CHAPTER V

COMPENSATION

5.1 OBJECTIVES

The discussion up to this point has focused on methods used to analyze the performance of a feedback system with a given set of parameters. The results of such analysis frequently show that the performance of the feedback system is unacceptable for a given application because of such deficiencies as low desensitivity, slow speed of response, or poor relative stability. The process of modifying the system to improve performance is called *compensation*.

Compensation usually reduces to a trial-and-error procedure, with the experience of the designer frequently playing a major role in the eventual outcome. One normally assumes a particular form of compensation and then evaluates the performance of the system to see if objectives have been met. If the performance remains inadequate, alternate methods of compensation are tried until either objectives are met, or it becomes evident that they cannot be achieved.

The type of compensation that can be used in a specific application is usually highly dependent on the components that form the system. The general principles that guide the compensation process will be described in this chapter. Most of these ideas will be reviewed and reinforced in later chapters after representative amplifier topologies and applications have been introduced.

5.2 SERIES COMPENSATION

One way to change the performance of a feedback system is to alter the transfer function of either its forward-gain path or its feedback path. This technique of modifying a series element in a single-loop system is called *series compensation*. The changes may involve the d-c gain of an element or its dynamics or both.

5.2.1 Adjusting the D-C Gain

One conceptually straightforward modification that can be made to the loop transmission is to vary its d-c or midband value a_0f_0. This modification has a direct effect on low-frequency desensitivity, since we have seen that the attenuation to changes in forward-path gain provided by feedback is equal to $1 + a_0 f_0$.

The closed-loop dynamics are also dependent on the magnitude of the low-frequency loop transmission. The example involving Fig. 4.6 showed how root-locus methods are used to determine the relationship between $a_0 f_0$ and the damping ratio of a dominant pole pair. A second approach to the control of closed-loop dynamics by adjusting $a_0 f_0$ for a specific value of M_p was used in the example involving Fig. 4.24.

An assumption common to both of these previous examples was that the value of $a_0 f_0$ could be selected without altering the singularities included in the loop transmission. For certain types of feedback systems independence of the d-c magnitude and the dynamics of the loop transmission is realistic. The dynamics of servomechanisms, for example, are generally dominated by mechanical components with bandwidths of less than 100 Hz. A portion of the d-c loop transmission of a servomechanism is often provided by an electronic amplifier, and these amplifiers can provide frequency-independent gain into the high kilohertz or megahertz range. Changing the amplifier gain changes the value of $a_0 f_0$ but leaves the dynamics associated with the loop transmission virtually unaltered.

This type of independence is frequently absent in operational amplifiers. In order to increase gain, stages may have to be added, producing significant changes in dynamics. Lowering the gain of an amplifying stage may also change dynamics because, for example, of a relationship between the input capacitance and voltage gain of a common-emitter amplifier. A further practical difficulty arises in that there is generally no predictable way to change the d-c open-loop gain of available discrete- or integrated-circuit operational amplifiers from the available terminals.

An alternative approach involves modification of the d-c loop transmission by means of the feedback network connected around the amplifier. The connection of Fig. 5.1*a* illustrates one possibility. The block diagram for this amplifier, assuming negligible loading at either input or output, is shown in part *b* of this figure, while the block diagram after reduction to unity-feedback form is shown in part *c*. If the shunt resistance R from the inverting input to ground is an open circuit, the d-c value of the loop transmission is completely determined by a_0 and the ideal closed-loop gain $-R_2/R_1$. However, inclusion of R provides an additional degree of freedom so that the d-c loop transmission and the ideal gain can be changed independently.

This technique is illustrated for a unity-gain inverter ($R_1 = R_2$) and

$$a(s) = \frac{10^6}{(s + 1)(10^{-5}s + 1)} \tag{5.1}$$

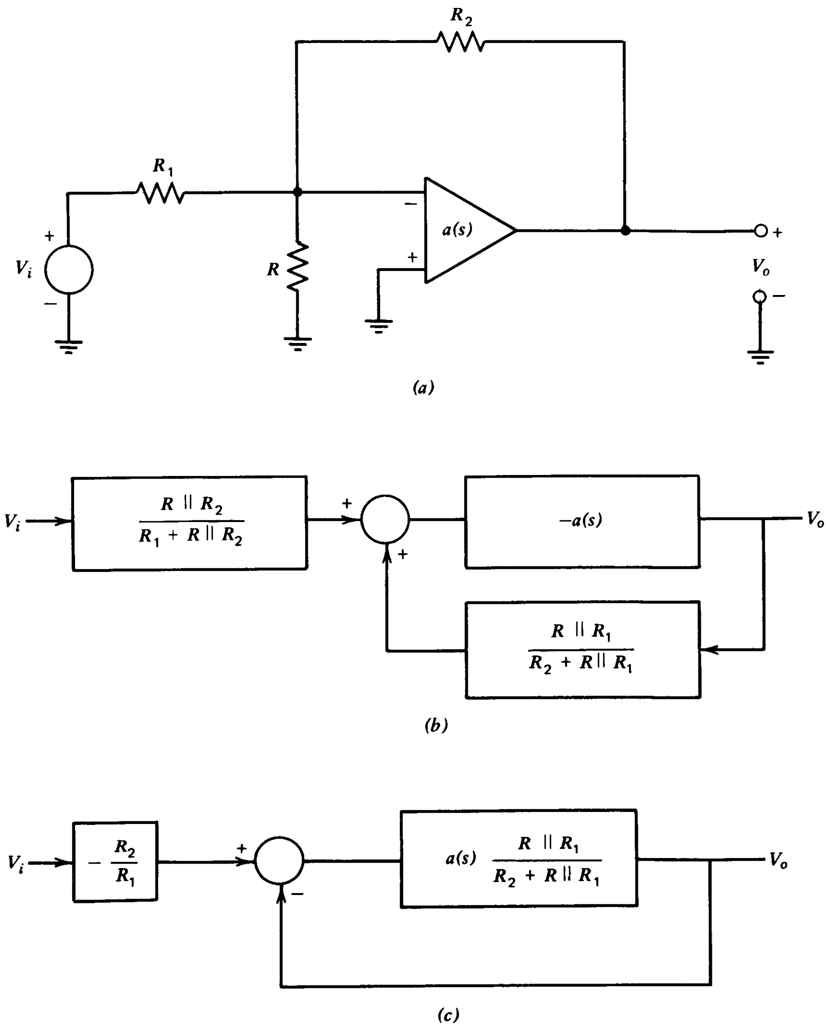


Figure 5.1 Inverter. (*a*) Circuit. (*b*) Block diagram. (*c*) Block diagram reduced to unity-feedback form.

A Bode plot of this transfer function is shown in Fig. 5.2. If R is an open circuit, the magnitude of the loop transmission is one at approximately 2.15×10^5 radians per second, since the magnitude of $a(s)$ at this frequency is equal to the factor of two attenuation provided by the R_1-R_2 network. The phase margin of the system is 25°, and Fig. 4.26*a* shows that the closed-loop damping ratio is 0.22. Since Fig. 4.26 was generated assuming this type of loop transmission, it yields exact results in this case. If the resistor

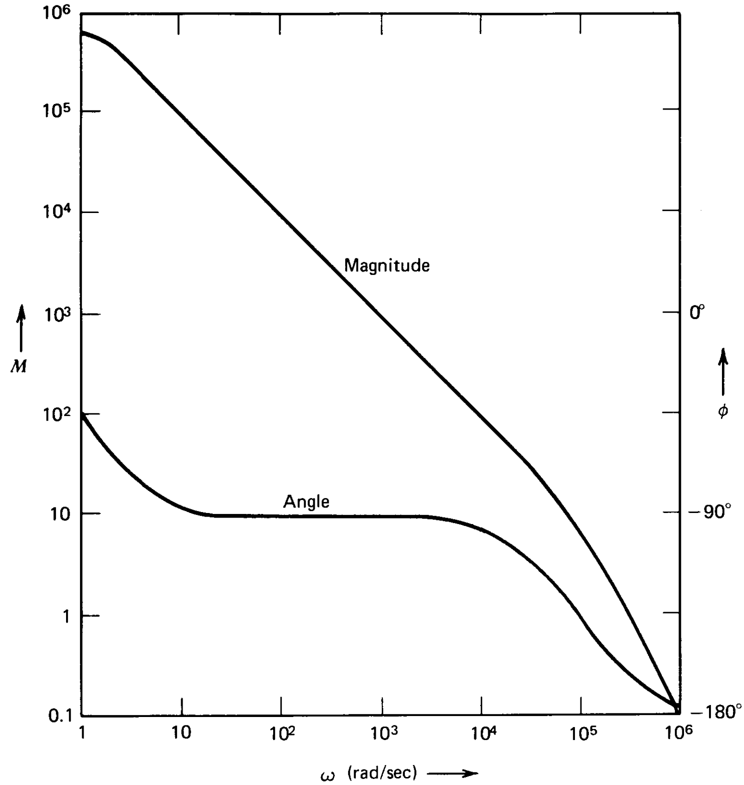


Figure 5.2 Bode plot of $10^6/[(s + 1)(10^{-5}s + 1)]$.

R is made equal to $0.2R_1$, the loop-transmission unity-gain frequency is lowered to 10^5 radians per second by the factor-of-seven attenuation provided by the network, and phase margin and damping ratio are increased to 45° and 0.42, respectively. One penalty paid for this type of attenuation at the input terminals of the amplifier is that the voltage offset and noise at the output of the amplifier are increased for a given offset and noise at the amplifier input terminals (see Problem P5.2).

5.2.2 Creating a Dominant Pole

Elementary considerations show that a single-pole loop transmission results in a stable system for any amount of negative feedback, and that the closed-loop bandwidth of such a system increases with increasing a_0f_0. Similarly, if the loop transmission in the vicinity of the unity-gain frequency is dominated by one pole, ample phase margin is easily obtained. Because of the ease of stabilizing approximately single-pole systems, many types of compensation essentially reduce to making one pole dominate the loop transmission.

One brute-force method for making one pole dominate the loop transmission of an amplifier is simply to connect a capacitor from a node in the signal path to ground. If a large enough capacitor is used, the gain of the amplifier will drop below one at a frequency where other amplifier poles can be ignored. The obvious disadvantage of this approach to compensation is that it may drastically reduce the closed-loop bandwidth of the system.

A feedback system designed to hold the value of its output constant independent of disturbances is called a *regulator*. Since the output need not track a rapidly varying input, closed-loop bandwidth is an unimportant parameter. If a dominant pole is included in the output portion of a regulator, the low-pass characteristics of this pole may actually improve system performance by attenuating disturbances even in the absence of feedback.

One possible type of voltage regulator is shown in simplified form in Fig. 5.3. An operational amplifier is used to compare the output voltage with a fixed reference. The operational amplifier drives a series regulator stage that consists of a transistor with an emitter resistor. The series regulator isolates the output of the circuit from an unregulated source of voltage. The load includes a parallel resistor-capacitor combination and a disturbing current source. The current source is included for purposes of analysis and will be used to determine the degree to which the circuit rejects load-current changes. The dominant pole in the system is assumed to occur because of the load, and it is further assumed that the operational amplifier and series transistor contribute no dynamics at frequencies where the loop-transmission magnitude exceeds one.

The block diagram of Fig. 5.3*b* models the regulator if it is assumed that the common-base current gain of the transistor is one and that the resistor R is large compared to the reciprocal of the transistor transconductance. This diagram verifies the single-pole nature of the system loop transmission.

As mentioned earlier, the objective of the circuitry is to minimize changes in load voltage that result from changes in the disturbing current and the unregulated voltage. The disturbance-to-output closed-loop transfer functions that indicate how well the regulator achieves this objective are

$$\frac{V_l}{I_d} = \frac{R/a_0}{RC_Ls/a_0 + (1 + R/a_0R_L)} \tag{5.2}$$

and

$$\frac{V_l}{V_u} = \frac{1/a_0}{RC_Ls/a_0 + (1 + R/a_0R_L)} \tag{5.3}$$

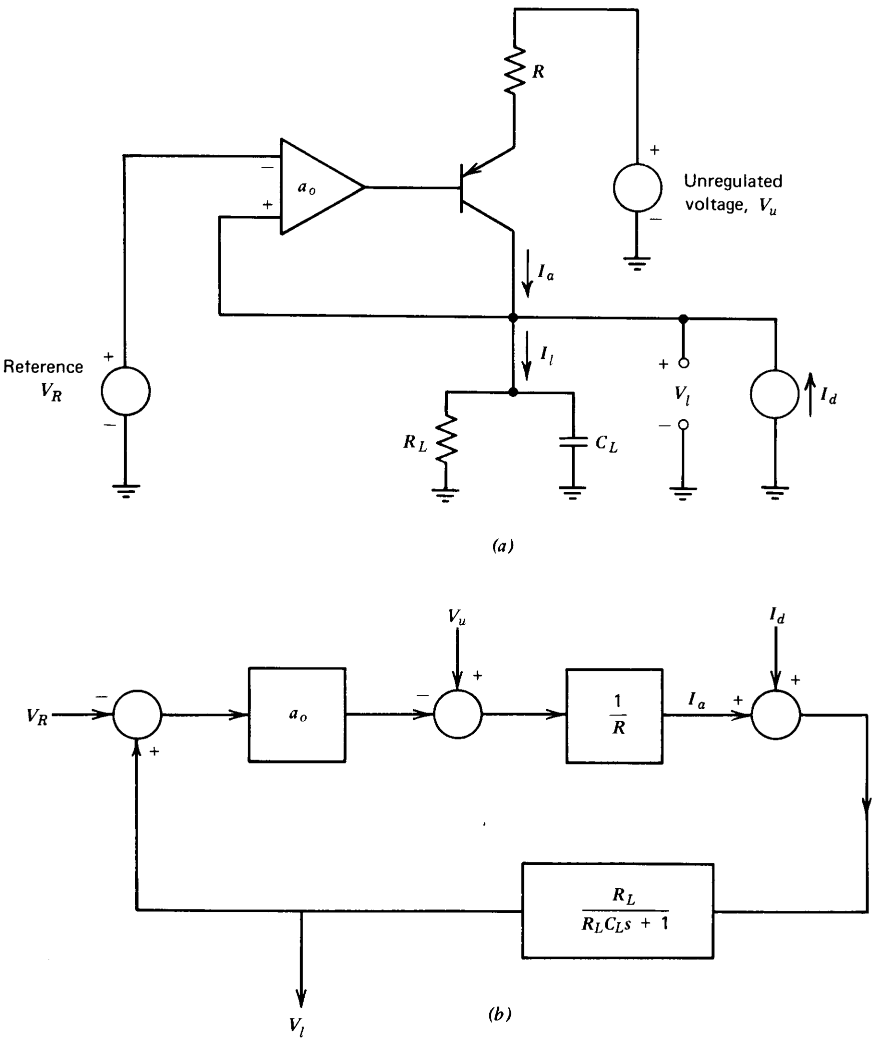


Figure 5.3 Voltage regulator. (*a*) Circuit. (*b*) Block diagram.

