# CHAPTER II

# PROPERTIES AND MODELING OF FEEDBACK SYSTEMS

## 2.1 INTRODUCTION

A *control system* is a system that regulates an output variable with the objective of producing a given relationship between it and an input variable or of maintaining the output at a fixed value. In a feedback control system, at least part of the information used to change the output variable is derived from measurements performed on the output variable itself. This type of *closed-loop* control is often used in preference to *open-loop* control (where the system does not use output-variable information to influence its output) since feedback can reduce the sensitivity of the system to externally applied disturbances and to changes in system parameters. Familiar examples of feedback control systems include residential heating systems, most high-fidelity audio amplifiers, and the iris-retina combination that regulates light entering the eye.

There are a variety of textbooks[1] available that provide detailed treatment on servomechanisms, or feedback control systems where at least one of the variables is a mechanical quantity. The emphasis in this presentation is on feedback amplifiers in general, with particular attention given to feedback connections which include operational amplifiers.

The operational amplifier is a component that is used almost exclusively in feedback connections; therefore a detailed knowledge of the behavior of feedback systems is necessary to obtain maximum performance from these amplifiers. For example, the open-loop transfer function of many operational amplifiers can be easily and predictably modified by means of external

[1] G. S. Brown and D. P. Cambell, *Principles of Servomechanisms*, Wiley, New York, 1948; J. G. Truxal, *Automatic Feedback Control System Synthesis*, McGraw-Hill, New York, 1955; H. Chestnut and R. W. Mayer, *Servomechanisms and Regulating System Design*, Vol. 1, 2nd Ed., Wiley, New York, 1959; R. N. Clark, *Introduction to Automatic Control Systems*, Wiley, New York, 1962; J. J. D'Azzo and C. H. Houpis, *Feedback Control System Analysis and Synthesis*, 2nd Ed., McGraw-Hill, New York, 1966; B. C. Kuo, *Automatic Control Systems*, 2nd Ed., Prentice-Hall, Englewood Cliffs, New Jersey, 1967; K. Ogata, *Modern Control Engineering*, Prentice-Hall, Englewood Cliffs, New Jersey, 1970.

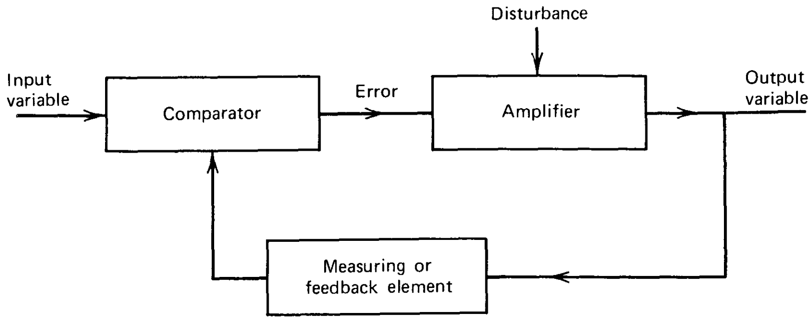



**Figure 2.1** A typical feedback system.

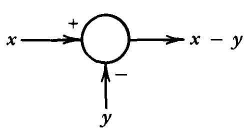
components. The choice of the open-loop transfer function used for a particular application must be based on feedback principles.

## 2.2 SYMBOLOGY

Elements common to many electronic feedback systems are shown in Fig. 2.1. The input signal is applied directly to a comparator. The output signal is determined and possibly operated upon by a feedback element. The difference between the input signal and the modified output signal is determined by the comparator and is a measure of the error or amount by which the output differs from its desired value. An amplifier drives the output in such a way as to reduce the magnitude of the error signal. The system output may also be influenced by disturbances that affect the amplifier or other elements.

We shall find it convenient to illustrate the relationships among variables in a feedback connection, such as that shown in Fig. 2.1, by means of *block diagrams*. A block diagram includes three types of elements.

1. A *line* represents a variable, with an arrow on the line indicating the direction of information flow. A line may split, indicating that a single variable is supplied to two or more portions of the system.
2. A *block* operates on an input supplied to it to provide an output.
3. Variables are added algebraically at a summation point drawn as follows:



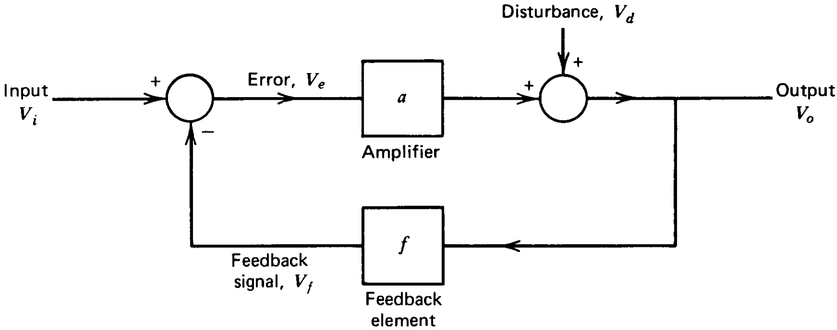



**Figure 2.2** Block diagram for the system of Fig. 2.1.

One possible representation for the system of Fig. 2.1, assuming that the input, output, and disturbance are voltages, is shown in block-diagram form in Fig. 2.2. (The voltages are all assumed to be measured with respect to references or grounds that are not shown.) The block diagram implies a specific set of relationships among system variables, including:

1. The error is the difference between the input signal and the feedback signal, or $V_e = V_i - V_f$.
2. The output is the sum of the disturbance and the amplified error signal, or $V_o = V_d + aV_e$.
3. The feedback signal is obtained by operating on the output signal with the feedback element, or $V_f = fV_o$.

The three relationships can be combined and solved for the output in terms of the input and the disturbance, yielding

$$V_o = \frac{aV_i}{1 + af} + \frac{V_d}{1 + af} \tag{2.1}$$

## 2.3 ADVANTAGES OF FEEDBACK

There is a frequent tendency on the part of the uninitiated to associate almost magical properties to feedback. Closer examination shows that many assumed benefits of feedback are illusory. The principal advantage is that feedback enables us to reduce the sensitivity of a system to changes in gain of certain elements. This reduction in sensitivity is obtained only in exchange for an increase in the magnitude of the gain of one or more of the elements in the system.

In some cases it is also possible to reduce the effects of disturbances applied to the system. We shall see that this moderation can always, at least conceptually, be accomplished without feedback, although the feedback approach is frequently a more practical solution. The limitations of this technique preclude reduction of such quantities as noise or drift at the input of an amplifier; thus feedback does not provide a method for detecting signals that cannot be detected by other means.

Feedback provides a convenient method of modifying the input and output impedance of amplifiers, although as with disturbance reduction, it is at least conceptually possible to obtain similar results without feedback.

### 2.3.1 Effect of Feedback on Changes in Open-Loop Gain

As mentioned above, the principal advantage of feedback systems compared with open-loop systems is that feedback provides a method for reducing the sensitivity of the system to changes in the gain of certain elements. This advantage can be illustrated using the block diagram of Fig. 2.2. If the disturbance is assumed to be zero, the closed-loop gain for the system is

$$\frac{V_o}{V_i} = \frac{a}{1 + af} \triangleq A \tag{2.2}$$

(We will frequently use the capital letter $A$ to denote closed-loop gain, while the lower-case $a$ is normally reserved for a forward-path gain.)

The quantity $af$ is the negative of the *loop transmission* for this system. The loop transmission is determined by setting all external inputs (and disturbances) to zero, breaking the system at any point inside the loop, and determining the ratio of the signal returned by the system to an applied test input.[2] If the system is a *negative feedback system*, the loop transmission is negative. The negative sign on the summing point input that is included in the loop shown in Fig. 2.2 indicates that the feedback is negative for this system if $a$ and $f$ have the same sign. Alternatively, the inversion necessary for negative feedback might be supplied by either the amplifier or the feedback element.

Equation 2.2 shows that negative feedback lowers the magnitude of the gain of an amplifier since as $f$ is increased from zero, the magnitude of the closed-loop gain decreases if $a$ and $f$ have this same sign. The result is general and can be used as a test for negative feedback.

It is also possible to design systems with positive feedback. Such systems are not as useful for our purposes and are not considered in detail.

The closed-loop gain expression shows that as the loop-transmission magnitude becomes large compared to unity, the closed-loop gain ap-

[2] An example of this type of calculation is given in Section 2.4.1.

proaches the value $1/f$. The significance of this relationship is as follows. The amplifier will normally include active elements whose characteristics vary as a function of age and operating conditions. This uncertainty may be unavoidable in that active elements are not available with the stability required for a given application, or it may be introduced as a compromise in return for economic or other advantages.

Conversely, the feedback network normally attenuates signals, and thus can frequently be constructed using only passive components. Fortunately, passive components with stable, precisely known values are readily available. If the magnitude of the loop transmission is sufficiently high, the closed-loop gain becomes dependent primarily on the characteristics of the feedback network.

This feature can be emphasized by calculating the fractional change in closed-loop gain $d(V_o/V_i)/(V_o/V_i)$ caused by a given fractional change in amplifier forward-path gain $da/a$, with the result

$$\frac{d(V_o/V_i)}{(V_o/V_i)} = \frac{da}{a}\left(\frac{1}{1 + af}\right) \tag{2.3}$$

Equation 2.3 shows that changes in the magnitude of $a$ can be attenuated to insignificant levels if $af$ is sufficiently large. The quantity $1 + af$ that relates changes in forward-path gain to changes in closed-loop gain is frequently called the *desensitivity* of a feedback system. Figure 2.3 illustrates this desensitization process by comparing two amplifier connections intended to give an input-output gain of 10. Clearly the input-output gain is identically equal to $a$ in Fig. 2.3*a*, and thus has the same fractional change in gain as does $a$. Equations 2.2 and 2.3 show that the closed-loop gain for the system of Fig. 2.3*b* is approximately 9.9, and that the fractional change in closed-loop gain is less than 1% of the fractional change in the forward-path gain of this system.

The desensitivity characteristic of the feedback process is obtained only in exchange for excess gain provided in the system. Returning to the example involving Fig. 2.2, we see that the closed-loop gain for the system is $a/(1 + af)$, while the forward-path gain provided by the amplifier is $a$. The desensitivity is identically equal to the ratio of the forward-path gain to closed-loop gain. Feedback connections are unique in their ability to automatically trade excess gain for desensitivity.

It is important to underline the fact that changes in the gain of the feedback element have direct influence on the closed-loop gain of the system, and we therefore conclude that it is necessary to observe or measure the output variable of a feedback system accurately in order to realize the advantages of feedback.

