog/digital instrumentation. The success with this material suggests a third possible variation of the course that stresses applications, with feedback and circuit concepts added as necessary to clarify the applications. I have not yet had the opportunity to structure an entire course in this way.

It is a pleasure to acknowledge several of the many individuals who contributed directly or indirectly to this book. High on the list are three teachers and colleagues, Dr. F. Williams Sarles, Jr., Professor Campbell L. Searle, and Professor Leonard A. Gould, who are largely responsible for my own understanding and appreciation of the presented material.

Two students, Jeffrey T. Millman and Samuel H. Maslak, devoted substantial effort to reviewing and improving the book.

Most of the original manuscript and its many revisions were typed and illustrated by Mrs. Janet Lague and Mrs. Rosalind Wood. Miss Susan Garland carefully proofread the final copy.

**James K. Roberge**

*Cambridge, Massachusetts  
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