If sinusoidal disturbances are considered, the magnitude of either disturbance-to-output transfer function is a maximum at d-c, and decreases with increasing frequency because of the low-pass characteristics of the load. Increasing C_L improves performance, since it lowers the frequency at which the disturbance is attenuated significantly compared to its d-c value. If it is assumed that arbitrary loads can be connected to the regu-

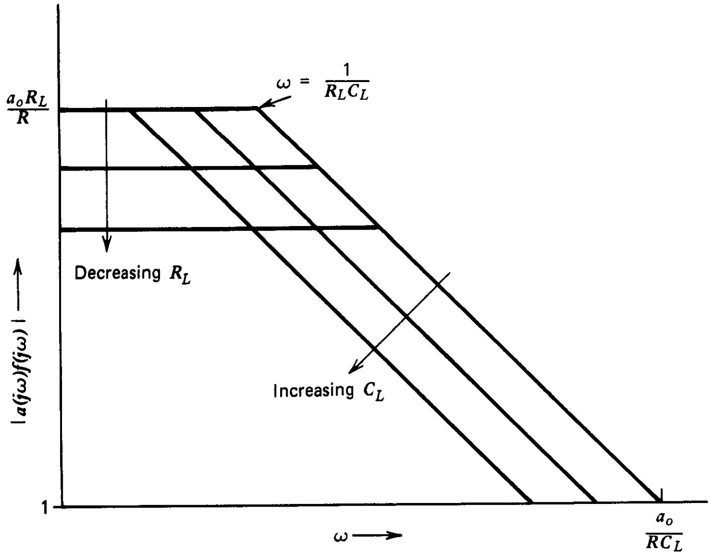


Figure 5.4 Effect of changing load parameters on the Bode plot of a voltage regulator.

lator (which is the usual situation, if, for example, this circuit is used as a laboratory power supply), the values of R_L and C_L must be considered variable. The minimum value of C_L can be constrained by including a capacitor with the regulation circuitry. The load-capacitor value increases as external loads are connected to the regulator because of the decoupling capacitors usually associated with these loads. Similarly, R_L decreases with increasing load to some minimum value determined by loading limitations.

The compensation provided by the pole at the output of the regulator maintains stability as R_L and C_L change, as illustrated in the Bode plot of Fig. 5.4. (The negative of the loop transmission for this plot is $a_0R_L/R(R_LC_Ls + 1)$, determined directly from Fig. 5.3*b*.) Note that the unity-gain frequency can be limited by constraining the maximum value of the a_0/RC_L ratio, and thus crossover can be forced before other system elements affect dynamics. The phase margin of the system remains close to 90° as R_L and C_L vary over wide limits.

5.2.3 Lead and Lag Compensation

If the designer is free to modify the dynamics of the loop transmission as well as its low-frequency magnitude, he has considerably more control over the closed-loop performance of the system. The rather simple modification of making a single pole dominate has already been discussed.

The types of changes that can be made to the dynamics of the loop transmission are constrained, even in purely mathematical systems. It is tempting to think that systems could be improved, for example, by adding positive phase shift to the loop transmission without changing its magnitude characteristics. This modification would clearly improve the phase margin of a system. Unfortunately, the magnitude and angle characteristics of physically realizable transfer functions are not independent, and transfer functions that provide positive phase shift also have a magnitude that increases with increasing frequency. The magnitude increase may result in a higher system crossover frequency, and the additional negative phase shift that results from other elements in the loop may negate hoped-for advantages.

The way that series compensation is implemented and the types of compensating transfer functions that can be obtained in practical systems are even further constrained by the hardware realities of the feedback system being compensated. The designer of a servomechanism normally has a wide variety of compensating transfer functions available to him, since the electrical networks and amplifiers usually used to compensate servomechanisms have virtually unlimited bandwidth relative to the mechanical portions of the system. Conversely, we should remember that the choices of the feedback-amplifier designer are more restricted because the ways that the transfer function of an amplifier can be changed, particularly near its unity-gain frequency where transistor bandwidth limitations dominate performance, are often severely constrained.

Two distinct types of transfer functions are normally used for the series compensation of feedback systems, and these types can either be used separately or can be combined in one system. A *lead transfer function* can be realized with the network shown in Fig. 5.5. The transfer function of this network is

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\alpha}\left[\frac{\alpha\tau s + 1}{\tau s + 1}\right] \tag{5.4}$$

where $\alpha = (R_1 + R_2)/R_2$ and $\tau = (R_1 \parallel R_2)C$. As the name implies, this network provides positive or leading phase shift of the output signal relative to the input signal at all frequencies. Lead-network parameters are usually selected to locate its singularities near the crossover frequency of the system being compensated. The positive phase shift of the network then improves the phase margin of the system. In many cases, the lead network has negligible effect on the magnitude characteristics of the compensated system at or below the crossover frequency, since we shall see that a

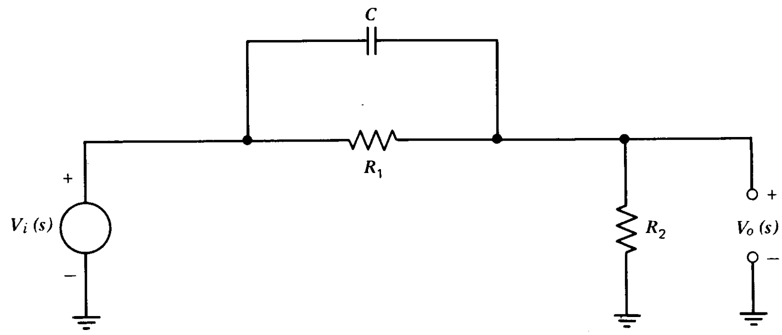


Figure 5.5 Lead network.

lead network provides substantial phase shift before its magnitude increases significantly.

The *lag network* shown in Fig. 5.6 has the transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{\tau s + 1}{\alpha \tau s + 1} \tag{5.5}$$

where $\alpha = (R_1 + R_2)/R_2$ and $\tau = R_2C$. The singularities of this type of network are usually located well below crossover in order to reduce the crossover frequency of a system so that the negative phase shift associated with other elements in the system is reduced at the unity-gain frequency. This effect is possible because of the attenuation of the lag network at frequencies above both its singularities.

The maximum magnitude of the phase angle associated with either of these transfer functions is

$$\phi_{max} = \sin^{-1}\left[\frac{\alpha - 1}{\alpha + 1}\right] \tag{5.6}$$

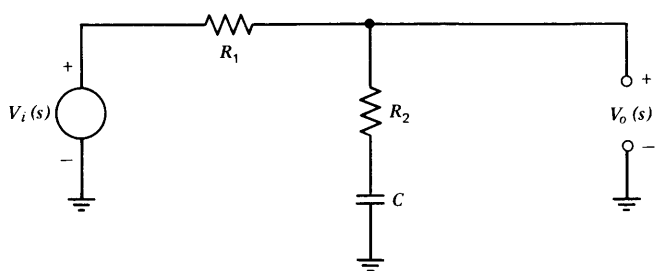


Figure 5.6 Lag network.

and this magnitude occurs at the geometric mean of the frequencies of the two singularities. The gain of either network at its maximum-phase-shift frequency is $1/\sqrt{\alpha}$.

The magnitudes and angles of lead transfer functions for α values of 5, 10, and 20, are shown in Bode-plot form in Fig. 5.7. Figure 5.8 shows corresponding curves for lag transfer functions. The corner frequencies for the poles of the plotted functions are normalized to one in these figures.

As mentioned earlier, an important feature of the lead transfer function is that it provides substantial positive phase shift over a range of frequencies below its zero location without a significant increase in magnitude. The reason stems from a basic property of real-axis singularities. At frequencies below the zero location, this singularity dominates the lead transfer function, so

$$\frac{V_o(s)}{V_i(s)} \simeq \frac{1}{\alpha}(\alpha\tau s + 1) \tag{5.7}$$

The magnitude and angle of this function are

$$M = \frac{1}{\alpha}[\sqrt{1 + (\alpha\tau\omega)^2}] \tag{5.8a}$$

$$\phi = \tan^{-1}\alpha\tau\omega \tag{5.8b}$$

At a small fraction of the zero location, $\alpha\tau\omega \ll 1$, so

$$M \simeq \frac{1}{\alpha}\left[1 + \frac{(\alpha\tau\omega)^2}{2}\right] \tag{5.9a}$$

$$\phi \simeq \alpha\tau\omega \tag{5.9b}$$

Since the angle increases linearly with frequency in this region while the magnitude increases quadratically, the angle change is relatively larger at a given frequency. The same sort of reasoning applies even if the zero is located at or slightly below crossover. Figure 5.7 shows that the positive phase shift of a lead transfer function with a reasonable value of α is approximately 40° at its zero location, while the magnitude increase is only a factor of 1.4. Much of this advantage is lost at frequencies beyond the geometric mean of the singularities, since the positive phase shift decreases beyond this frequency, while the magnitude continues to increase.

We should recognize that an isolated zero can be used in place of a lead transfer function, and that this type of transfer function actually has phase-shift characteristics superior to those of the zero-pole pair. However, the unlimited high-frequency gain implied by an isolated zero is clearly unachievable, at least at sufficiently high frequencies. Thus the form of the

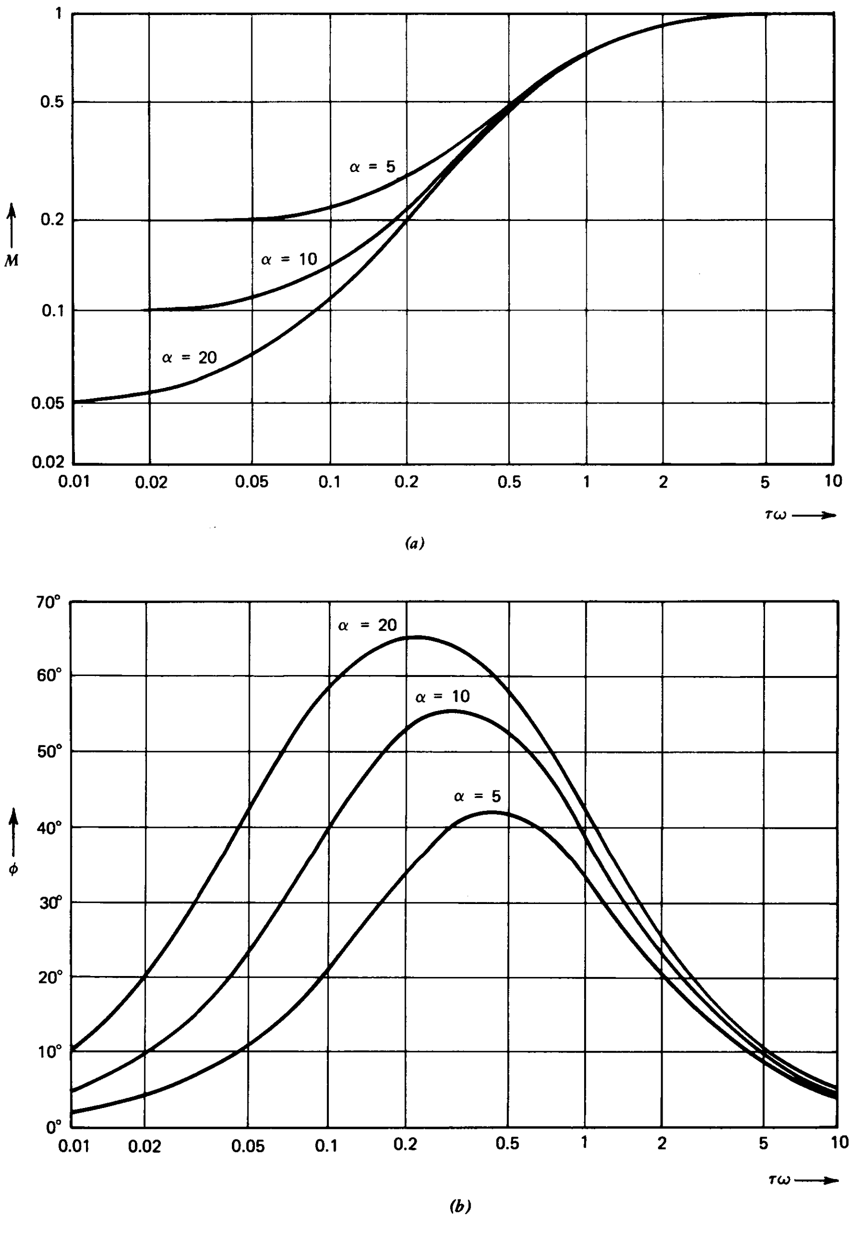
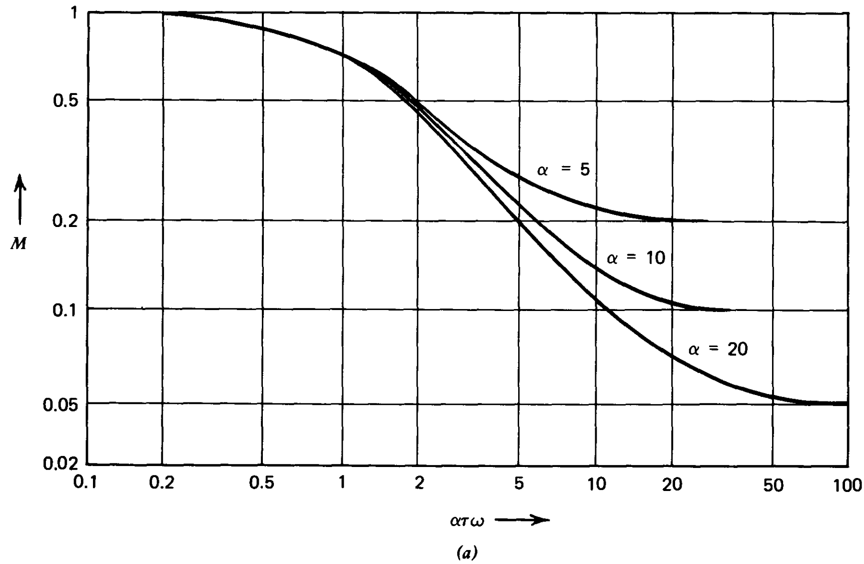


Figure 5.7 Lead network characteristics for $V_o(s)/V_i(s) = (1/\alpha)[(\alpha\tau s + 1)/(\tau s + 1)]$. (*a*) Magnitude. (*b*) Angle.