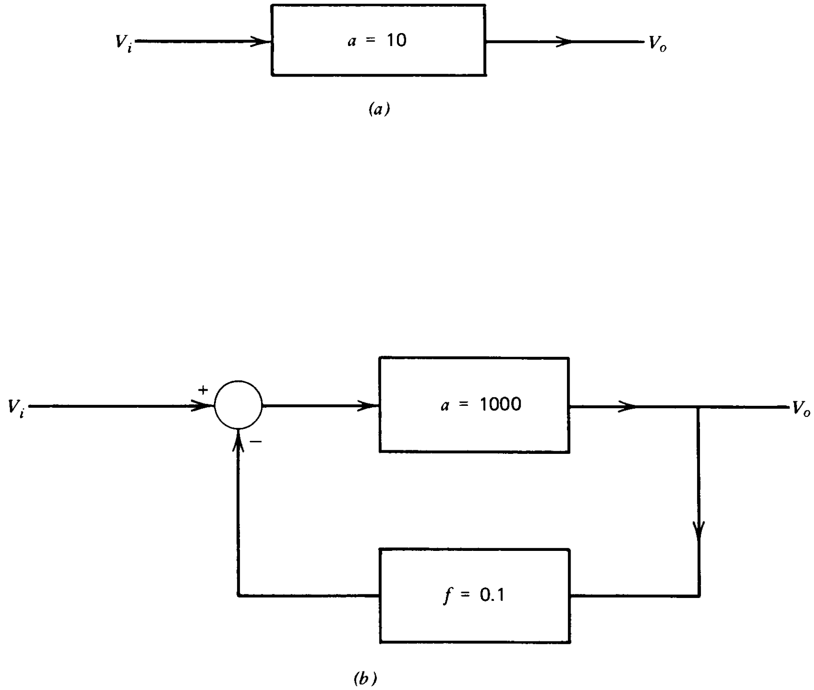



**Figure 2.3** Amplifier connections for a gain of ten. (*a*) Open loop. (*b*) Closed loop.

### 2.3.2 Effect of Feedback on Nonlinearities

Because feedback reduces the sensitivity of a system to changes in open-loop gain, it can often moderate the effects of nonlinearities. Figure 2.4 illustrates this process. The forward path in this connection consists of an amplifier with a gain of 1000 followed by a nonlinear element that might be an idealized representation of the transfer characteristics of a power output stage. The transfer characteristics of the nonlinear element show these four distinct regions:

1. A deadzone, where the output remains zero until the input magnitude exceeds 1 volt. This region models the crossover distortion associated with many types of power amplifiers.
2. A linear region, where the incremental gain of the element is one.
3. A region of soft limiting, where the incremental gain of the element is lowered to 0.1.

4. A region of hard limiting or saturation where the incremental gain of the element is zero.

The performance of the system can be determined by recognizing that, since the nonlinear element is piecewise linear, all transfer relationships must be piecewise linear. The values of all the variables at a breakpoint can be found by an iterative process. Assume, for example, that the variables associated with the nonlinear element are such that this element is at its breakpoint connecting a slope of zero to a slope of +1. This condition only occurs for $v_A = 1$ and $v_B = 0$. If $v_B = v_O = 0$, the signal $v_F$ must be zero, since $v_F = 0.1\ v_O$. Similarly, with $v_A = 1$, $v_E = 10^{-3}v_A = 10^{-3}$. Since the relationships at the summing point imply $v_E = v_I - v_F$, or $v_I = v_E + v_F$, $v_I$ must equal $10^{-3}$. The values of variables at all other breakpoints can be found by similar reasoning. Results are summarized in Table 2.1.

**Table 2.1** Values of Variables at Breakpoints for System of Fig. 2.4

| $v_I$ | $v_E = v_I - v_F$ | $v_A = 10^3 v_E$ | $v_B = v_O$ | $v_F = 0.1 v_O$ |
|---|---|---|---|---|
| $< -0.258$ | $v_I + 0.250$ | $10^3 v_I + 250$ | $-2.5$ | $-0.25$ |
| $-0.258$ | $-0.008$ | $-8$ | $-2.5$ | $-0.25$ |
| $-0.203$ | $-0.003$ | $-3$ | $-2$ | $-0.2$ |
| $-10^{-3}$ | $-10^{-3}$ | $-1$ | 0 | 0 |
| $10^{-3}$ | $10^{-3}$ | 1 | 0 | 0 |
| 0.203 | 0.003 | 3 | 2 | 0.2 |
| 0.258 | 0.008 | 8 | 2.5 | 0.25 |
| $> 0.258$ | $v_I - 0.250$ | $10^3 v_I - 250$ | 2.5 | 0.25 |

The input-output transfer relationship for the system shown in Fig. 2.4*c* is generated from values included in Table 2.1. The transfer relationship can also be found by using the incremental forward gain, or 1000 times the incremental gain of the nonlinear element, as the value for $a$ in Eqn. 2.2. If the magnitude of signal $v_A$ is less than 1 volt, $a$ is zero, and the incremental closed-loop gain of the system is also zero. If $v_A$ is between 1 and 3 volts, $a$ is $10^3$, so the incremental closed-loop gain is 9.9. Similarly, the incremental closed-loop gain is 9.1 for $3 < v_A < 8$.

Note from Fig. 2.4*c* that feedback dramatically reduces the width of the deadzone and the change in gain as the output stage soft limits. Once the amplifier saturates, the incremental loop transmission becomes zero, and as a result feedback cannot improve performance in this region.

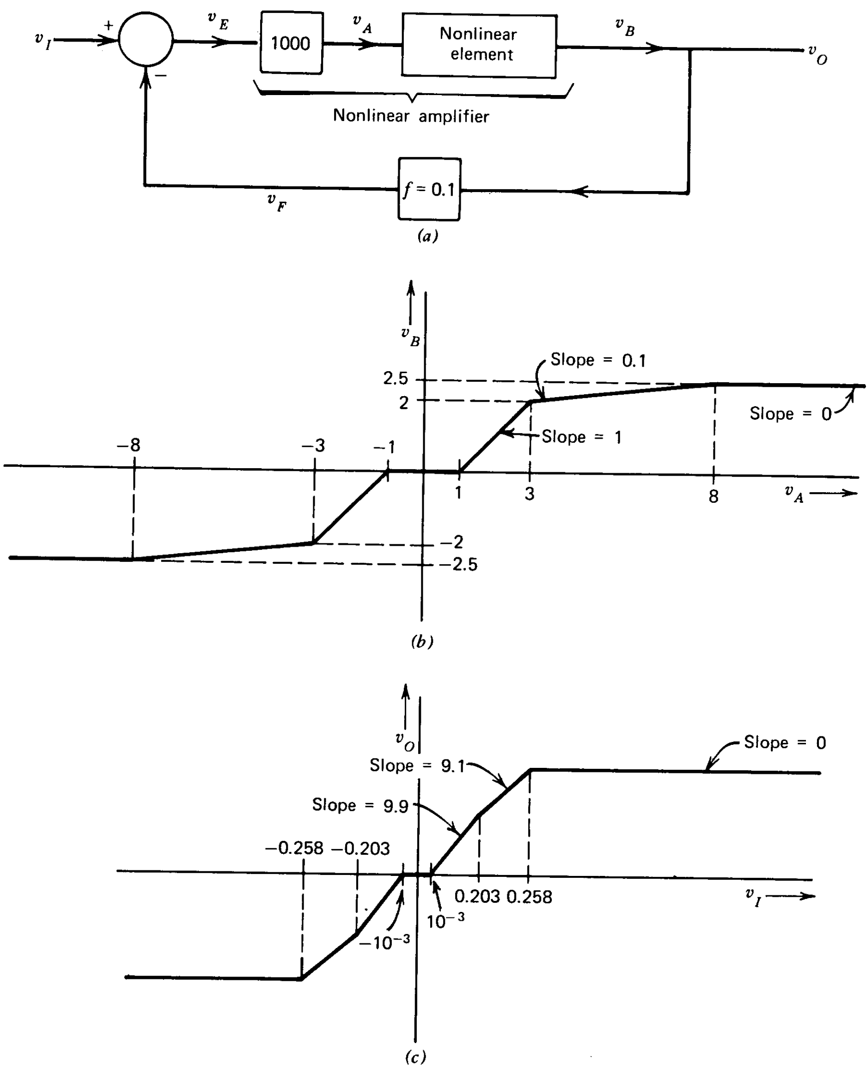
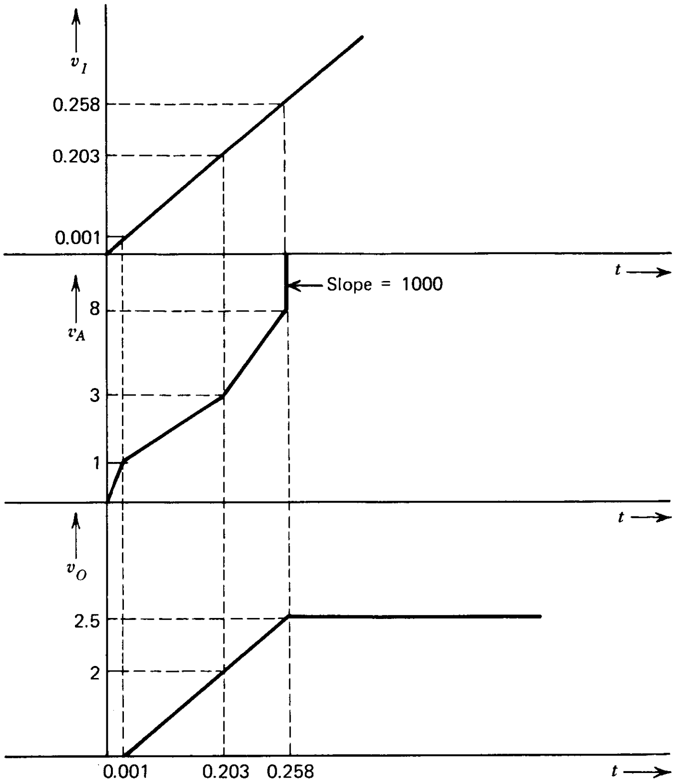



**Figure 2.4** The effects of feedback on a nonlinearity. (*a*) System. (*b*) Transfer characteristics of the nonlinear element. (*c*) System transfer characteristics (closed loop). (Not to scale.) (*d*) Waveforms for $v_I(t)$ a unit ramp. (Not to scale.)




(d)

**Figure 2.4**—Continued

Figure 2.4*d* provides insight into the operation of the circuit by comparing the output of the system and the voltage $v_A$ for a unit ramp input. The output remains a good approximation to the input until saturation is reached. The signal into the nonlinear element is "predistorted" by feedback in such a way as to force the output from this element to be nearly linear.

The technique of employing feedback to reduce the effects of nonlinear elements on system performance is a powerful and widely used method that evolves directly from the desensitivity to gain changes provided by feedback. In some applications, feedback is used to counteract the unavoidable nonlinearities associated with active elements. In other applications, feedback is used to maintain performance when nonlinearities result from economic compromises. Consider the power amplifier that provided the motivation for the previous example. The designs for linear power-handling stages are complex and expensive because compensation for the base-to-emitter voltages of the transistors and variations of gain with operating point must be included. Economic advantages normally result if linearity of the power-handling stage is reduced and low-power voltage-gain stages (possibly in the form of an operational amplifier) are added prior to the output stage so that feedback can be used to restore system linearity.

While this section has highlighted the use of feedback to reduce the effects of nonlinearities associated with the forward-gain element of a system, feedback can also be used to produce nonlinearities with well-controlled characteristics. If the feedback element in a system with large loop transmission is nonlinear, the output of the system becomes approximately $v_O = f^{-1}(v_I)$. Here $f^{-1}$ is the inverse of the feedback-element transfer relationship, in the sense that $f^{-1}[f(V)] = V$. For example, transistors or diodes with exponential characteristics can be used as feedback elements around an operational amplifier to provide a logarithmic closed-loop transfer relationship.

### 2.3.3 Disturbances in Feedback Systems

Feedback provides a method for reducing the sensitivity of a system to certain kinds of disturbances. This advantage is illustrated in Fig. 2.5. Three different sources of disturbances are applied to this system. The disturbance $V_{d1}$ enters the system at the same point as the system input, and might represent the noise associated with the input stage of an amplifier. Disturbance $V_{d2}$ enters the system at an intermediate point, and might represent a disturbance from the hum associated with the poorly filtered voltage often used to power an amplifier output stage. Disturbance $V_{d3}$ enters at the amplifier output and might represent changing load characteristics.

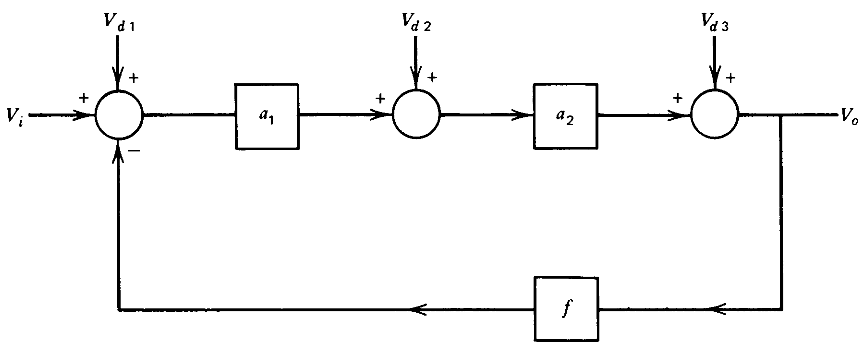



**Figure 2.5** Feedback system illustrating effects of disturbances.

The reader should convince himself that the block diagram of Fig. 2.5 implies that the output voltage is related to input and disturbances as

$$V_o = \frac{a_1 a_2[(V_i + V_{d1}) + (V_{d2}/a_1) + (V_{d3}/a_1 a_2)]}{1 + a_1 a_2 f} \tag{2.4}$$

Equation 2.4 shows that the disturbance $V_{d1}$ is not attenuated relative to the input signal. This result is expected since $V_i$ and $V_{d1}$ enter the system at the same point, and reflects the fact that feedback cannot improve quantities such as the noise figure of an amplifier. The disturbances that enter the amplifier at other points are attenuated relative to the input signal by amounts equal to the forward-path gains between the input and the points where the disturbances are applied.

It is important to emphasize that the forward-path gain preceding the disturbance, rather than the feedback, results in the relative attenuation of the disturbance. This feature is illustrated in Fig. 2.6. This open-loop system, which follows the forward path of Fig. 2.5 with an attenuator, yields the same output as the feedback system of Fig. 2.5. The feedback system is nearly always the more practical approach, since the open-loop system requires large signals, with attendant problems of saturation and power dissipation, at the input to the attenuator. Conversely, the feedback realization constrains system variables to more realistic levels.

### 2.3.4 Summary

This section has shown how feedback can be used to desensitize a system to changes in component values or to externally applied disturbances. This desensitivity can only be obtained in return for increases in the gains of various components of the system. There are numerous situations where this type of trade is advantageous. For example, it may be possible to replace a costly, linear output stage in a high-fidelity audio amplifier with a cheaper unit and compensate for this change by adding an inexpensive stage of low-level amplification.

The input and output impedances of amplifiers are also modified by feedback. For example, if the output variable that is fed back is a voltage, the

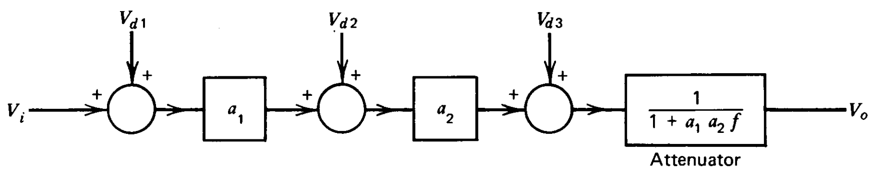



**Figure 2.6** Open-loop system illustrating effects of disturbances.

feedback tends to stabilize the value of this voltage and reduce its dependence on disturbing load currents, implying that the feedback results in lower output impedance. Alternatively, if the information fed back is proportional to output current, the feedback raises the output impedance. Similarly, feedback can limit input voltage or current applied to an amplifier, resulting in low or high input impedance respectively. A quantitative discussion of this effect is reserved for Section 2.5.

A word of caution is in order to moderate the impression that performance improvements always accompany increases in loop-transmission magnitude. Unfortunately, the loop transmission of a system cannot be increased without limit, since sufficiently high gain invariably causes a system to become *unstable*. A *stable* system is defined as one for which a bounded output is produced in response to a bounded input. Conversely, an unstable system exhibits runaway or oscillatory behavior in response to a bounded input. Instability occurs in high-gain systems because small errors give rise to large corrective action. The propagation of signals around the loop is delayed by the dynamics of the elements in the loop, and as a consequence high-gain systems tend to overcorrect. When this overcorrection produces an error larger than the initiating error, the system is unstable. This important aspect of the feedback problem did not appear in this section since the dynamics associated with various elements have been ignored. The problem of stability will be investigated in detail in Chapter 4.

## 2.4 BLOCK DIAGRAMS

A block diagram is a graphical method of representing the relationships among variables in a system. The symbols used to form a block diagram were introduced in Section 2.2. Advantages of this representation include the insight into system operation that it often provides, its clear indication of various feedback loops, and the simplification it affords to determining the transfer functions that relate input and output variables of the system. The discussion in this section is limited to linear, time-invariant systems, with the enumeration of certain techniques useful for the analysis of nonlinear systems reserved for Chapter 6.

### 2.4.1 Forming the Block Diagram

Just as there are many complete sets of equations that can be written to describe the relationships among variables in a system, so there are many possible block diagrams that can be used to represent a particular system. The choice of block diagram should be made on the basis of the insight it lends to operation and the ease with which required transfer functions can be evaluated. The following systematic method is useful for circuits where all variables of interest are node voltages.

1. Determine the node voltages of interest. The selected number of voltages does not have to be equal to the total number of nodes in the circuit, but it must be possible to write a complete, independent set of equations using the selected voltages. One line (which may split into two or more branches in the final block diagram) will represent each of these variables, and these lines may be drawn as isolated segments.

2. Determine each of the selected node voltages as a weighted sum of the other selected voltages and any inputs or disturbances that may be applied to the circuit. This determination requires a set of equations of the form

$$V_j = \sum_{n \neq j} a_{nj} V_n + \sum_m b_{mj} E_m \tag{2.5}$$

where $V_k$ is the $k$th node voltage and $E_k$ is the $k$th input or disturbance.

3. The variable $V_j$ is generated as the output of a summing point in the block diagram. The inputs to the summing point come from all other variables, inputs, and disturbances via blocks with transmissions that are the $a$'s and $b$'s in Eqn. 2.5. Some of the blocks may have transmissions of zero, and these blocks and corresponding summing-point inputs can be eliminated.

The set of equations required in Step 2 can be determined by writing node equations for the complete circuit and solving the equation written about the $j$th node for $V_j$ in terms of all other variables. If a certain node voltage $V_k$ is not required in the final block diagram, the equation relating $V_k$ to other system voltages is used to eliminate $V_k$ from all other members of the set of equations. While this degree of formality is often unnecessary, it always yields a correct block diagram, and should be used if the desired diagram cannot easily be obtained by other methods.

As an example of block diagram construction by this formal approach, consider the common-emitter amplifier shown in Fig. 2.7*a*. (Elements used for bias have been eliminated for simplicity.) The corresponding small-signal equivalent circuit is obtained by substituting a hybrid-pi[3] model for the transistor and is shown in Fig. 2.7*b*. Node equations are[4]

[3] The hybrid-pi model will be used exclusively for the analysis of bipolar transistors operating in the linear region. The reader who is unfamiliar with the development or use of this model is referred to P. E. Gray and C. L. Searle, *Electronic Principles: Physics, Models, and Circuits*, Wiley, New York, 1969.

[4] $G$'s and $R$'s (or $g$'s and $r$'s) are used to identify corresponding conductances and resistances, while $Y$'s and $Z$'s (or $y$'s and $z$'s) are used to identify corresponding admittances and impedances. Thus for example, $G_A = 1/R_A$ and $z_b = 1/y_b$.

$$
\begin{aligned}
G_S V_i &= (G_S + g_x) V_a - g_x V_b \\
0 &= -g_x V_a + [(g_x + g_\pi) + (C_\mu + C_\pi)s] V_b - C_\mu s V_o \\
0 &= (g_m - C_\mu s) V_b + (G_L + C_\mu s) V_o
\end{aligned}
\tag{2.6}
$$

If the desired block diagram includes all three node voltages, Eqn. 2.6 is arranged so that each member of the set is solved for the voltage at the node about which the member was written. Thus,

$$
\begin{aligned}
V_a &= \frac{g_x}{g_a} V_b + \frac{G_S}{g_a} V_i \\
V_b &= \frac{g_x}{y_b} V_a + \frac{C_\mu s}{y_b} V_o \\
V_o &= \frac{(C_\mu s - g_m)}{y_o} V_b
\end{aligned}
\tag{2.7}
$$

Where

$$
\begin{aligned}
g_a &= G_S + g_x \\
y_b &= [(g_x + g_\pi) + (C_\mu + C_\pi)s] \\
y_o &= G_L + C_\mu s
\end{aligned}
$$

The block diagram shown in Fig. 2.7*c* follows directly from this set of equations.

Figure 2.8 is the basis for an example that is more typical of our intended use of block diagrams. A simple operational-amplifier model is shown connected as a noninverting amplifier. It is assumed that the variables of interest are the voltages $V_b$ and $V_o$. The voltage $V_o$ can be related to the other selected voltage, $V_b$, and the input voltage, $V_i$, by superposition.

with $V_i = 0$,

$$V_o = -aV_b \tag{2.8}$$

while with $V_b = 0$,

$$V_o = aV_i \tag{2.9}$$

The equation relating $V_o$ to other selected voltages and inputs is simply the superposition of the responses represented by Eqns. 2.8 and 2.9, or

$$V_o = aV_i - aV_b \tag{2.10}$$

The voltage $V_b$ is independent of $V_i$ and is related to $V_o$ as

$$V_b = \frac{Z_1}{Z_1 + Z_2} V_o \tag{2.11}$$

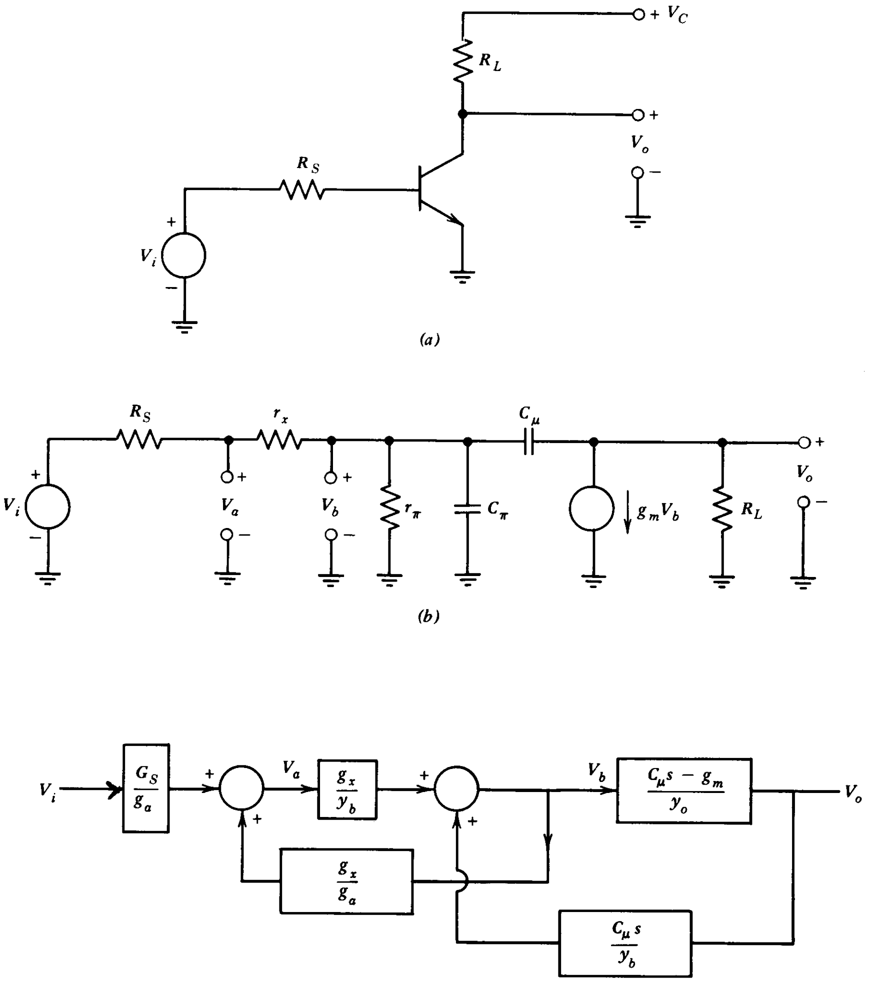



**Figure 2.7** Common-emitter amplifier. (*a*) Circuit. (*b*) Incremental equivalent circuit. (*c*) Block diagram.

Equations 2.10 and 2.11 are readily combined to form the block diagram shown in Fig. 2.8*b*.