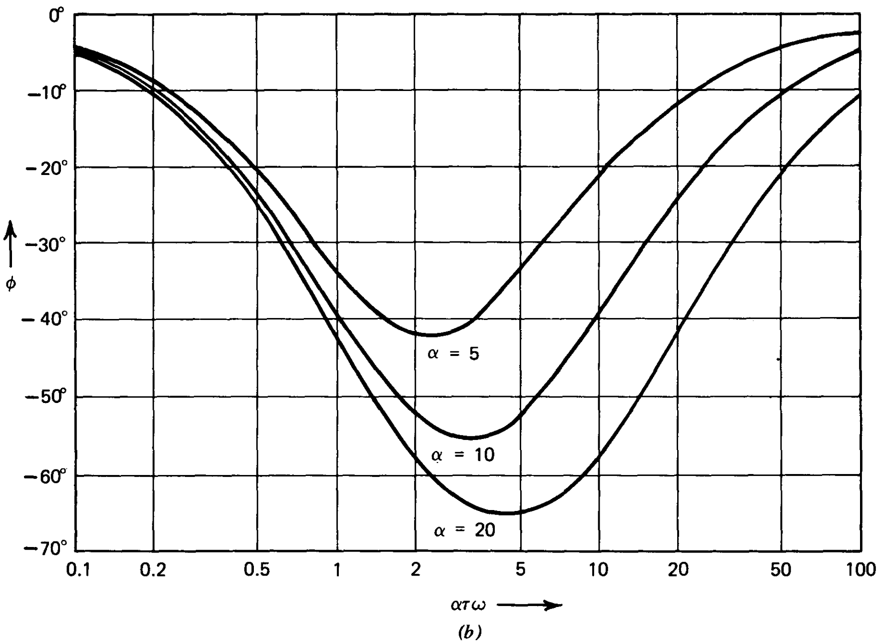


Figure 5.8 Lag network characteristics for $V_o(s)/V_i(s) = (\tau s + 1)/(\alpha\tau s + 1)$. (*a*) Magnitude. (*b*) Angle.

lead transfer function introduced earlier reflects the realities of physical systems.

The important feature of the lag transfer function illustrated in Fig. 5.8 is that at frequencies well above the zero location, it provides a magnitude attenuation equal to the ratio of the two singularity locations and negligible phase shift. It can thus be used to reduce the magnitude of the loop transmission without significantly adding to the negative phase shift of this transmission at moderate frequencies.

5.2.4 Example

Lead and lag networks were originally developed for use in servomechanisms, and provide a powerful means for compensation when their singularities can be located arbitrarily with respect to other system poles and when independent adjustment of the low-frequency loop-transmission magnitude is possible. Even without this flexibility, which is usually absent with operational-amplifier circuits, lead or lag compensation can provide effective control of closed-loop performance in certain configurations. As an example, consider the noninverting gain-of-ten amplifier connection shown in Fig. 5.9. It is assumed that the input admittance and output impedance of the operational amplifier are small. The open-loop transfer function of the operational amplifier is[1]

$$a(s) = \frac{5 \times 10^5}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)} \tag{5.10}$$

and it is assumed that the user cannot alter this function. When connected as shown in Fig. 5.9 the value of f is 0.1, and thus the negative of the loop transmission is

$$a(s)f(s) = \frac{5 \times 10^4}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)} \tag{5.11}$$

[1] While an analytic expression is used for $a(s)$ in this example, the reader should realize that the open-loop transfer function of an operational amplifier will generally not be available in this form. Note, however, that an experimentally determined Bode plot is completely acceptable for all of the required manipulations, and that this information can always be determined.

The general characteristics of the assumed open-loop transfer function are typical of many operational amplifiers, in that this quantity is dominated by a single pole at low frequencies. At frequencies closer to the unity-gain frequency, additional negative phase shift results from effects related to transistor limitations. As we shall see in later sections, these effects constrain the ultimate performance capabilities of the amplifier.

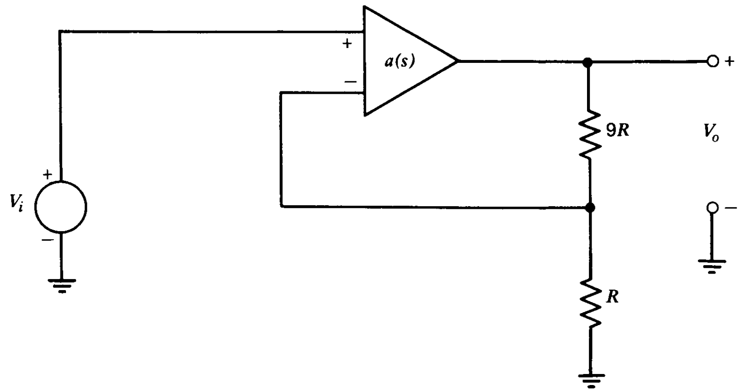


Figure 5.9 Gain-of-ten amplifier.

The closed-loop gain is

$$\frac{V_o(s)}{V_i(s)} = A(s) = \frac{a(s)}{1 + a(s)f(s)}$$

$$\simeq \frac{10}{2 \times 10^{-14}s^3 + 2.2 \times 10^{-9}s^2 + 2 \times 10^{-5}s + 1} \tag{5.12}$$

A Bode plot of Eqn. 5.11 (Fig. 5.10) shows that the system crossover frequency is 2.1×10^4 radians per second, its phase margin is 13°, and the gain margin is 2.

While the problem statement precludes altering $a(s)$, we can introduce a lead transfer function into the loop transmission by including a capacitor across the upper resistor in the feedback network. The topology is shown in Fig. 5.11*a*, with a block diagram shown in Fig. 5.11*b*. The negative of the loop transmission for the system is

$$a'(s)f'(s) = \frac{5 \times 10^4(9RCs + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(0.9RCs + 1)} \tag{5.13}$$

Several considerations influence the selection of the *R*-*C* product that locates the singularities of the lead network. As mentioned earlier, the objective of a lead network is to provide positive phase shift in the vicinity of the crossover frequency, and maximum positive phase shift from the network results if crossover occurs at the geometric mean of the zero-pole pair. However, the network singularities and the crossover frequently cannot be adjusted independently for this system, since if the zero of the lead network is located at a frequency below about 3×10^4 radians per second, the crossover frequency increases. An increase in crossover frequency in-

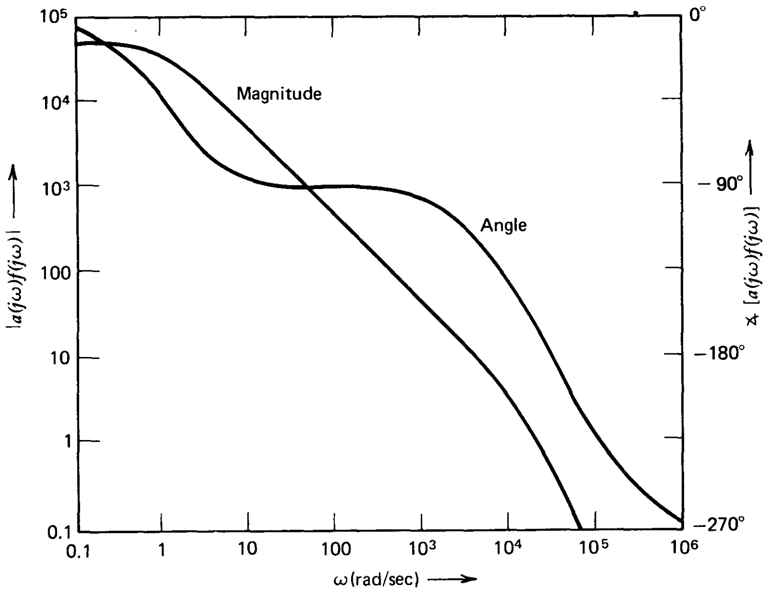


Figure 5.10 Bode plot for uncompensated grain-of-ten amplifier. $af = 5 \times 10^4 / [(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)]$.

creases the negative phase shift of the amplifier at this frequency, offsetting in part the positive phase shift of the network. A related consideration involves the effect of the lead network on the ideal closed-loop gain of the amplifier since the network is introduced in the feedback path and the ideal gain is reciprocally related to the feedback transfer function. If the lead-network zero is located at a low frequency, a low-frequency closed-loop pole that reduces the closed-loop bandwidth of the system results.

A reasonable compromise in this case is to locate the zero of the lead network near the unity-gain frequency, in an attempt to obtain positive phase shift from the network without a significant increase in the crossover frequency. The choice $RC = 4.44 \times 10^{-6}$ seconds locates the zero at 2.5×10^4 radians per second. A Bode plot of Eqn. 5.13 for this value of RC is shown in Fig. 5.12. The unity-gain frequency is increased slightly to 2.5×10^4 radians per second, while the phase margin is increased to the respectable value of 47°. Gain margin is 14.

A lag transfer function can be introduced into the forward path of the amplifier by shunting a series resistor-capacitor network between its input terminals as shown in Fig. 5.13*a*. Note that the same loop transmission could be obtained by shunting the R-valued resistor with the R_1-C network,

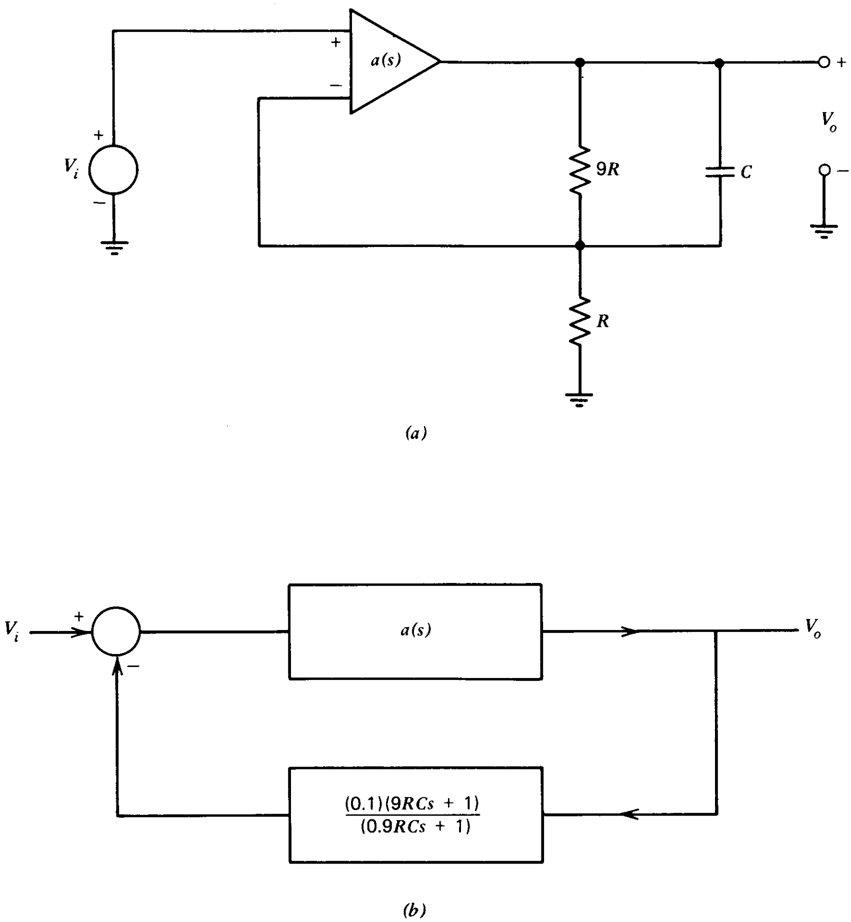


Figure 5.11 Gain-of-ten amplifier with lead network in feedback path. (*a*) Circuit. (*b*) Block diagram.

since both the bottom end of the R-valued resistor and the noninverting input of the amplifier are connected to incrementally grounded points. If this later option were used, the R_1-C network would introduce the lag transfer function into the feedback path of the topology. Consequently, the ideal closed-loop transfer function would include the reciprocal of the lag function. Since the singularities of lag networks are generally located at low frequencies, the closed-loop transfer function could be adversely influenced at frequencies of interest. (See Problem P5.7.)