It is possible to form a block diagram that provides somewhat greater insight into the operation of the circuit by replacing Eqn. 2.10 by the pair of equations

$$V_a = V_i - V_b \tag{2.12}$$

and

$$V_o = aV_a \tag{2.13}$$

Note that the original set of equations were not written including $V_a$, since $V_a$, $V_b$, and $V_i$ form a Kirchhoff loop and thus cannot all be included in an independent set of equations.

The alternate block diagram shown in Fig. 2.8*c* is obtained from Eqns. 2.11, 2.12, and 2.13. In this block diagram it is clear that the summing point models the function provided by the differential input of the operational amplifier. This same block diagram would have evolved had $V_a$ and $V_o$ been initially selected as the amplifier voltages of interest.

The loop transmission for any system represented as a block diagram can always be determined by setting all inputs and disturbances to zero, breaking the block diagram at any point inside the loop, and finding the signal returned by the loop in response to an applied test signal. One possible point to break the loop is illustrated in Fig. 2.8*c*. With $V_i = 0$, it is evident that

$$\frac{V_o}{V_t} = \frac{-aZ_1}{Z_1 + Z_2} \tag{2.14}$$

The same result is obtained for the loop transmission if the loop in Fig. 2.8*c* is broken elsewhere, or if the loop in Fig. 2.8*b* is broken at any point.

Figure 2.9 is the basis for a slightly more involved example. Here a fairly detailed operational-amplifier model, which includes input and output impedances, is shown connected as an inverting amplifier. A disturbing current generator is included, and this generator can be used to determine the closed-loop output impedance of the amplifier $V_o/I_d$.

It is assumed that the amplifier voltages of interest are $V_a$ and $V_o$. The equation relating $V_a$ to the other voltage of interest $V_o$, the input $V_i$, and the disturbance $I_d$, is obtained by superposition (allowing all other signals to be nonzero one at a time and superposing results) as in the preceding example. The reader should verify the results

$$V_a = \frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2} V_i + \frac{Z_i \parallel Z_1}{Z_2 + Z_i \parallel Z_1} V_o \tag{2.15}$$

and

$$V_o = \frac{-aZ_2 + Z_o}{Z_2 + Z_o} V_a + (Z_o \parallel Z_2)I_d \tag{2.16}$$

The block diagram of Fig. 2.9*b* follows directly from Eqns. 2.15 and 2.16.

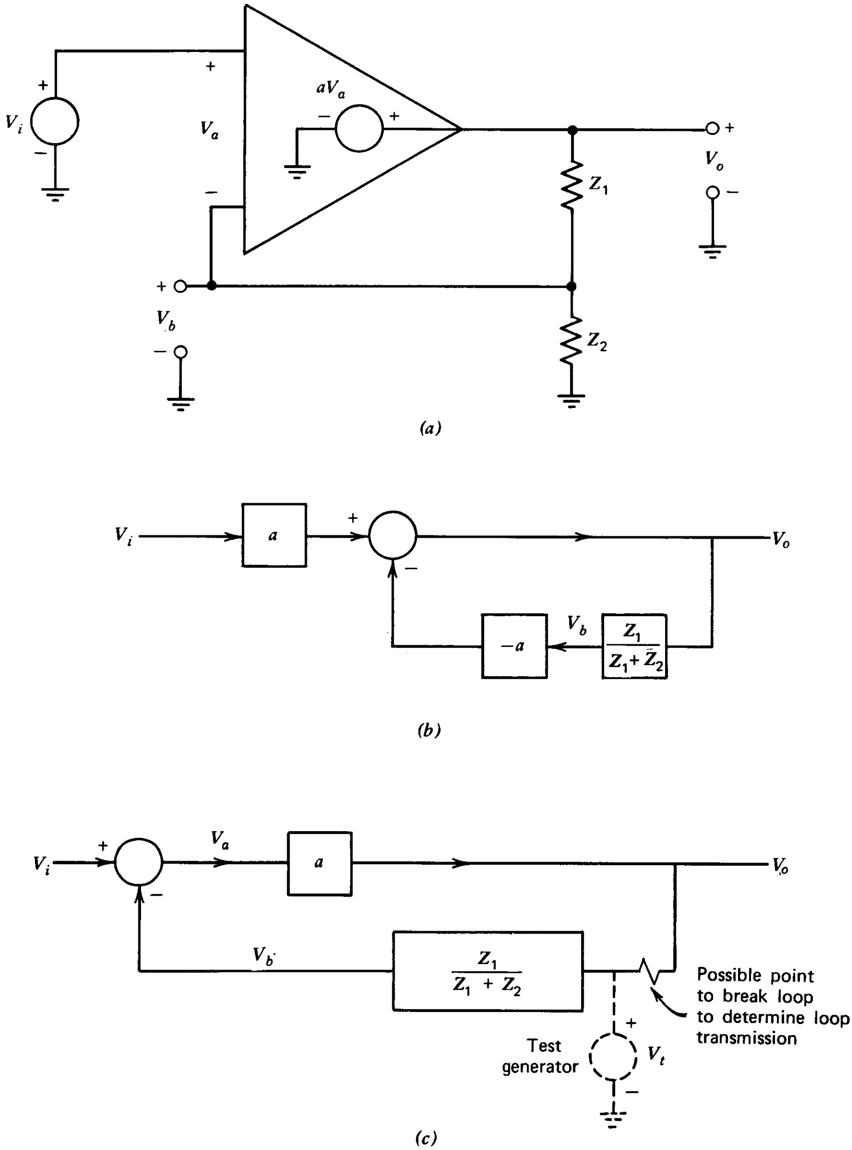
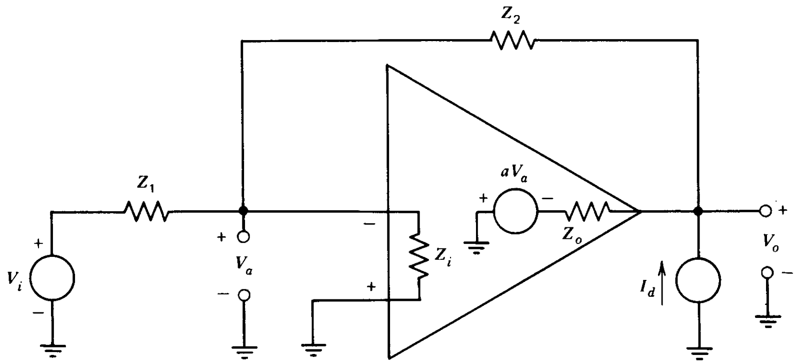
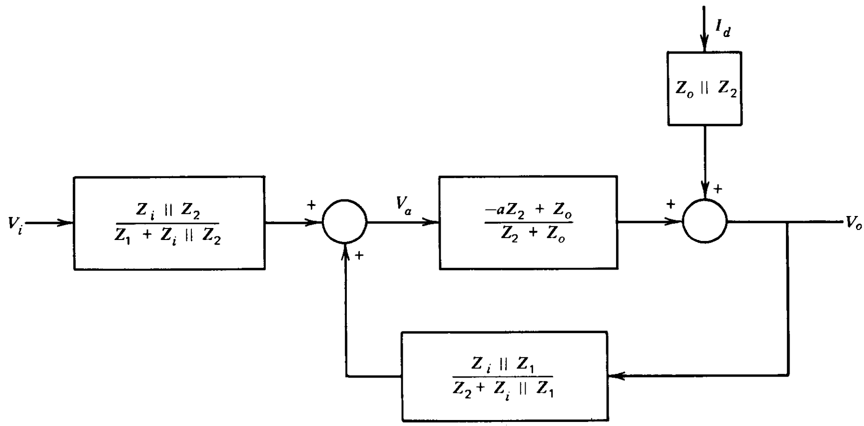



**Figure 2.8** Noninverting amplifier. (*a*) Circuit. (*b*) Block diagram. (*c*) Alternative block diagram.




(a)




(b)

**Figure 2.9** Inverting amplifier. (*a*) Circuit. (*b*) Block diagram.

### 2.4.2 Block-Diagram Manipulations

There are a number of ways that block diagrams can be restructured or reordered while maintaining the correct gain expression between an input or disturbance and an output. These modified block diagrams could be obtained directly by rearranging the equations used to form the block diagram or by using other system variables in the equations. Equivalences that can be used to modify block diagrams are shown in Fig. 2.10.

It is necessary to be able to find the transfer functions relating outputs to inputs and disturbances or the relations among other system variables from the block diagram of the system. These transfer functions can always be found by appropriately applying various equivalences of Fig. 2.10 until a single-loop system is obtained. The transfer function can then be determined by loop reduction (Fig. 2.10*h*). Alternatively, once the block diagram has been reduced to a single loop, important system quantities are evident. The loop transmission as well as the closed-loop gain approached for large loop-transmission magnitude can both be found by inspection.

Figure 2.11 illustrates the use of equivalences to reduce the block diagram of the common-emitter amplifier previously shown as Fig. 2.7*c*. Figure 2.11*a* is identical to Fig. 2.7*c*, with the exceptions that a line has been replaced with a unity-gain block (see Fig. 2.10*a*) and an intermediate variable $V_c$ has been defined. These changes clarify the transformation from Fig. 2.11*a* to 2.11*b*, which is made as follows. The transfer function from $V_c$ to $V_b$ is determined using the equivalance of Fig. 2.10*h*, recognizing that the feedback path for this loop is the product of the transfer functions of blocks 1 and 2. The transfer function $V_b/V_c$ is included in the remaining loop, and the transfer function of block 1 links $V_o$ to $V_b$.

The equivalences of Figs. 2.10*b* and 2.10*h* using the identification of transfer functions shown in Fig. 2.11*b* (unfortunately, as a diagram is reduced, the complexities of the transfer functions of residual blocks increase) are used to determine the overall transfer function indicated in Fig. 2.11*c*.