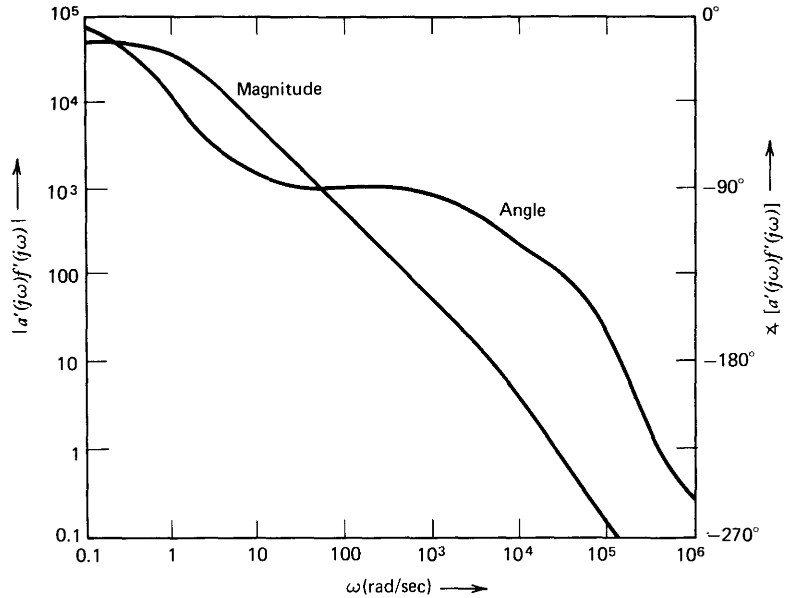


Figure 5.12 Bode plot for lead-compensated gain-of-ten amplifier. $a'f' = 5 \times 10^4(4 \times 10^{-5}s + 1)/[(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(4 \times 10^{-6}s + 1)]$.

The system block diagram for the topology of Fig. 5.13*a* is shown in Fig. 5.13*b*. In this case, the lag transfer function appears in both the feedback path and a forward path outside the loop. The block diagram can be rearranged as shown in Fig. 5.13*c*, and this final diagram shows that including the R_1-C network between amplifier inputs leaves the ideal closed-loop gain unchanged. The negative of the loop transmission for Fig. 5.13*c* is

$$a''(s)f''(s) = 0.1 \frac{(\tau s + 1)}{(\alpha\tau s + 1)} a(s) \tag{5.14}$$

where

$$\alpha = \frac{R_1 + 0.9R}{R_1} \quad \text{and} \quad \tau = R_1C$$

As mentioned earlier, the singularities of a lag transfer function are generally located well below the system crossover frequency so that the lag network does not deteriorate phase margin significantly. A frequently used rule of thumb suggests locating the zero of the lag network at one-tenth of the crossover frequency that results following compensation, since this value yields a maximum negative phase contribution of 5.7° from the network at crossover. We also, rather arbitrarily, decide to choose the lag-network parameters to yield a phase margin of approximately 47°, the same

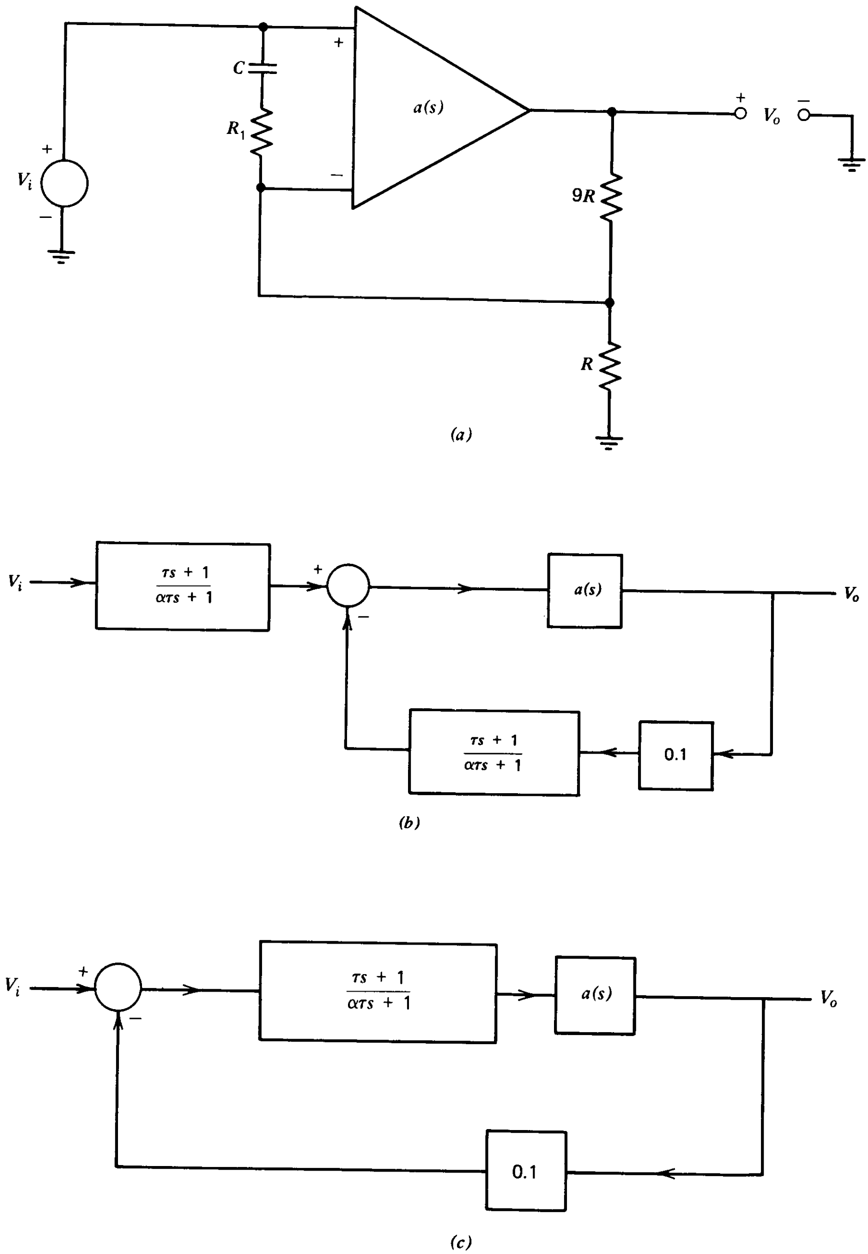


Figure 5.13 Gain-of-ten amplifier with lag compensation. (*a*) Circuit. (*b*) Block diagram. (*c*) Block diagram following rearrangement.

value as that of the system compensated with a lead network. The Bode plot of the system without compensation, Fig. 5.10, aids in selecting lag-network parameters. This plot indicates an uncompensated phase angle of $-128°$ and an uncompensated magnitude of 6.2 at a frequency of 6.7×10^3 radians per second. If the value of 6.2 is the chosen high-frequency attenuation α of the lag network, the compensated crossover frequency will be 6.7×10^3 radians per second. The 5° of negative phase shift anticipated from a properly located lag network combines with the $-128°$ of phase shift of the system prior to compensation to yield a compensated phase margin of 47°. The zero of the lag network is located at 6.7×10^2 radians per second, a factor 10 below crossover. These design objectives are met with $R_1 = 0.173R$ and $R_1C = 1.5 \times 10^{-3}$ seconds. With these values, the negative of the loop transmission is

$$a''(s)f''(s) = \frac{5 \times 10^4(1.5 \times 10^{-3}s + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(9.3 \times 10^{-3}s + 1)} \tag{5.15}$$

This transfer function, plotted in Fig. 5.14, indicates predicted values for crossover frequency and phase margin. The gain margin is 15.

Two other modifications of the loop transmission result in Bode plots that are similar to that of the lag-compensated system in the vicinity of the crossover frequency. One possibility is to lower the value of a_0f_0 by a factor of 6.2 (see Section 5.2.1). The required reduction can be accomplished by simply using the shunt-resistor value determined for lag compensation directly across the input terminals of the operational amplifier. This modification results in the same crossover frequency as that of the lag-compensated amplifier, and has several degrees more phase margin since it does not have the slight negative phase shift associated with the lag network at crossover. Unfortunately, the lowered a_0f_0 results in a lower value for desensitivity compared with that of the lag-compensated amplifier at all frequencies below the zero of the network.

A second possibility is to move the lowest-frequency pole of the loop transmission back by a factor of 6.2. This modification might be made to the amplifier itself, or could be accomplished by appropriate selection of lag-network components. The effect on parameters in the vicinity of crossover is essentially identical to that of reducing a_0f_0. Desensitivity is retained at d-c with this method, but is lowered at intermediate frequencies compared to that provided by lag compensation. These two approaches to compensating the amplifier described here are investigated in detail in Problem P5.8.

The discussion of series compensation up to this point has focused on the use of the frequency-domain concepts of phase margin, gain margin,

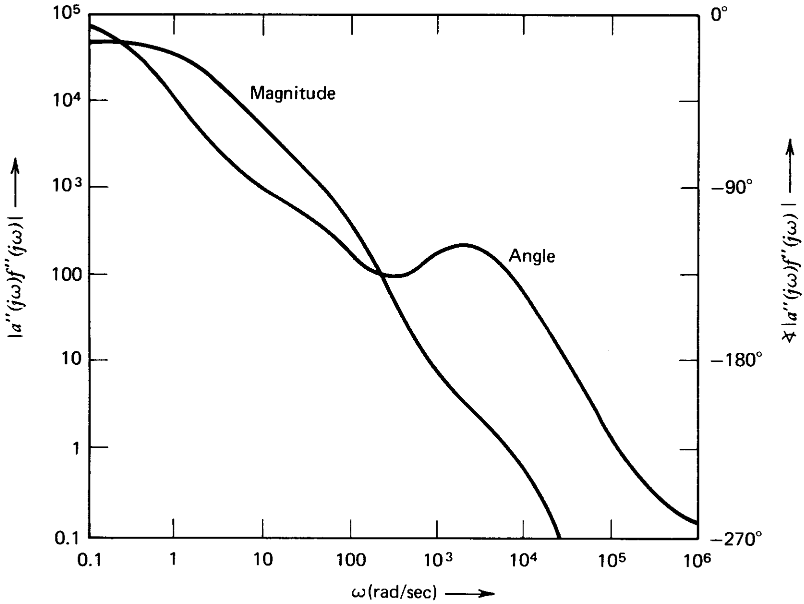


Figure 5.14 Bode plot for lag compensated gain-of-ten amplifier. $a''f'' = 5 \times 10^4(1.5 \times 10^{-3}s + 1)/[(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(9.3 \times 10^{-3}s + 1)]$.

and crossover frequency to determine compensating-network parameters. Root-locus methods cannot be used directly since the value of a_0f_0 is not varied to effect compensation. However, the root-locus sketches for the uncompensated, lead-compensated, and lag-compensated systems shown in Fig. 5.15 do lend a degree of insight into system behavior. (There is significant distortion in these sketches, since it is not convenient to present sketches accurately where the singularities are located several decades apart.)

The root-locus diagram of Fig. 5.15*a* illustrates the change in closed-loop pole location as a function of a_0f_0 for the uncompensated system. Adding the lead network (Fig. 5.15*b*) shifts the dominant branches to the left and, thus, improves the damping ratio of this pair of poles for a given value of a_0f_0.

The effect of lag compensation is somewhat more subtle. The root-locus diagram of Fig. 5.15*c* is virtually identical to that of Fig. 5.15*a* except in the immediate vicinity of the lag-network singularity pair. However, a gain calculation using rule 8 (Section 4.3.1) shows that the value of a_0f_0 required to reach a given damping ratio for the dominant pair is higher by approximately a factor of α when the lag network is included.

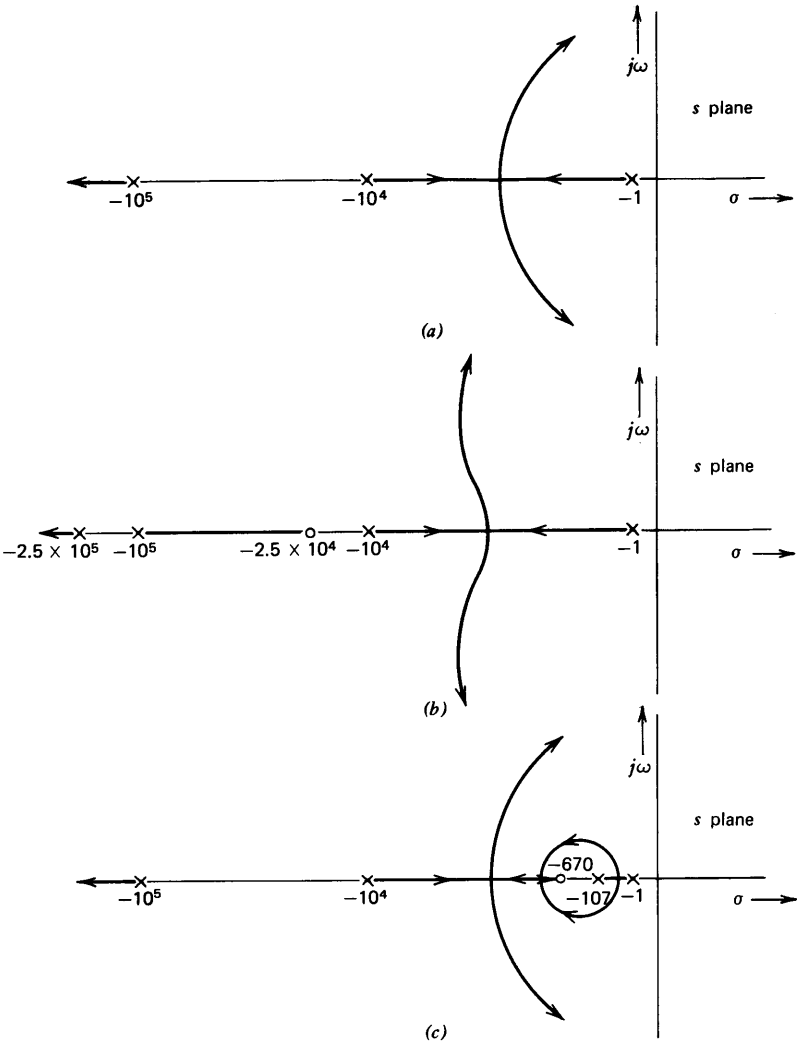


Figure 5.15 Root-locus diagrams illustrating compensation of gain-of-ten amplifier. (*a*) Uncompensated. (*b*) Lead compensated. (*c*) Lag compensated.