The inverting-amplifier connection (Fig. 2.9) is used as another example of block-diagram reduction. The transfer function relating $V_o$ to $V_i$ in Fig. 2.9*b* can be reduced to single-loop form by absorbing the left-hand block in this diagram (equivalence in Fig. 2.10*d*). Figure 2.12 shows the result of this absorption after simplifying the feedback path algebraically, eliminating the disturbing input, and using the equivalence of Fig. 2.10*e* to introduce an inversion at the summing point. The gain of this system approaches the reciprocal of the feedback path for large loop transmission; thus the ideal closed-loop gain is

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \tag{2.17}$$

The forward gain for this system is

$$\begin{aligned}\frac{V_o}{V_e} &= \left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right]\left[\frac{-aZ_2 + Z_o}{Z_2 + Z_o}\right] \\ &= \left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right]\left[\frac{-aZ_2}{Z_2 + Z_o}\right] + \left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right]\left[\frac{Z_o}{Z_2 + Z_o}\right]\end{aligned} \tag{2.18}$$

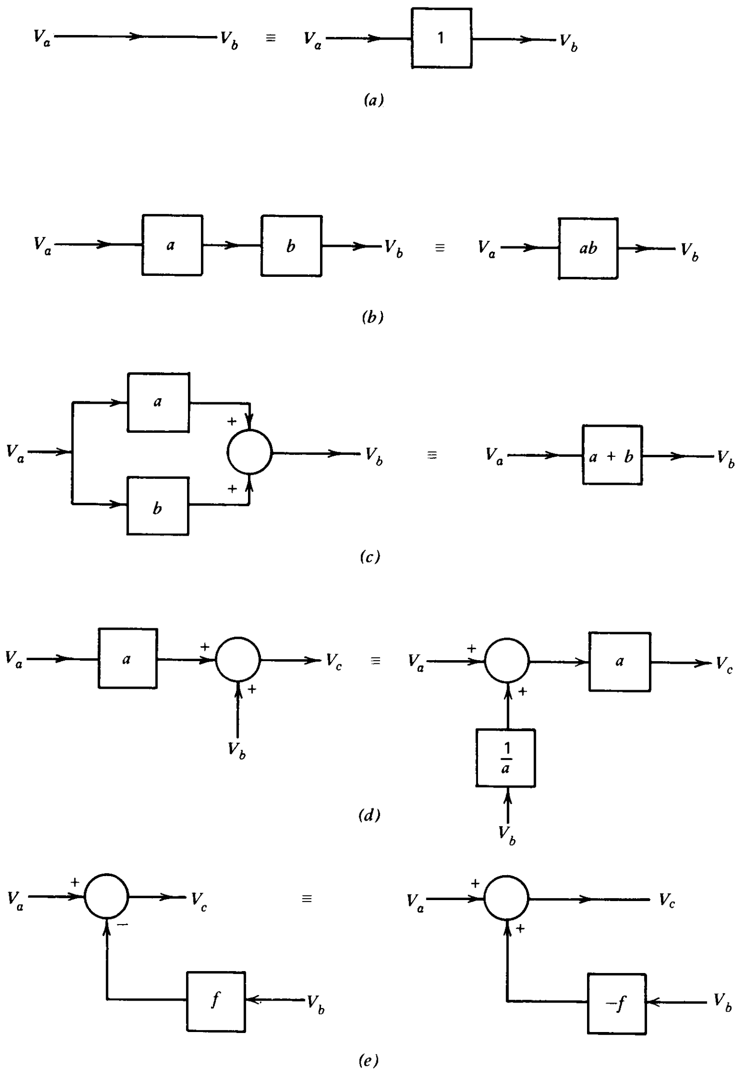



**Figure 2.10** Block-diagram equivalences. (*a*) Unity gain of line. (*b*) Cascading. (*c*) Summation. (*d*) Absorption. (*e*) Negation. (*f*) Branching. (*g*) Factoring. (*h*) Loop reduction.

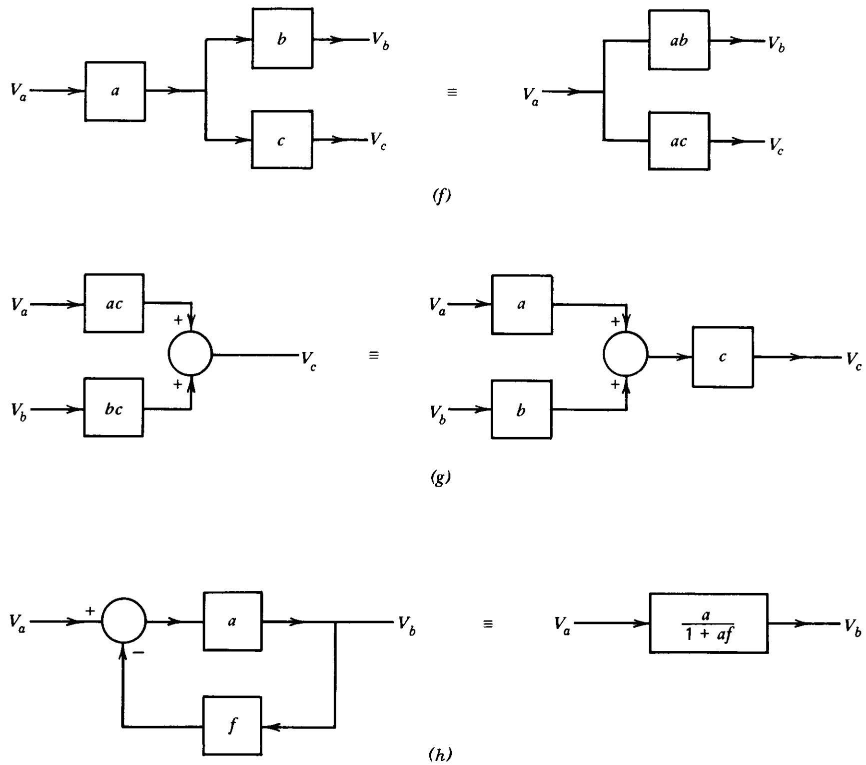



**Figure 2.10**—Continued

The final term on the right-hand side of Eqn. 2.18 reflects the fact that some fraction of the input signal is coupled directly to the output via the feedback network, even if the amplifier voltage gain $a$ is zero. Since the impedances included in this term are generally resistive or capacitive, the magnitude of this coupling term will be less than one at all frequencies. Similarly, the component of loop transmission attributable to this direct path, determined by setting $a = 0$ and opening the loop is

$$\left.\frac{V_f}{V_e}\right|_{a=0} = \left[\frac{Z_1}{Z_2}\right]\left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right]\left[\frac{Z_o}{Z_2 + Z_o}\right]$$

$$= \left[\frac{Z_i Z_1}{Z_i Z_1 + Z_i Z_2 + Z_1 Z_2}\right]\left[\frac{Z_o}{Z_2 + Z_o}\right] \quad (2.19)$$

and will be less than one in magnitude at all frequencies when the impedances involved are resistive or capacitive.

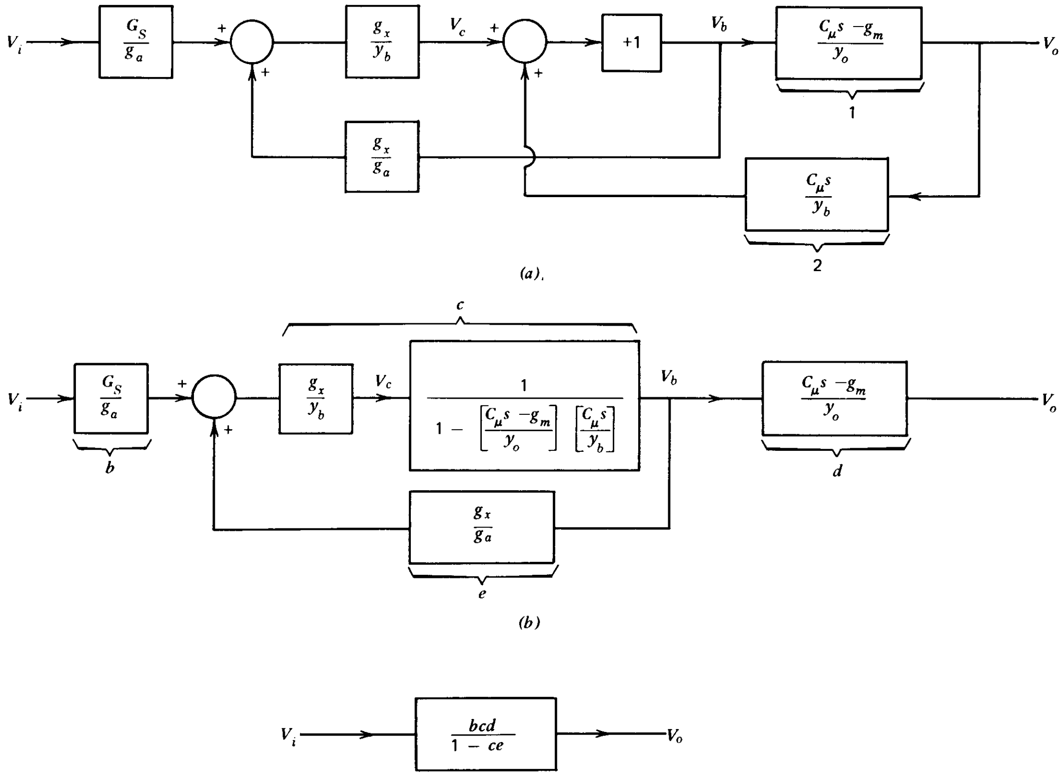



**Figure 2.11** Simplification of common-emitter block diagram. (*a*) Original block diagram. (*b*) After eliminating loop generating $V_b$. (*c*) Reduction to single block.

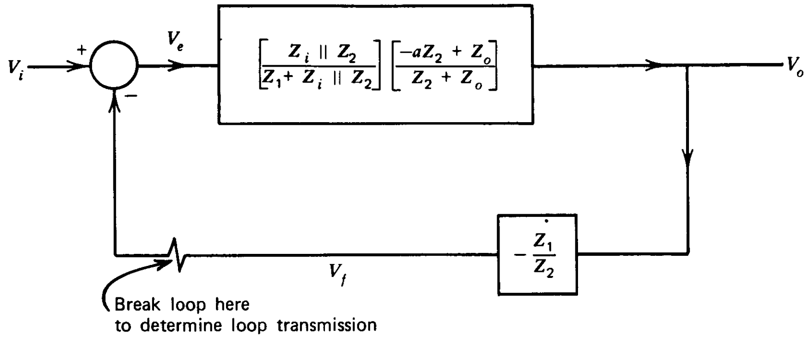



**Figure 2.12** Reduced diagram for inverting amplifier.

If the loop-transmission magnitude of the operational-amplifier connection is large compared to one, the component attributable to direct coupling through the feedback network (Eqn. 2.19) must be insignificant. Consequently, the forward-path gain of the system can be approximated as

$$\frac{V_o}{V_e} \simeq \left[\frac{-aZ_2}{Z_2 + Z_o}\right]\left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right] \tag{2.20}$$

in this case. The corresponding loop transmission becomes

$$\frac{V_f}{V_e} \simeq \left[\frac{-aZ_1}{Z_2 + Z_o}\right]\left[\frac{Z_i \parallel Z_2}{Z_1 + Z_i \parallel Z_2}\right] \tag{2.21}$$

It is frequently found that the loop-transmission term involving direct coupling through the feedback network can be neglected in practical operational-amplifier connections, reflecting the reasonable hypothesis that the dominant gain mechanism is the amplifier rather than the passive network. While this approximation normally yields excellent results at frequencies where the amplifier gain is large, there are systems where stability calculations are incorrect when the approximation is used. The reason is that stability depends largely on the behavior of the loop transmission at frequencies where its magnitude is close to one, and the gain of the amplifier may not dominate at these frequencies.