Root contours can also be used to show the effects of varying a single parameter of either the lead or the lag network. This design approach is explored in Problems P5.9 and P5.10.

5.2.5 Evaluation of the Effects of Compensation

There are several ways to demonstrate the improvement in performance provided by compensation. Since the parameters of the compensating transfer function are usually determined with the aid of loop-transmission Bode plots, one simple way to evaluate various types of compensation is to compare the desensitivity obtained from them. The considerations used to determine lead- and lag-compensation parameters for an operational amplifier connected to provide a gain of 10 were described in detail in Section 5.2.4. The resulting loop transmissions, repeated here for convenience, are

$$a'(s)f'(s) = \frac{5 \times 10^4(4 \times 10^{-5}s + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(4 \times 10^{-6}s + 1)} \tag{5.16}$$

and

$$a''(s)f''(s) = \frac{5 \times 10^4(1.5 \times 10^{-3}s + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(9.3 \times 10^{-3}s + 1)} \tag{5.17}$$

for the lead- and lag-compensated cases, respectively. The phase-margin obtained by either method is approximately 47°.

It was mentioned that the stability of the uncompensated amplifier could be improved by either lowering a_0f_0 by a factor of 6.2, resulting in

$$a'''(s)f(s)''' = \frac{8.1 \times 10^3}{(s + 1)(10^{-4}\, s + 1)(10^{-5}\, s + 1)} \tag{5.18}$$

or by lowering the location of the first pole by the same factor, yielding

$$a''''(s)f''''(s) = \frac{5 \times 10^4}{(6.2\, s + 1)(10^{-4}\, s + 1)(10^{-5}\, s + 1)} \tag{5.19}$$

Either of these approaches results in a crossover frequency identical to that of the lag-compensated system and a phase margin of approximately 52°.

The magnitude portions of the loop transmissions for these four cases are compared in Fig. 5.16. The relative desensitivities that are achieved at various frequencies, as well as the relative crossover frequencies, are evident in this figure.

An alternative way to evaluate various compensation techniques is to compare the error coefficients that are obtained using them. This approach is explored in Problem P5.11. As expected, systems with greater desensitivity generally also have smaller-magnitude error coefficients.

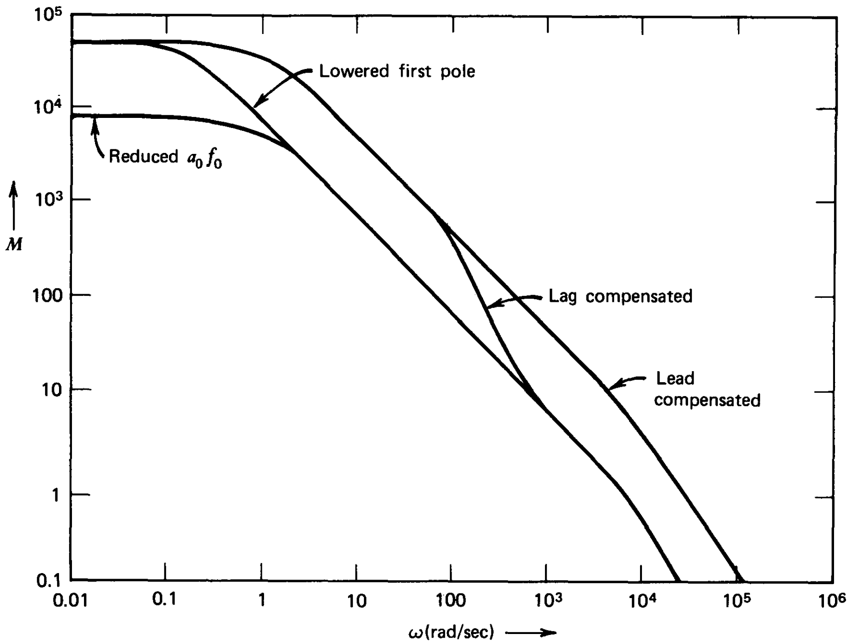


Figure 5.16 Effects of various types of compensation on loop-transmission magnitude.

The discussion of compensation up to now has focused on the use of Bode plots, since this is usually the quickest way to find compensating parameters. However, design objectives are frequently stated in terms of transient response, and the inexperienced designer often feels an act of faith is required to accept the principle that systems with properly chosen values for phase margin, gain margin, and crossover frequency will produce satisfactory transient responses. The step responses shown in Fig. 5.17 are offered as an aid to establishing this necessary faith.

Figure 5.17*a* shows the step response of the gain-of-ten amplifier without compensation. The large peak overshoot and poor damping of the ringing reflect the low phase margin of the system. The overshoot and damping for the lead compensated, lag compensated, and reduced $a_0 f_0$ cases (Figs. 5.17*b*, 5.17*c*, and 5.17*d*, respectively) are significantly improved, as anticipated in view of the much higher phase margins of these connections. The step response obtained by lowering the frequency of the first pole in the loop is not shown, since it is indistinguishable from Fig. 5.17*d*.

Certain features of these step responses are evident from the figures. The peak overshoot exhibited by the amplifier with reduced $a_0 f_0$ is slightly

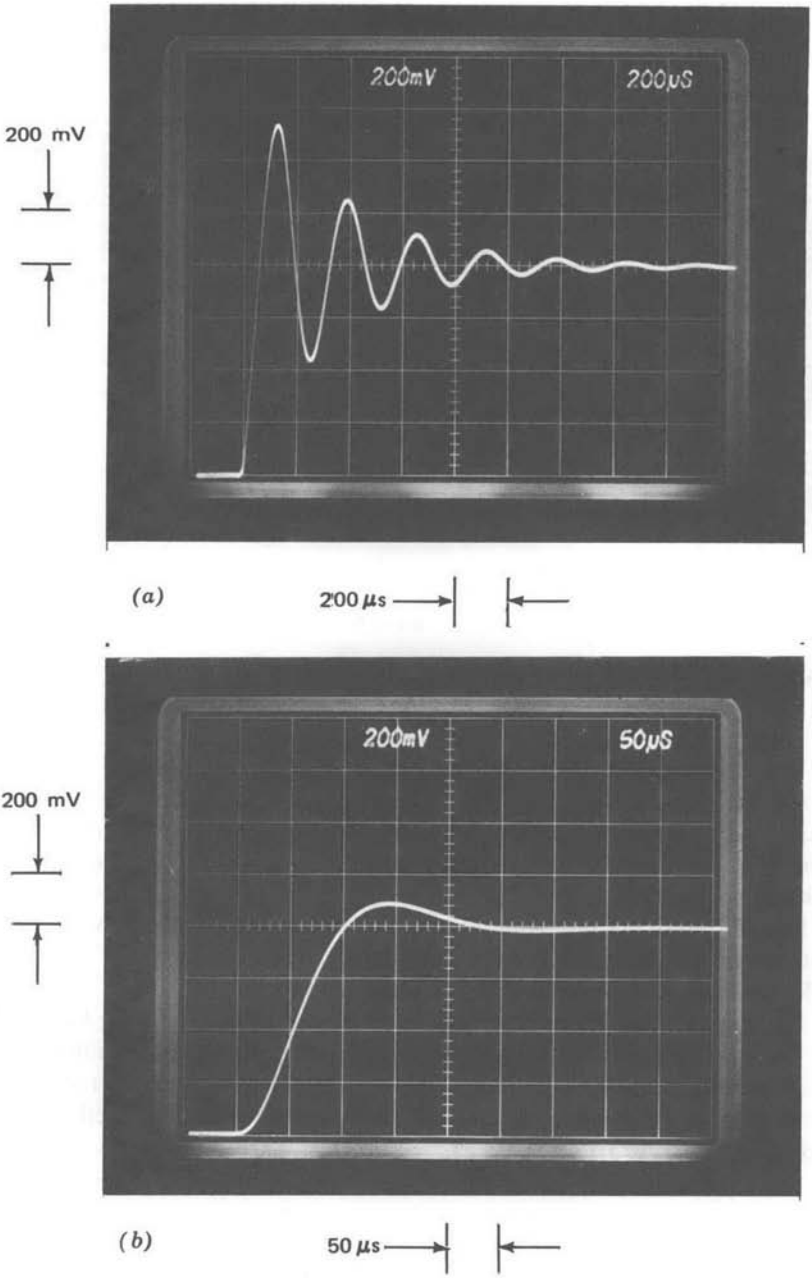
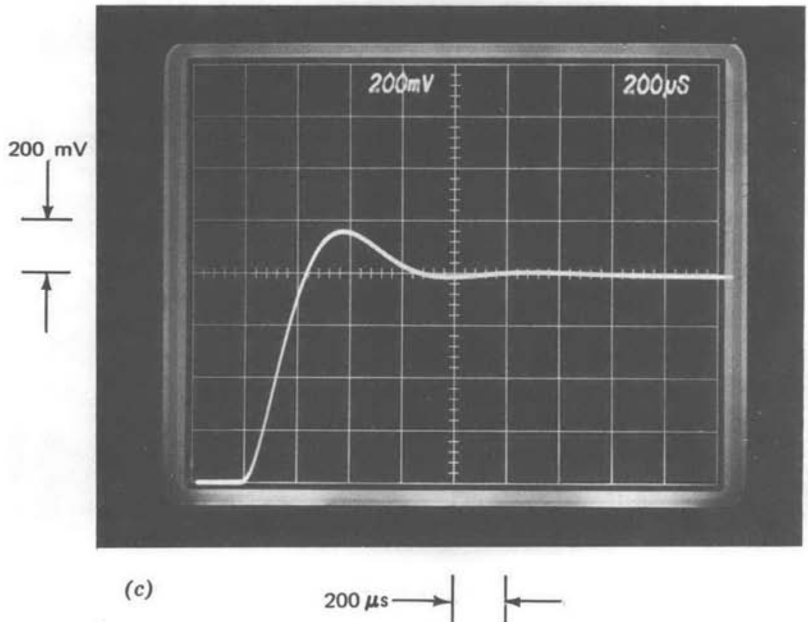


Figure 5.17 Response of gain-of-ten amplifier to an 80-mV step. (*a*) No compensation. (*b*) Lead compensated. (*c*) Lag compensated. (*d*) Lowered a_0f_0. (*e*) Lead compensation in forward path. (*f*) Second-order approximation to (*c*).



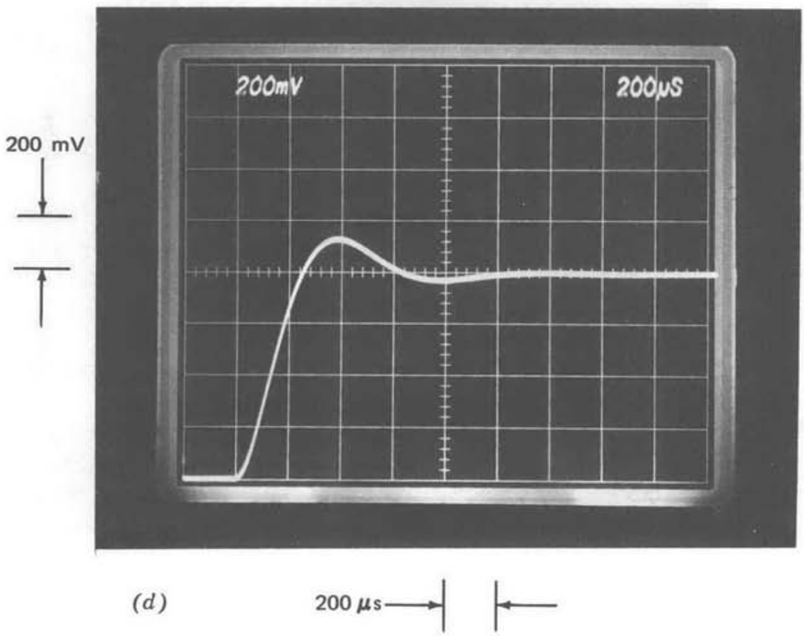
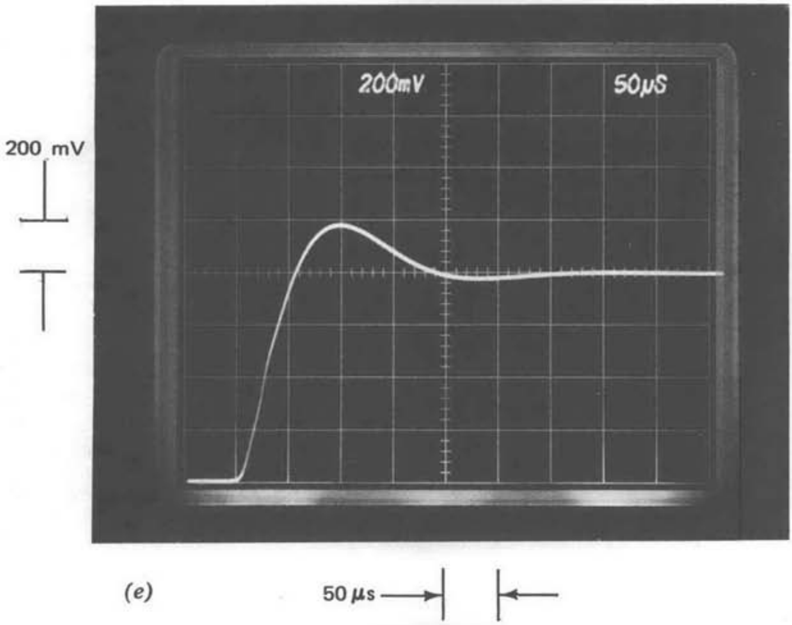
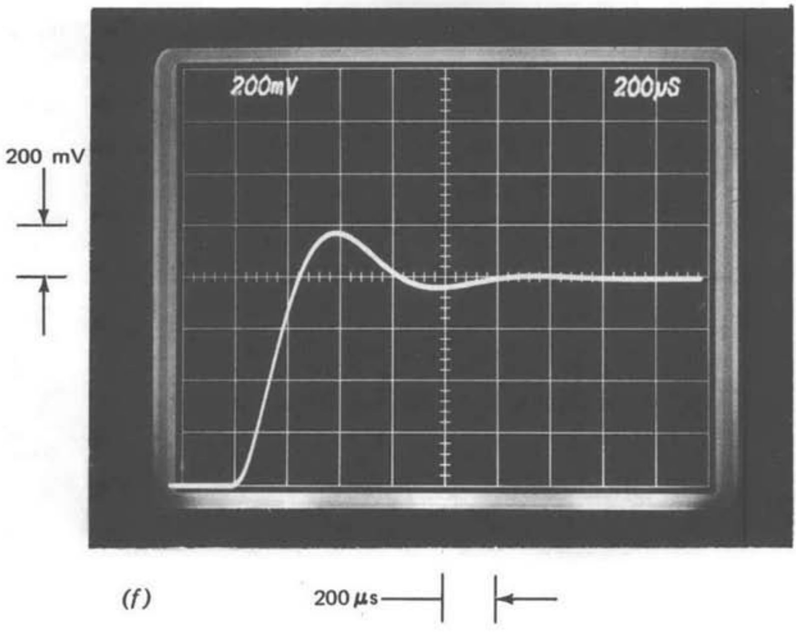


Figure 5.17—Continued



(e)



(f)

Figure 5.17—Continued

less than that of the amplifier with lag compensation, reflecting slightly higher phase margin. Similarly, the rise time of lag-compensated amplifier is very slightly faster, again reflecting the influence of relative phase margin on the performance of these two systems with identical crossover frequencies. The smaller peak overshoot of the lead-compensated system does not imply greater relative stability for this amplifier, but rather occurs because of the influence of the lead network in the feedback path on the ideal closed-loop gain.

Figure 5.17*e* shows the step response that results if lead compensation is provided in the forward path rather than in the feedback path. Thus the loop transmission for this transient response is identical to that of Fig. 5.17*b* (Eqn. 5.16), but the feedback path for the system illustrated in Fig. 5.17*e* is frequency independent. While forward-path lead compensation was prohibited by the problem statement of the earlier examples, Fig. 5.17*e* provides a more realistic indication of relative stability than does Fig. 5.17*b*, since Fig. 5.17*e* is obtained from a system with a frequency-independent ideal gain. The difference between these two systems with identical loop transmissions arises because of differences in the closed-loop zero locations (see Section 4.3.4).

The peak overshoot and relative damping of Figs. 5.17*c* and 5.17*e* are virtually identical, demonstrating that, at least for this example, equal values of phase margin result in equal relative stability for the lead- and lag-compensated systems. The rise time of Fig. 5.17*e* is approximately one-quarter that of Fig. 5.17*c*, and this ratio is virtually identical to the ratio of the crossover frequencies of the two amplifiers.

The step response of Fig. 5.17*f* is that of a second-order system with $\zeta = 0.45$ and $\omega_n = 8.5 \times 10^3$ radians per second. These values were obtained using Fig. 4.26*a* to determine a second-order approximating system to the lag-compensated amplifier. The similarity of Figs. 5.17*c* and 5.17*f* is another example of the accuracy that is frequently obtained when complex systems are approximated by first- or second-order ones. The loop transmission for the lag-compensated system (Eqn. 5.17) includes four poles and one zero. However, this quantity has only a single-pole roll off between 6.7×10^2 radians per second and the crossover frequency, with a second pole in the vicinity of crossover. It can thus be well approximated as a system with two widely separated poles, the model from which Fig. 4.26 was developed.

5.2.6 Related Considerations

Several additional comments concerning the relative benefits of different series compensation methods are in order. The evaluation of performance in the previous example seems to imply advantages for lead compensation. The lead-compensated amplifier appears superior if desensitivity at various frequencies, error-coefficient magnitude, or speed of transient response is used as the indicator of performance. Furthermore, if the lead transfer function is included in the feedback path, the amplifier exhibits better-damped transient responses than can be obtained from other types of compensation selected to yield equivalent phase margin. The advantages associated with lead compensation primarily reflect the higher value for crossover frequency and the correspondingly higher closed-loop bandwidth that is frequently possible with this method. It should be emphasized, however, that bandwidth in excess of requirements usually deteriorates overall performance. Larger bandwidth increases the noise susceptibility of an amplifier and frequently leads to greater stability problems because of stray inductance or capacitance.

Lead compensation usually aggravates the stability problem if the loop also includes elements that provide large negative phase shift over a wide frequency range without a corresponding magnitude attenuation. (While the constraints of physical realizability preclude elements that provide positive phase shift without an amplitude increase, the less useful converse described above occurs with distressing frequency.) For example, consider a system that combines a frequency-independent gain in a loop with a τ-second time delay such as that provided by a delay line. The negative of the loop transmission for this system is

$$a(s)f(s) = a_0 e^{-s\tau} \tag{5.20}$$

The time delay is an element that has a gain magnitude of one at all frequencies and a negative phase shift that is linearly related to frequency. The Nyquist diagram (Fig. 5.18) for this system shows that it is unstable for $a_0 > 1$. The use of lead compensation compounds the problem, since the positive phase shift of the lead network cannot counteract the unlimited negative phase shift of the time delay, while the magnitude increase of the lead function further lowers the maximum low frequency desensitivity consistent with stable operation.

The correct approach is to use a dominant pole to decrease the magnitude of the loop transmission before the phase shift of time delay becomes excessive. The limiting case of an integrator (pole at the origin) works well, and this modification results in

$$a(s)f(s) = \frac{a_0}{s} e^{-s\tau} \tag{5.21}$$

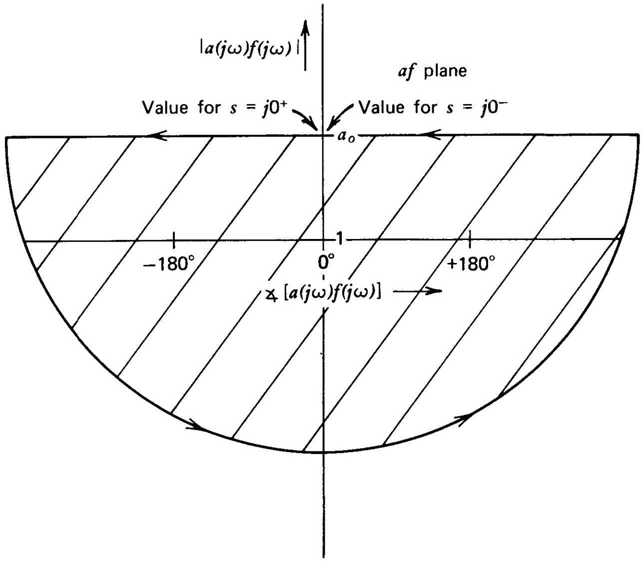


Figure 5.18 Nyquist test for $a(s)f(s) = a_0e^{-s\tau}$.

The desensitivity of this function is infinite at d-c. The reader should convince himself that the system is absolutely stable for any positive value of $a_0 < \pi/2\tau$, and that at least 45° of phase margin is obtained with positive $a_0 < \pi/4\tau$.

The use of lag compensation introduces a type of error that compromises its value in some applications. If the step response of a lag-compensated amplifier is examined in sufficient detail, it is often found to include a long time-constant, small-amplitude "tail," which may increase inordinately the time required to settle to a small fraction of final value. Similarly, while the error coefficient e_1 may be quite small, the time required for the ramp error to reach its steady-state value may seem incompatible with the amplifier crossover frequency.

As an aid to understanding this problem, consider a system with $f(s) = 1$ and

$$a(s) = \frac{1000(0.1s + 1)}{s(s + 1)} \tag{5.22}$$

This transfer function is an idealized representation of a system that combines a single dominant pole with lag compensation to improve desensitivity. The zero of the lag network is located a factor of 10 below the crossover frequency. The closed-loop transfer function is

$$A(s) = \frac{a(s)}{1 + a(s)f(s)} = \frac{(0.1s + 1)}{10^{-3}s^2 + 0.101s + 1}$$
$$= \frac{(0.1s + 1)}{(0.09s + 1)(0.011s + 1)} \tag{5.23}$$

The response of this system to a unit step is easily evaluated via Laplace techniques, with the result

$$v_o(t) = 1 - 1.126e^{-t/0.011} + 0.126e^{-t/0.09} \tag{5.24}$$

This step response reaches 10% of final value in 0.02 second, a reasonable value in view of the 100 radian per second crossover frequency of the system. However, the time required to reach 1% of final value is 0.23 second because of the final term in Eqn. 5.24. Note that if $a(s)$ is changed to $100/s$, a transfer function with the same unity-gain frequency as Eqn. 5.22 and *less* gain magnitude at all frequencies below 10 radians per second, the time required for the system step response to reach 1% of final value is approximately 0.05 second.

The root-locus diagram for the system (Fig. 5.19) clarifies the situation. The system has a closed-loop zero with a corner frequency at 10 radians per second since the zero shown in the diagram is a forward-path singularity. The feedback forces one closed-loop pole close to this zero. The resultant closely spaced pole-zero doublet adds a long-time-constant tail

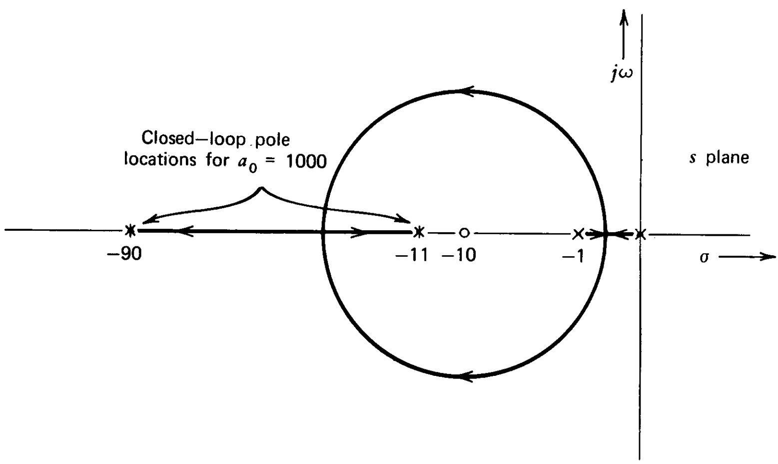


Figure 5.19 Root-locus diagram for $a(s)f(s) = a_0(0.1s + 1)/[s(s + 1)]$.

to the otherwise well-behaved system transient response. The reader should recall that it is precisely this type of doublet that deteriorates the step response of a poorly compensated oscilloscope probe. Since linear system relationships require that the ramp response be the integral of the step response, the time required for the ramp error to reach final value is similarly delayed.

Similar calculations show that as the lag transfer function is moved further below crossover, the amplitude of the tail decreases, but its time constant increases. We conclude that while lag compensation is a powerful technique for improving desensitivity, it must be used with care when the time required for the step response to settle to a small fraction of its final value or the time required for the ramp error to reach final value is constrained.

It should be emphasized that a closed-loop pole will generally be located close to any open-loop zero with a break frequency below the crossover frequency. Thus the type of tail associated with lag compensation can also result with, for example, lead compensation that often includes a zero below crossover. The performance difference results because the zero and the closed-loop pole that approaches it to form a doublet are usually located

Table 5.1 Comparison of Series-Compensating Methods

Type	Special Considerations	Advantages	Disadvantages
Reduced $a_0 f_0$		Simplicity.	Lowest desensitivity.
Create dominant pole	Lower the frequency of the existing dominant pole if possible. Locate at the output of a regulator.	Can improve noise immunity of system. Usually the type of choise for a regulator.	Lowers bandwidth.
Lag	Locate well below crossover frequency.	Better desensitivity than either of above.	May add undesirable "tail" to transient response.
Lead	Locate zero near crossover frequency.	Greatest desensitivity. Lowest error coefficients. Fastest transient response.	Increases sensitivity to noise. Cannot be used with fixed elements that contribute excessive negative phase shift.

close to the crossover frequency for lead compensation. Thus the decay time of the resultant tail, which is determined by the closed-loop pole in question, does not greatly lengthen the settling time of the system.

It is difficult to develop generalized rules concerning compensation, since the proper approach is highly dependent on the fixed elements included in the loop, on the types of inputs anticipated, on the performance criterion chosen, and on numerous other factors. In spite of this reservation, Table 5.1 is an attempt to summarize the most important features of the four types of series compensation described in this section.

5.3 FEEDBACK COMPENSATION

Series compensation is accomplished by adding a cascaded element to a single-loop feedback system. Feedback compensation is implemented by adding a feedback element which creates a two-loop system. One possible topology is illustrated in Fig. 5.20. The closed-loop transfer function for this system is

$$\frac{V_o}{V_i} = \frac{a_1 a_2/(1 + a_2 f_2)}{1 + a_1 a_2 f_1/(1 + a_2 f_2)} \tag{5.25}$$

A series-compensated system with a feedback element identical to the major-loop feedback element of Fig. 5.20 is shown in Fig. 5.21. The two feedback elements are identical since it is assumed that the same ideal closed-loop transfer function is required from the two systems. The closed-loop transfer function for the series-compensated system is

$$\frac{V_o}{V_i} = \frac{a_3 a_4}{1 + a_3 a_4 f_1} \tag{5.26}$$

The closed-loop transfer functions of the feedback- and series-compensated systems will be equal if f_2 is selected so that

$$a_3 = \frac{a_1 a_2}{(1 + a_2 f_2) a_4} \tag{5.27a}$$

or

$$f_2 = \frac{a_1 a_2 - a_3 a_4}{a_2 a_3 a_4} \tag{5.27b}$$

The above analysis suggests that one way to select appropriate feedback compensation is first to determine the series compensation that yields acceptable performance and then convert to equivalent feedback compensation. In practice, this approach is normally *not* used, but rather the series compensation is determined to exploit potential advantages of this method.