### 2.4.3 The Closed-Loop Gain

It is always possible to determine the gain that relates any signal in a block diagram to an input or a disturbance by manipulating the block diagram until a single path connects the two quantities of interest. Alternatively, it is possible to use a method developed by Mason[5] to calculate gains directly from an unreduced block diagram.

In order to determine the gain between an input or disturbance and any other points in the diagram, it is necessary to identify two topological features of a block diagram. A *path* is a continuous succession of blocks, lines, and summation points that connect the input and signal of interest and along which no element is encountered more than once. Lines may be traversed only in the direction of information flow (with the arrow). It is possible in general to have more than one path connecting an input to an output or other signal of interest. The *path gain* is a product of the gains of all elements in a path. A *loop* is a closed succession of blocks, lines, and summation points traversed with the arrows, along which no element is encountered more than once per cycle. The *loop gain* is the product of gains of all elements in a loop. It is necessary to include the inversions indicated by negative signs at summation points when calculating path or loop gains.

The general expression for the gain or transmission of a block diagram is

$$T = \frac{\sum_a P_a \left(1 - \sum_b L_b + \sum_{c,d} L_c L_d - \sum_{e,f,g} L_e L_f L_g + \cdots -\right)}{1 - \sum_h L_h + \sum_{i,j} L_i L_j - \sum_{k,l,m} L_k L_l L_m + \cdots -} \tag{2.22}$$

The numerator of the gain expression is the sum of the gains of all paths connecting the input and the signal of interest, with each path gain scaled by a *cofactor*. The first sum in a cofactor includes the gains of all loops that do not touch (share a common block or summation point with) the path; the second sum includes all possible products of loop gains for loops that do not touch the path or each other taken two at a time; the third sum includes all possible triple products of loop gains for loops that do not touch the path or each other; etc.

The denominator of the gain expression is called the *determinant* or *characteristic equation* of the block diagram, and is identically equal to one minus the loop transmission of the complete block diagram. The first sum in the characteristic equation includes all loop gains; the second all possible products of the gains of nontouching loops taken two at a time; etc.

Two examples will serve to clarify the evaluation of the gain expression. Figure 2.13 provides the first example. In order to apply Mason's gain formula for the transmission $V_o/V_i$, the paths and loops are identified and their gains are evaluated. The results are:

$$P_1 = ace$$

[5] S. J. Mason and H. J. Zimmermann, *Electronic Circuits, Signals, and Systems*, Wiley, New York, 1960, Chapter 4, "Linear Signal-Flow Graphs."

$$P_2 = ag$$
$$P_3 = -h$$
$$L_1 = -ab$$
$$L_2 = cd$$
$$L_3 = -ef$$
$$L_4 = -acei$$

The topology of Fig. 2.13 shows that path $P_1$ shares common blocks with and therefore touches all loops. Path $P_2$ does not touch loops $L_2$ or $L_3$, while path $P_3$ does not touch any loops. Similarly, loops $L_1$, $L_2$, and $L_3$ do not touch each other, but all touch loop $L_4$. Equation 2.22 evaluated for this system becomes

$$\frac{V_o}{V_i} = \frac{\begin{array}{l} P_1 + P_2(1 - L_2 - L_3 + L_2L_3) \\ \quad + P_3(1 - L_1 - L_2 - L_3 - L_4 + L_1L_2 + L_2L_3 + L_1L_3 - L_1L_2L_3) \end{array}}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_2 + L_2L_3 + L_1L_3 - L_1L_2L_3} \tag{2.23}$$

A second example of block-diagram reduction and some reinforcement of the techniques used to describe a system in block-diagram form is provided by the set of algebraic equations

$$X + Y + Z = 6 \tag{2.24}$$
$$X + Y - Z = 0$$
$$2X + 3Y + Z = 11$$

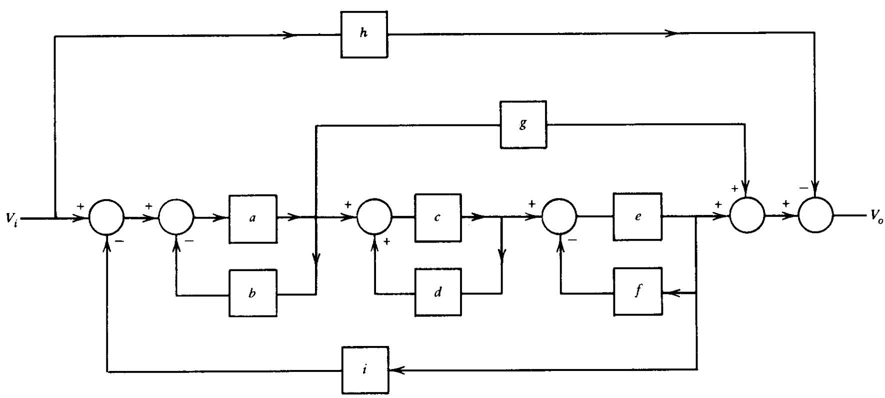



**Figure 2.13** Block diagram for gain-expression example.

In order to represent this set of equations in block-diagram form, the three equations are rewritten

$$
\begin{aligned}
X &= \qquad\quad - Y - Z + 6 \\
Y &= -X \qquad\quad + Z \\
Z &= -2X - 3Y \qquad + 11
\end{aligned}
\tag{2.25}
$$

This set of equations is shown in block-diagram form in Fig. 2.14. If we use the identification of loops in this figure, loop gains are

$$
\begin{aligned}
L_1 &= 1 \\
L_2 &= -3 \\
L_3 &= -3 \\
L_4 &= 2 \\
L_5 &= 2
\end{aligned}
$$

Since all loops touch, the determinant of any gain expression for this system is

$$1 - L_1 - L_2 - L_3 - L_4 - L_5 = 2 \tag{2.26}$$

(This value is of course identically equal to the determinant of the coefficients of Eqn. 2.24.)

Assume that the value of $X$ is required. The block diagram shows one path with a transmission of $+1$ connecting the excitation with a value of 6 to $X$. This path does not touch $L_2$. There are also two paths (roughly parallèling $L_3$ and $L_5$) with transmissions of $-1$ connecting the excitation with a value of 11 to $X$. These paths touch all loops. Linearity allows us to combine the $X$ responses related to the two excitations, with the result that

$$X = \frac{6[1 - (-3)] - 11 - 11}{2} = 1 \tag{2.27}$$

The reader should verify that this method yields the values $Y = 2$ and $Z = 3$ for the other two dependent variables.

## 2.5 EFFECTS OF FEEDBACK ON INPUT AND OUTPUT IMPEDANCE

The gain-stabilizing and linearizing effects of feedback have been described earlier in this chapter. Feedback also has important effects on the input and output impedances of an amplifier, with the type of modification dependent on the topology of the amplifier-feedback network combination.

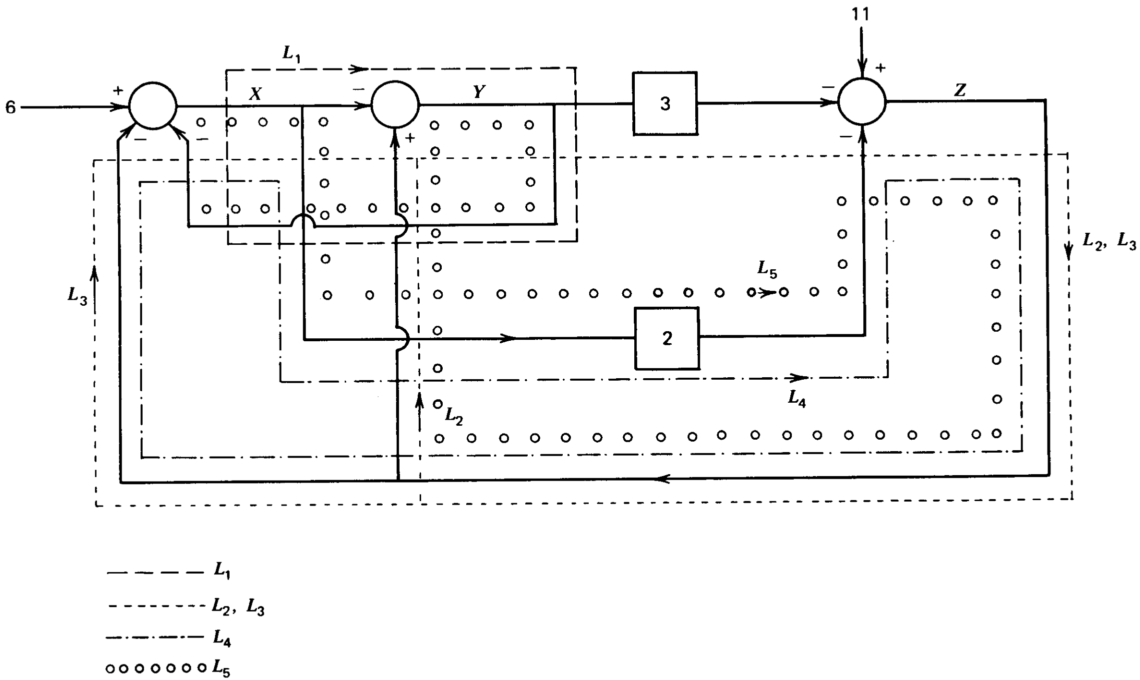



**Figure 2.14** Block diagram of Eqn. 2.25.

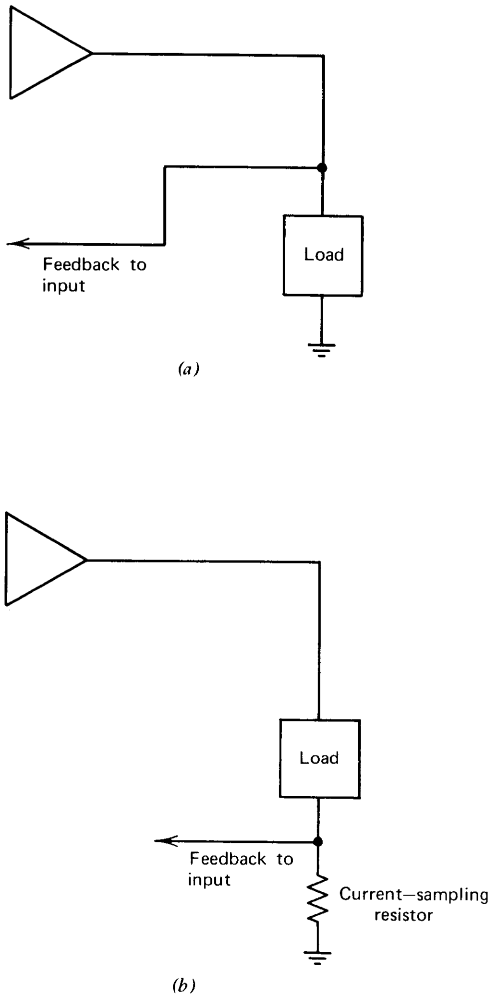



**Figure 2.15** Two possible output topologies. (*a*) Feedback of load-voltage information. (*b*) Feedback of load-current information.

Figure 2.15 shows how feedback might be arranged to return information about either the voltage applied to the load or the current flow through it. It is clear from physical arguments that these two output topologies must alter the impedance facing the load in different ways. If the information fed back to the input concerns the output voltage, the feedback tends to reduce changes in output voltage caused by disturbances (changes in load current), thus implying that the output impedance of the amplifier shown in Fig. 2.15*a* is reduced by feedback. Alternatively, if information about load current is fed back, changes in output current caused by disturbances (changes in load voltage) are reduced, showing that this type of feedback raises output impedance.