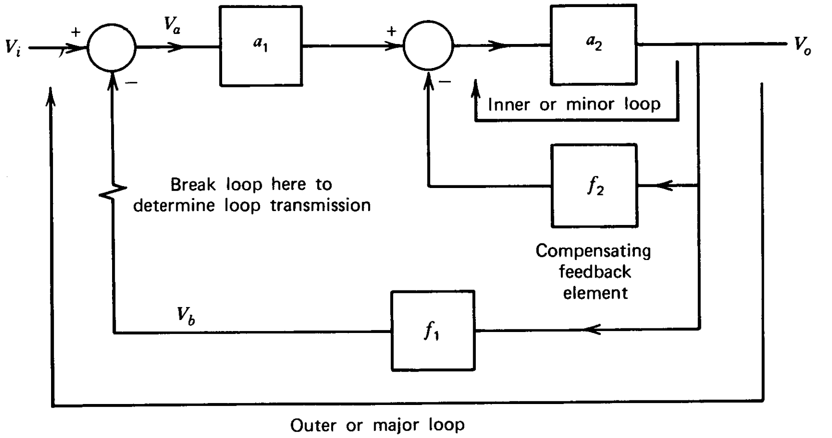


Figure 5.20 Topology for feedback compensation.

We shall see that if an operational amplifier is designed to accept feedback compensation, the use of this technique often results in performance superior to that which can be achieved with series compensation. The frequent advantage of feedback compensation is not a consequence of any error in the mathematics that led to the equivalence of Eqn. 5.27 but instead is a result of practical factors that do not enter into these calculations. For example, the compensating network required to obtain specified closed-loop performance is often easier to determine and implement and may be less sensitive to variations in other amplifier parameters in the case of a feedback-compensated amplifier. Similarly, problems associated with nonlinearities and noise are often accentuated by series compensation, yet may actually be reduced by feedback compensation.

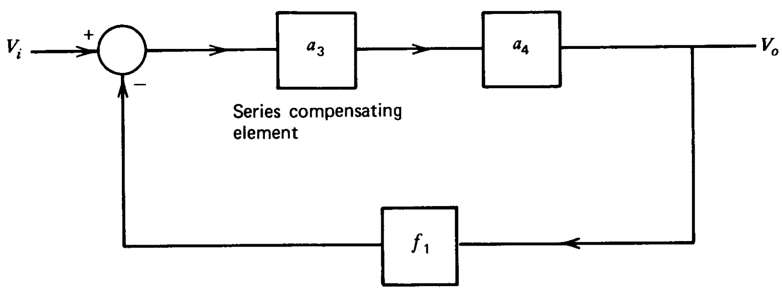


Figure 5.21 Series-compensated system.

The approach to finding the type of feedback compensation that should be used in a given application is to consider the negative of the loop transmission for the system of Fig. 5.20. This quantity is

$$\frac{V_b}{V_a} = a_1 f_1 \frac{a_2}{1 + a_2 f_2} \tag{5.28}$$

If the inner loop is stable (i.e., if $1 + a_2 f_2$ has no zeros in the right half of the s plane), then

$$\frac{V_b(j\omega)}{V_a(j\omega)} \simeq \frac{a_1(j\omega) f_1(j\omega)}{f_2(j\omega)} \qquad |a_2(j\omega) f_2(j\omega)| \gg 1 \tag{5.29a}$$

and

$$\frac{V_b(j\omega)}{V_a(j\omega)} \simeq a_1(j\omega) f_1(j\omega) a_2(j\omega) \qquad |a_2(j\omega) f_2(j\omega)| \ll 1 \tag{5.29b}$$

In practice, system parameters are frequently selected so that the magnitude of the transmission of the minor loop is large at frequencies where the magnitude of the major loop transmission is close to one. The approximation of Eqn. 5.29a can then be used to determine a value for f_2 that insures stability for the system.

A simple example of feedback compensation is provided by the operational-amplifier model shown in Fig. 5.22*a*. The model is an idealization of a common amplifier topology that will be investigated in detail in subsequent sections. The amplifier modeled includes a first stage with wide bandwidth compared to the rest of the circuit driving into a second stage that has relatively low input impedance and that dominates the uncompensated dynamics of the amplifier. The compensation is provided by a two-port network that is connected around the second stage and that forms a minor loop. This network is constrained to be passive. A block diagram for the amplifier is shown in Fig. 5.22*b*. The quantity Y_c is the short-circuit transfer admittance of the compensating network, I_n/V_n.[2]

If no compensation is used, the open-loop transfer function for the amplifier is

$$\frac{V_o(s)}{V_i(s)} = -\frac{10^6}{(10^{-3}s + 1)^2} \tag{5.30}$$

If a wire is connected from the output of the amplifier back to its input, creating a major loop with $f = 1$, the phase margin of the resultant system is approximately 0.12°.

[2] The convention used to define Y_c is at variance with normal two-port notation, which would change the reference direction for I_n. This form is used since it results in fewer minus signs in subsequent equations.

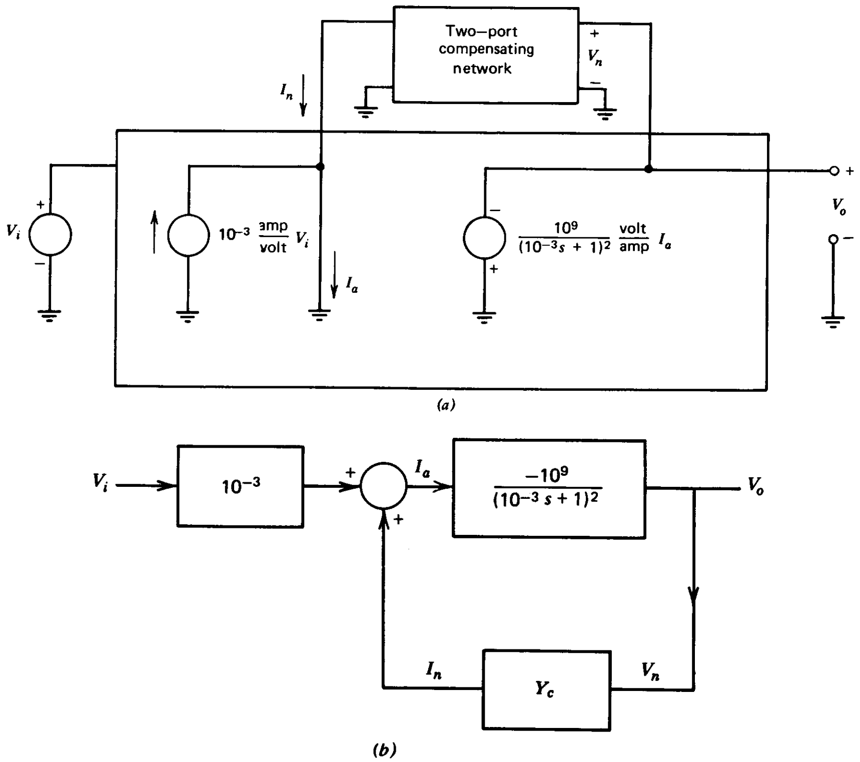


Figure 5.22 Operational amplifier. (*a*) Model. (*b*) Block diagram.

When feedback compensation is included, the block diagram shows that the amplifier transfer function is

$$\frac{V_o(s)}{V_i(s)} = \frac{-10^6/(10^{-3}s+1)^2}{1+10^9 Y_c/(10^{-3}s+1)^2} \tag{5.31}$$

One way to improve the phase margin of this amplifier when used in a feedback connection is to make $V_o(s)/V_i(s)$ dominated by a single pole. Equation 5.31 shows that

$$\frac{V_o(j\omega)}{V_i(j\omega)} \simeq \frac{-10^{-3}}{Y_c(j\omega)} \quad \text{when} \quad \left| \frac{10^9 Y_c(j\omega)}{(10^{-3}j\omega+1)^2} \right| \gg 1 \tag{5.32}$$

If a single capacitor C is used for the compensating network, $Y_c = Cs$ and

$$\frac{V_o(j\omega)}{V_i(j\omega)} \simeq \frac{-10^{-3}}{j\omega C} \tag{5.33}$$

for all frequencies such that

$$\left| \frac{10^9 Cj\omega}{(10^{-3}j\omega + 1)^2} \right| \gg 1$$

The exact expression for the amplifier open-loop transfer function with this compensation is

$$\frac{V_o(s)}{V_i(s)} = \frac{-10^6/(10^{-3}s + 1)^2}{1 + 10^9 Cs/(10^{-3}s + 1)^2}$$

$$= \frac{-10^6}{10^{-6}s^2 + (2 \times 10^{-3} + 10^9 C)s + 1} \tag{5.34}$$

If an 840-pF capacitor is used for C, the transfer function becomes

$$\frac{V_o(s)}{V_i(s)} = \frac{-10^6}{(0.84s + 1)(1.19 \times 10^{-6}s + 1)} \tag{5.35}$$

and a phase margin of at least 45° is assured for frequency-independent feedback with any magnitude less than one applied around the amplifier. With this value of compensating feedback element,

$$-\frac{V_o(j\omega)}{V_i(j\omega)} \simeq \frac{1.19 \times 10^6}{j\omega} = \frac{10^{-3}}{Cj\omega} = \frac{10^{-3}}{Y_c(j\omega)} \tag{5.36}$$

at any frequency between 1.19 radians per second and 0.84×10^6 radians per second. The two bounding frequencies are those at which the magnitude of the compensating loop transmission is one. The essential point is that minor-loop feedback controls the transfer function of the amplifier over nearly six decades of frequency. We also note that even though a dominant pole has been created by means of feedback compensation, the unity-gain frequency of the compensated amplifier (approximately 8×10^5 radians per second) remains close to the uncompensated value of 10^6 radians per second.

Feedback compensation is a powerful and frequently used compensating technique for modern operational amplifiers. Several examples of this type of compensation will be provided after the circuit topologies of representative amplifiers have been described.

PROBLEMS

P5.1

An operational amplifier has an open-loop transfer function

$$a(s) = \frac{2 \times 10^5}{(0.1s + 1)(10^{-5}s + 1)^2}$$

Design a connection that uses this amplifier to provide an ideal gain of -10. Include provision to lower the magnitude of the loop transmission so that the overshoot in response to a unit step is 10%. You may use the curves of Fig. 4.26 as an aid to determining the required attenuation.

P5.2

An operational amplifier is connected as shown in Fig. 5.23*a*. The value of α is adjusted to control the stability of the connection. Assume that noise associated with the amplifier can be modeled as shown in Fig. 5.23*b*. Evaluate the noise at the amplifier output as a function of α, neglecting loading at the input and the output of the amplifier. Note that an increase in the noise at the amplifier output implies a decrease in signal-to-noise ratio, since the gain from input to output is essentially independent of α.

P5.3

A certain feedback amplifier can be modeled as shown in Fig. 5.24. You may assume that the operational amplifier included in this diagram is ideal. Select a value for the capacitor C that results in a system phase margin of 45°.

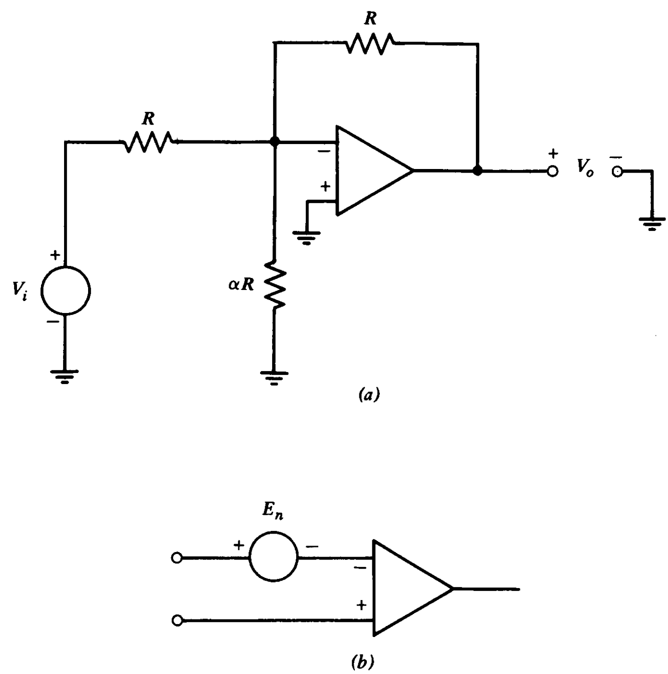


Figure 5.23 Evaluation of noise at the output of an inverting amplifier. (*a*) Inverter connection. (*b*) Method for modeling noise at amplifier input.

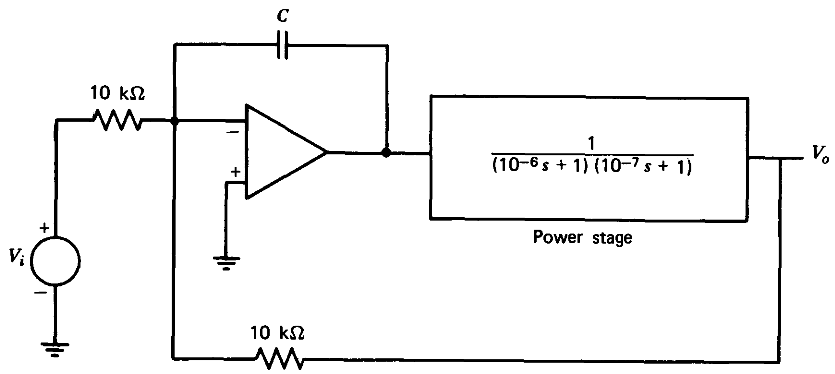


Figure 5.24 Feedback system with dominant pole.

P5.4

A speed-control system combines a high-power operational amplifier in a loop with a motor and a tachometer as shown in Fig. 5.25. The tachometer provides a voltage proportional to output shaft velocity, and this voltage is used as the feedback signal to effect speed control.