Two possible input topologies are shown in Fig. 2.16. In Fig. 2.16*a*, the input signal is applied in series with the differential input of the amplifier. If the amplifier characteristics are satisfactory, we are assured that any required output signal level can be achieved with a small amplifier input current. Thus the current required from the input-signal source will be small, implying high input impedance. The topology shown in Fig. 2.16*b* reduces input impedance, since only a small voltage appears across the parallel input-signal and amplifier-input connection.

The amount by which feedback scales input and output impedances is directly related to the loop transmission, as shown by the following example. An operational amplifier connected for high input and high output resistances is shown in Fig. 2.17. The input resistance for this topology is simply the ratio $V_i/I_i$. Output resistance is determined by including a voltage source in series with the load resistor and calculating the ratio of the change in the voltage of this source to the resulting change in load current, $V_l/I_l$. If it is assumed that the components of $I_l$ and the current through the sampling resistor $R_S$ attributable to $I_i$ are negligible (implying that the

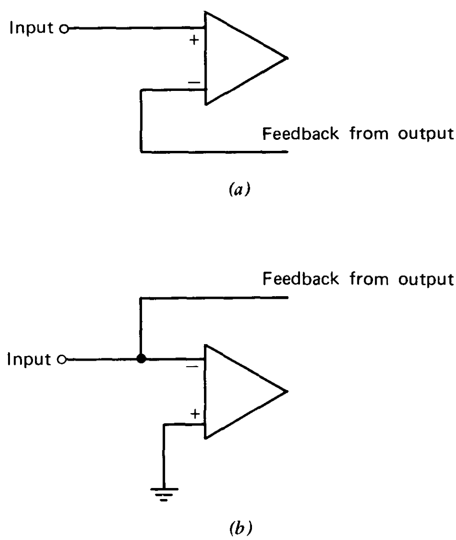



**Figure 2.16** Two possible input topologies. (*a*) Input signal applied in series with amplifier input. (*b*) Input signal applied in parallel with amplifier input.

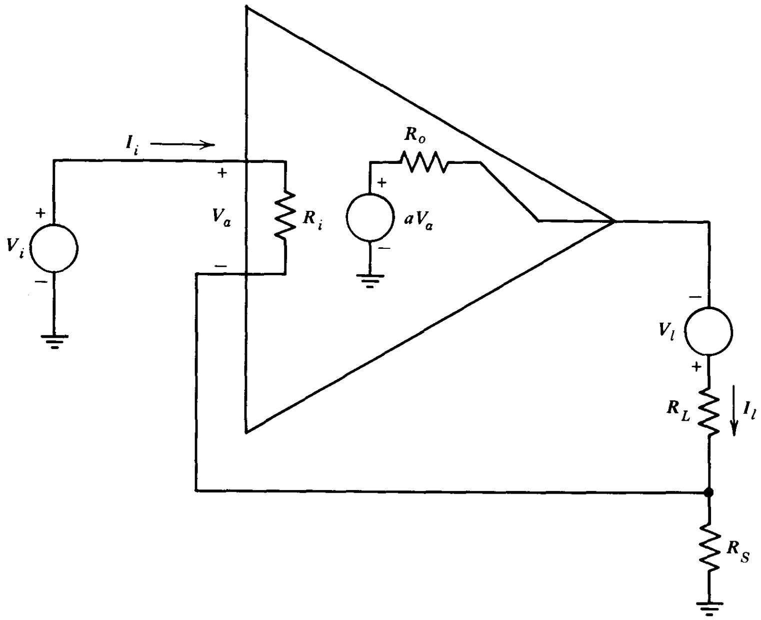



**Figure 2.17** Amplifier with high input and output resistances.

amplifier, rather than a passive network, provides system gain) and that $R_i \gg R_S$, the following equations apply.

$$V_a = V_i - R_S I_l \tag{2.28}$$

$$I_l = \frac{aV_a + V_l}{R_o + R_L + R_S} \tag{2.29}$$

$$I_i = \frac{V_a}{R_i} \tag{2.30}$$

These equations are represented in block-diagram form in Fig. 2.18. This block diagram verifies the anticipated result that, since the input voltage is compared with the output current sampled via resistor $R_S$, the ideal transconductance (ratio of $I_l$ to $V_i$) is simply equal to $G_S$. The input resistance is evaluated by noting that

$$\frac{I_i}{V_i} = \frac{1}{R_{\text{in}}} = \frac{1}{R_i\{1 + [aR_S/(R_o + R_L + R_S)]\}} \tag{2.31}$$

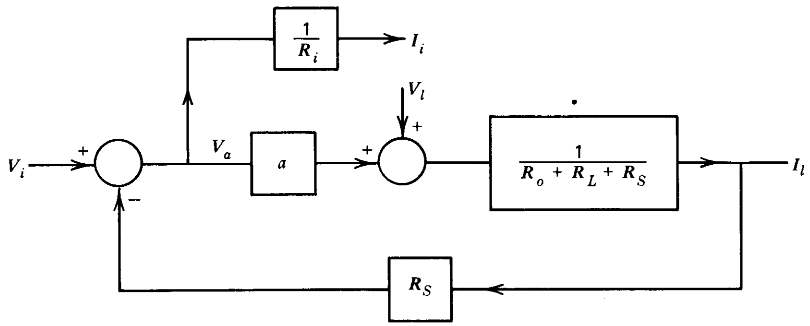



**Figure 2.18** Block diagram for amplifier of Fig. 2.17.

or

$$R_{\text{in}} = R_i\left(1 + \frac{aR_S}{R_o + R_L + R_S}\right) \tag{2.32}$$

The output resistance is determined from[6]

$$\frac{I_l}{V_l} = \frac{1}{R_{\text{out}}} = \frac{1}{(R_o + R_L + R_S)\{1 + [aR_S/(R_o + R_L + R_S)]\}} \tag{2.33}$$

yielding

$$R_{\text{out}} = (R_o + R_L + R_S)\left(1 + \frac{aR_S}{R_o + R_L + R_S}\right) \tag{2.34}$$

The essential features of Eqns. 2.32 and 2.34 are the following. If the system has no feedback (e.g., if $a = 0$), the input and output resistances become

$$R'_{\text{in}} = R_i \tag{2.35}$$

and

$$R'_{\text{out}} = R_o + R_L + R_S \tag{2.36}$$

Feedback increases both of these quantities by a factor of $1 + [aR_S/(R_o + R_L + R_S)]$, where $-aR_S/(R_o + R_L + R_S)$ is recognized as the loop transmission. Thus we see that the resistances in this example are increased by the same factor (one minus the loop transmission) as the desensitivity

[6] Note that the output resistance in this example is calculated by including a voltage source in series with the load resistor. This approach is used to emphasize that the loop transmission that determines output resistance is influenced by $R_L$. An alternative development might evaluate the resistance facing the load by replacing $R_L$ with a test generator.

increase attributable to feedback. The result is general, so that input or output impedances can always be calculated for the topologies shown in Figs. 2.15 or 2.16 by finding the impedance of interest with no feedback and scaling it (up or down according to topology) by a factor of one minus the loop transmission.

While feedback offers a convenient method for controlling amplifier input or output impedances, comparable (and in certain cases, superior) results are at least conceptually possible without the use of feedback. Consider, for example, Fig. 2.19, which shows three ways to connect an operational amplifier for high input impedance and unity voltage gain.

The follower connection of Fig. 2.19*a* provides a voltage gain

$$\frac{V_o}{V_i} = \frac{a}{1 + a} \tag{2.37}$$

or approximately unity for large values of *a*. The relationship between input impedance and loop transmission discussed earlier in this section shows that the input impedance for this connection is

$$\frac{V_i}{I_i'} = Z_i(1 + a) \tag{2.38}$$

The connection shown in Fig. 2.19*b* precedes the amplifier with an impedance that, in conjunction with the input impedance of the amplifier, attenuates the input signal by a factor of $1/(1 + a)$. This attenuation combines with the voltage gain of the amplifier itself to provide a composite voltage gain identical to that of the follower connection. Similarly, the series impedance of the attenuator input element adds to the input impedance of the amplifier itself so that the input impedance of the combination is identical to that of the follower.

The use of an ideal transformer as impedance-modifying element can lead to improved input impedance compared to the feedback approach. With a transformer turns ratio of $(a + 1){:}1$, the overall voltage gain of the transformer-amplifier combination is the same as that of the follower connection, while the input impedance is

$$\frac{V_i}{I_i''} = Z_i(1 + a)^2 \tag{2.39}$$

This value greatly exceeds the value obtained with the follower for large amplifier voltage gain.

The purpose of the above example is certainly not to imply that attenuators or transformers should be used in preference to feedback to modify impedance levels. The practical disadvantages associated with the two

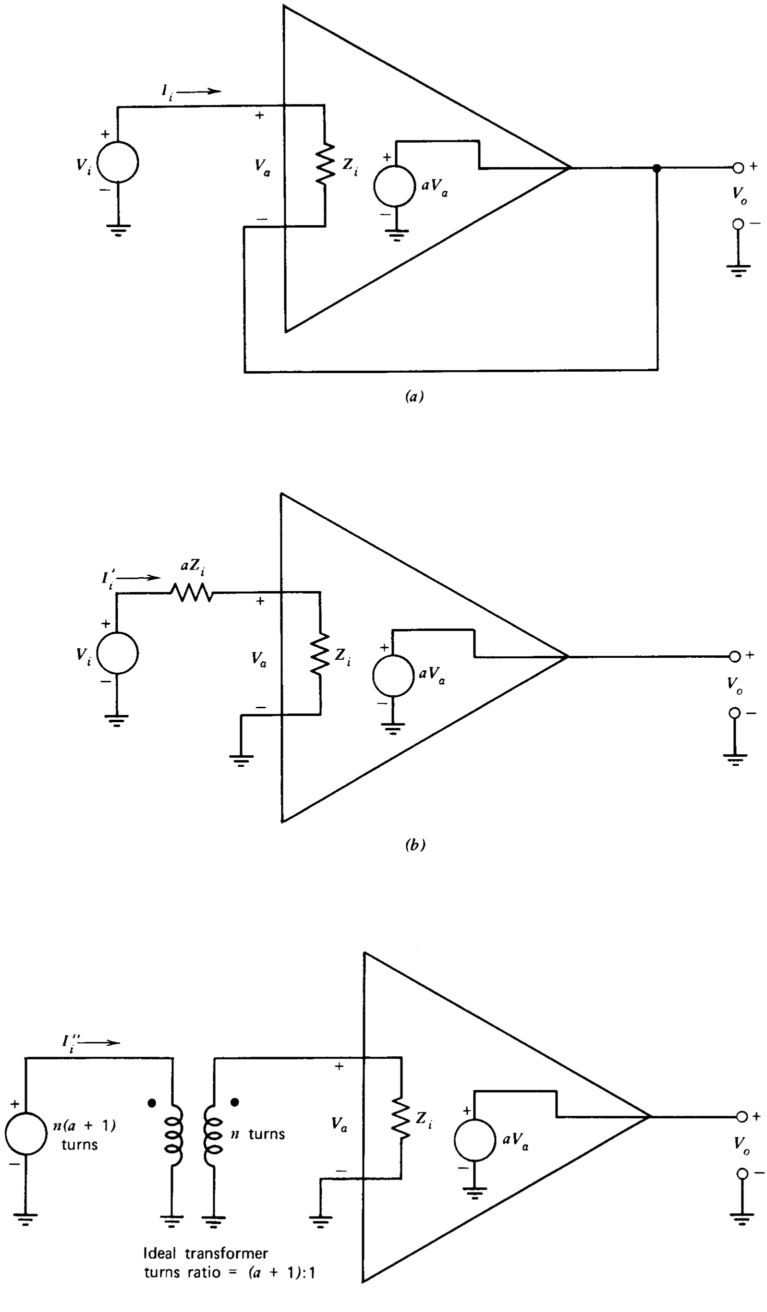



**Figure 2.19** Unity-gain amplifiers. (*a*) Follower connection. (*b*) Amplifier with input attenuator. (*c*) Amplifier with input transformer.

former approaches, such as the noise accentuation that accompanies large input-signal attenuation and the limited frequency response characteristic of transformers, often preclude their use. The example does, however, serve to illustrate that it is really the power gain of the amplifier, rather than the use of feedback, that leads to the impedance scaling. We can further emphasize this point by noting that the input impedance of the amplifier connection can be increased without limit by following it with a step-up transformer and increasing the voltage attenuation of either the network or the transformer that precedes the amplifier so that the overall gain is one. This observation is a reflection of the fact that the amplifier alone provides infinite power gain since it has zero output impedance.