(a) Draw a block diagram for this system that includes the effects of the disturbing torque.
(b) Determine compensating component values (R and C) as a function of J_L so that the system loop transmission is $-100/s$.
(c) Show that, with this type of loop transmission, the steady-state output velocity is independent of any constant load torque.
(d) Use an error-coefficient analysis to show that the system is less sensitive to time-varying disturbing torques when larger values of J_L are used. Assume that R and C are changed with J_L to maintain the loop transmission indicated in part b.

P5.5

Show that the network illustrated in Fig. 5.26 can be used to combine a lag transfer function with a lead transfer function located at a higher frequency. Determine network parameters that will result in the transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{(0.1s + 1)(10^{-2}s + 1)}{(s + 1)(10^{-3}s + 1)}$$

P5.6

The loop transmission of a feedback system can be approximated as

$$L(s) = -\frac{10^6}{s^2}$$

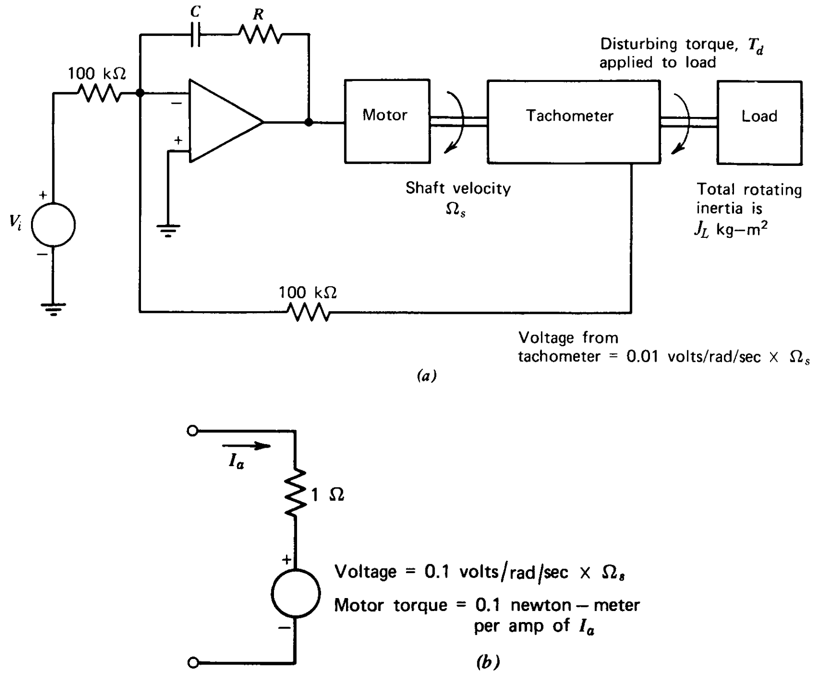


Figure 5.25 Speed-control system. (*a*) System diagram. (*b*) Motor model.

in the vicinity of the unity-gain frequency. Assume that a lead transfer function (Eqn. 5.4) with a value of $\alpha = 10$ can be added to the loop transmission. How should the transfer function be located to maximize phase margin? What values of phase margin and crossover frequency result?

P5.7

Use a block diagram to show that a lag transfer function can be introduced into the loop transmission of the gain-of-ten amplifier (Fig. 5.9) by shunting the R-valued resistor with an appropriate network.

(a) Choose network parameters so that the system loop transmission is given by Eqn. 5.15.

(b) Find the closed-loop transfer function and plot the closed-loop step response for the gain-of-ten amplifier using values found in part *a*, *assuming that the operational-amplifier characteristics are ideal.*

(c) Estimate the closed-loop step response for this connection assuming that the amplifier open-loop transfer function is as given by Eqn. 5.10.

(d) Compare the performance of the lag-compensated system developed in this problem with that shown in Fig. 5.13 considering both the sta-

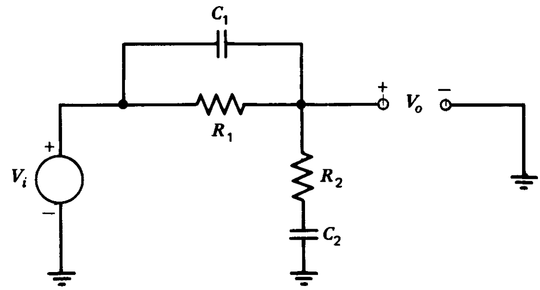


Figure 5.26 Lag-Lead network.

bility and the ideal closed-loop transfer function of the two connections.

P5.8

It was mentioned in Section 5.2.4 that alternative compensation possibilities for the gain-of-ten amplifier include lowering the magnitude of the loop transmission at all frequencies by a factor of 6.2 and lowering the location of the lowest-frequency pole in the loop transfer function by a factor of 6.2 by selecting appropriate lag-network parameters.

(a) Determine topologies and component values to implement both of these compensation schemes.
(b) Draw loop-transmission Bode plots for these two methods of compensation.
(c) Compare the relative stability produced by these methods with that provided by the lag compensation described in Section 5.2.4.

P5.9

The negative of the loop transmission for the lead-compensated gain-of-ten amplifier described in Section 5.2.4 is

$$a(s)f(s) = \frac{5 \times 10^4(10\tau s + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(\tau s + 1)}$$

where τ is determined by the resistor and capacitor values used in the feedback network (see Eqn. 5.13). Use root contours to evaluate the stability of the gain-of-ten amplifier as a function of the parameter τ. Find the value of τ that maximizes the damping ratio of the dominant pole pair. *Note.* Since it is necessary to factor third- and fourth-order polynomials in order to complete this problem, the use of machine computation is suggested. Numerical calculations are also suggested to evaluate the maximum damping ratio.

P5.10

The negative of the loop transmission for the lag-compensated amplifier is

$$a(s)f(s) = \frac{5 \times 10^4(\tau s + 1)}{(s + 1)(10^{-4}s + 1)(10^{-5}s + 1)(\alpha\tau s + 1)}$$

It was shown in Section 5.2.4 that reasonable stability results for $\alpha = 6.2$ and a value of τ that locates the lag-function zero a factor of 10 below crossover. Use root contours to evaluate stability as a function of the zero location $(1/\tau)$ for $\alpha = 6.2$. The note concerning the advisability of machine computation mentioned in Problem P5.9 applies to this calculation as well.

P5.11

Determine the first three error coefficients for the four loop transmissions of the gain-of-ten amplifier described by Eqns. 5.16 through 5.19. Assume that the lead compensation is obtained in the feedback path (see Section 5.2.4) while all other compensations can be considered to be located in the forward path.

P5.12

A feedback system includes a factor

$$\frac{(s^2/12) - (s/2) + 1}{(s^2/12) + (s/2) + 1}$$

in its loop transmission.

Assume that you have complete freedom in the choice of d-c loop-transmission magnitude and the selection of additional singularities in the loop transmission. Determine the type of compensation that will maximize the desensitivity of this system.

P5.13

Calculate the settling time (to 1% of final value for a step input) for the gain-of-ten amplifier with lag compensation (Eqn. 5.15). Contrast this value with that of a first-order system with an identical crossover frequency.

P5.14

A model for an operational amplifier using minor-loop compensation is shown in block-diagram form in Fig. 5.27.

(a) Assume that the series compensating element has a transfer function $a_c(s) = 1$. Find values for b and τ such that a major loop formed by feeding V_o directly back to V_i will have a crossover frequency of 10^3 radians per second, approximately 55° of phase margin, and maximum desensitivity at frequencies below crossover subject to these constraints.

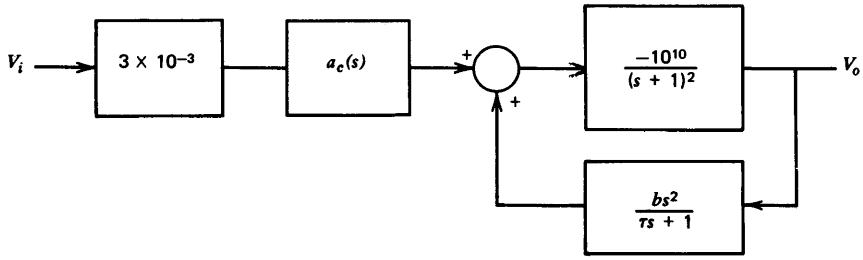


Figure 5.27 Operational-amplifier model.

Draw an open-loop Bode plot for the amplifier with these values for b and τ.

(b) Now assume that $b = 0$. Can you find a value for $a_c(s)$ that results in the same asymptotic open-loop magnitude characteristics as you obtained in part a, subject to the constraint that $|a_c(j\omega)| \leq 1$ for all ω?

P5.15

This problem includes a laboratory portion that can be performed with commonly available test equipment and that will give you experience compensating a system with well-defined dynamics. The experimental vehicle is the circuit shown in Fig. 5.28, which gives quite repeatable operational-amplifier-like characteristics. The suggested experiments use the configuration at relatively low frequencies, so that the inevitable stray circuit elements have little effect on the measured performance.

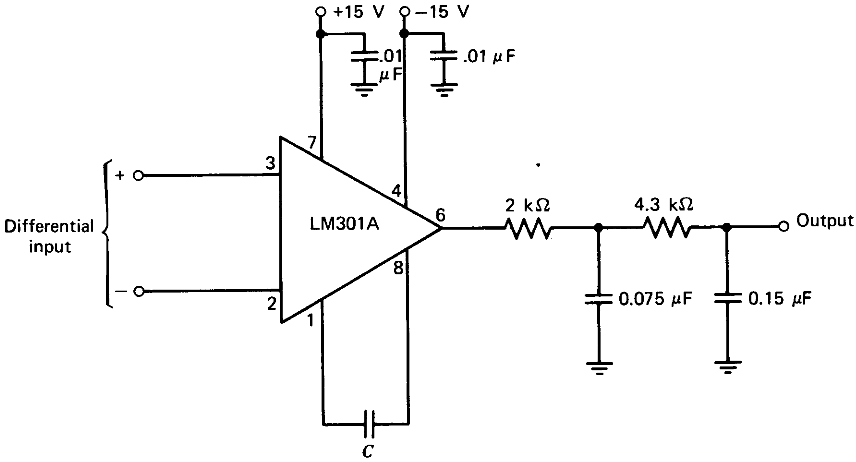
The dynamics of the circuit should first be standardized. Connect it as an inverting amplifier as shown in Fig. 5.29.

Select the capacitor C connected between pins 1 and 8 of the LM301A so that the configuration is just on the verge of instability. An estimated value should be around 5000 pF. Please remember that the amplifier reacts very poorly (usually by dying) if pins 1 or 5 are shorted to almost any potential.

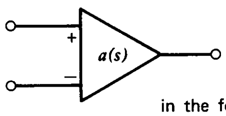
Note. The assumptions required for linear analysis are severely compromised if the peak-to-peak magnitude of the input signal exceeds approximately 50 mV. It is also necessary to have the driving source impedance low in this and other connections. A resistive divider attenuating the signal-generator output and located close to the amplifier is suggested.

After this standardization, it is claimed that if the loads applied to the amplifier are much higher than the output impedance of the network involving the 0.15 μF capacitor, etc., we can approximate $a(s)$ as

$$a(s) \simeq \frac{5 \times 10^4}{(s+1)(10^{-3}s+1)(10^{-4}s+1)}$$



Note. This complete circuit will be denoted as



in the following figures.

Figure 5.28 Amplifier with controlled dynamics. Pin numbers are for TO-99 and minidip packages.

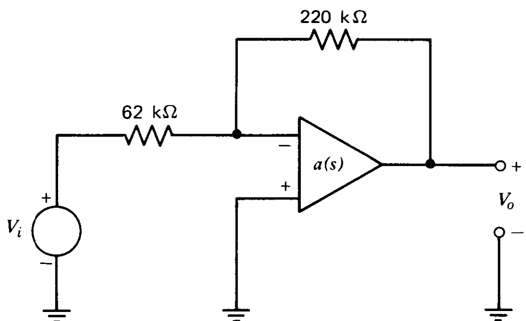


Figure 5.29 Inverting configuration.

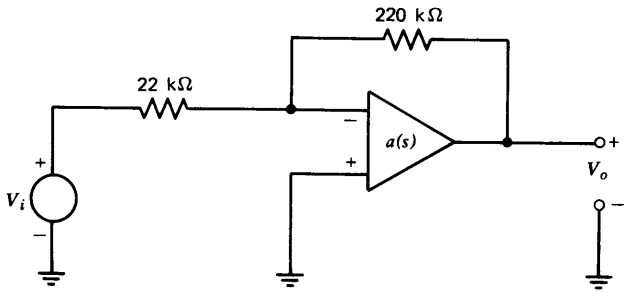


Figure 5.30 Inverting gain-of-ten amplifier.

for purposes of stability analysis. This transfer function is not unique and, in general, functions of the form

$$a(s) = \frac{5 \times 10^4 \tau}{(\tau s + 1)(10^{-3}s + 1)(10^{-4}s + 1)}$$

will yield equivalent results in your analysis providing $\tau \gg 10^{-3}$ seconds.

Supply a convincing argument why the above family of transfer functions properly represents the operational amplifier that you have just brought to the verge of oscillation. Note that simply showing the two given expressions are equivalent is *not* sufficient. You must show why they can be used to analyze the standardized circuit.

Use a Bode plot to determine the phase margin of the connection shown in Fig. 5.30 when the standardized amplifier is used. Predict a value for M_p based on the phase margin, and compare your prediction with measured results.

You are to compensate the system to improve its phase margin to 60° by reducing $a_0 f_0$ and by using lag and lead compensating techniques. You may not change the value of C or elements in the network connected to the output of the LM301A, nor load the network unreasonably to implement compensation.

Analytically determine the topology and element values you will use for each of the three forms of compensation. It may not be possible to meet the phase-margin objective using lead compensation alone; if you find this to be the case, you may reduce $a_0 f_0$ slightly so that the design goal can be achieved.

Compensate the amplifier in the laboratory and convince yourself that the step responses you measure are reasonable for systems with 60° of phase margin. Also correlate the rise times of the responses with your predicted values for crossover frequencies.