One rather philosophical way to accept this reality concerning impedance scaling is to realize that feedback is most frequently used because of its fundamental advantage of reducing the sensitivity of a system to changes in the gain of its forward-path element. The advantages of impedance scaling can be obtained *in addition* to desensitivity simply by choosing an appropriate topology.

## PROBLEMS

### P2.1

Figure 2.20 shows a block diagram for a linear feedback system. Write a complete, independent set of equations for the relationships implied by this diagram. Solve your set of equations to determine the input-to-output gain of the system.

### P2.2

Determine how the fractional change in closed-loop gain

$$\frac{d(V_o/V_i)}{V_o/V_i}$$

is related to fractional changes in $a_1$, $a_2$, and $f$ for the system shown in Fig. 2.21.

### P2.3

Plot the closed-loop transfer characteristics for the nonlinear system shown in Fig. 2.22.

### P2.4

The complementary emitter-follower connection shown in Fig. 2.23 is a simple unity-voltage-gain stage that has a power gain approximately equal to the current gain of the transistors used. It has nonlinear transfer charac-

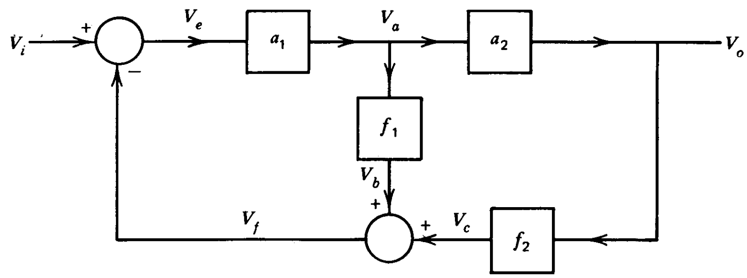



**Figure 2.20** Two-loop feedback system.

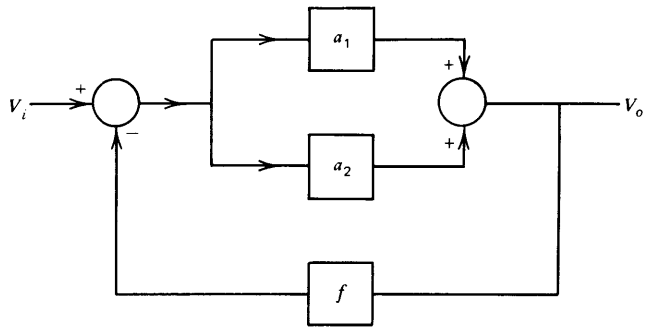


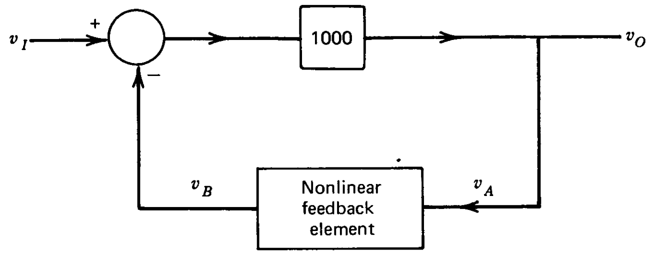

**Figure 2.21** Feedback system with parallel forward paths.

teristics, since it is necessary to apply approximately 0.6 volts to the base-to-emitter junction of a silicon transistor in order to initiate conduction.

(a) Approximate the input-output transfer characteristics for the emitter-follower stage.
(b) Design a circuit that combines this power stage with an operational amplifier and any necessary passive components in order to provide a closed-loop gain with an ideal value of +5.
(c) Approximate the actual input-output characteristics of your feedback circuit assuming that the open-loop gain of the operational amplifier is $10^5$.

**P2.5**

(a) Determine the incremental gain $v_o/v_i$ for $V_I = 0.5$ and 1.25 for the system shown in Fig. 2.24.



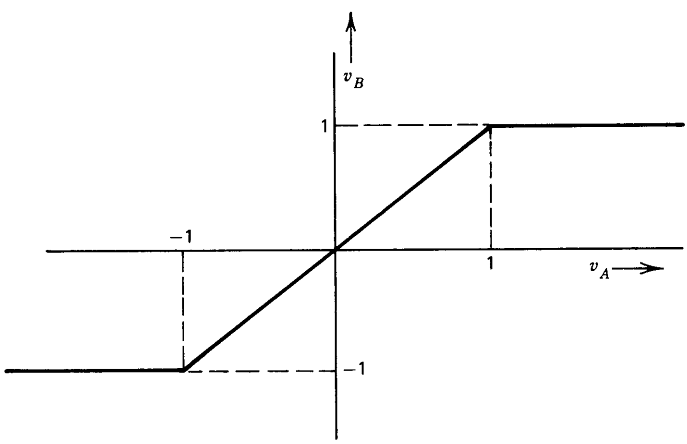




**Figure 2.22** Nonlinear feedback system. (*a*) System. (*b*) Transfer characteristics for nonlinear element.

(b) Estimate the signal $v_A$ for $v_I$, a unit ramp $[v_I(t) = 0, t < 0, = t, t > 0]$.

(c) For $v_I = 0$, determine the amplitude of the sinusoidal component of $v_O$.

**P2.6**

Determine $V_o$ as a function of $V_{i1}$ and $V_{i2}$ for the feedback system shown in Fig. 2.25.

**P2.7**

Draw a block diagram that relates output voltage to input voltage for an emitter follower. You may assume that the transistor remains linear, and

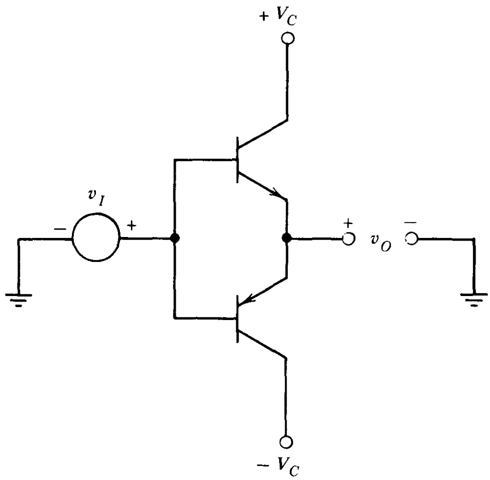



**Figure 2.23** Complementary emitter follower.

use a hybrid-pi model for the device. Include elements $r_\pi$, $r_x$, $C_\pi$, and $C_\mu$, in addition to the dependent generator, in your model. Reduce the block diagram to a single input-output transfer function.

**P2.8**

Draw a block diagram that relates $V_o$ to $V_i$ for the noninverting connection shown in Fig. 2.26. Also use block-diagram techniques to determine the impedance at the output, assuming that $Z_i$ is very large.

**P2.9**

A negative-feedback system used to rotate a roof-top antenna is shown in Fig. 2.27*a*.

The total inertia of the output member (antenna, motor armature, and pot wiper) is 2 kg·m². The motor can be modeled as a resistor in series with a speed-dependent voltage generator (Fig. 2.27*b*).

The torque provided by the motor that accelerates the total output-member inertia is 10 N·m per ampere of $I_a$. The polarity of the motor dependent generator is such that it tends to reduce the value of $I_a$ as the motor accelerates so that $I_a$ becomes zero for a motor shaft velocity equal to $V_m/10$ radians per second.

Draw a block diagram that relates $\theta_o$ to $\theta_i$. You may include as many intermediate variables as you wish, but be sure to include $V_m$ and $I_a$ in your diagram. Find the transfer function $\theta_o/\theta_i$.

Modify your diagram to include an output disturbance applied to the

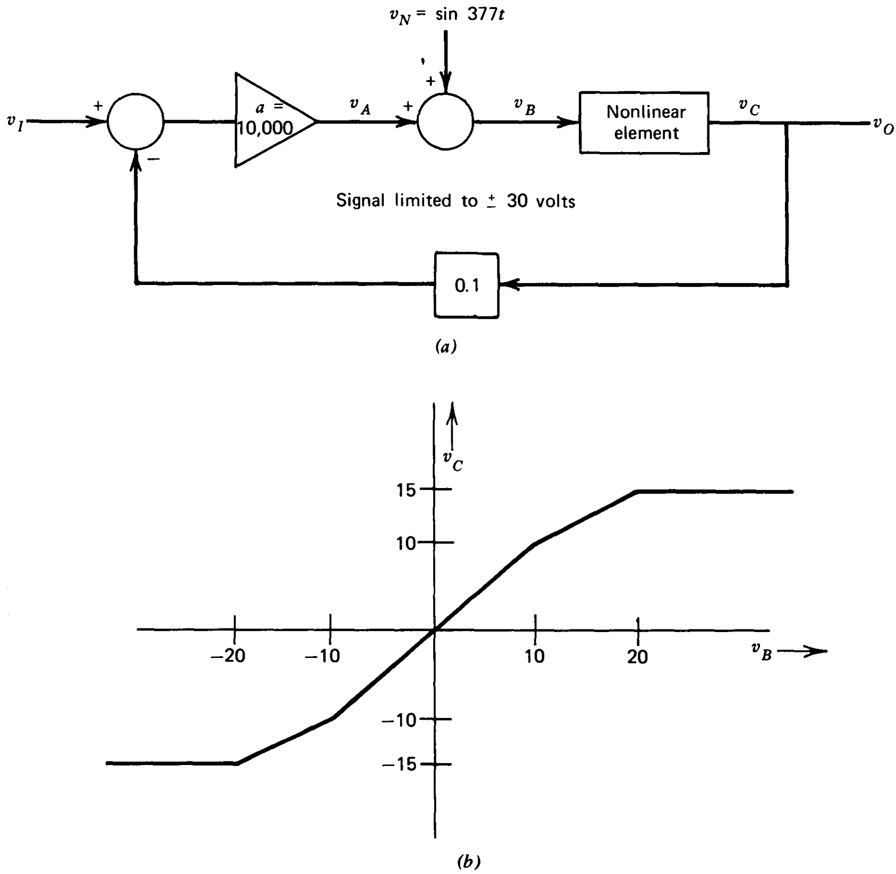



**Figure 2.24** Nonlinear system. (*a*) System. (*b*) Transfer characteristics for nonlinear element.

antenna by wind. Calculate the angular error that results from a 1 N·m disturbance.

**P2.10**

Draw a block diagram for this set of equations:

$$\begin{aligned} W + X \quad\quad\quad\quad &= 3 \\ X + Y \quad\quad &= 5 \\ Y + Z &= 7 \\ 2W + X + Y + Z &= 11 \end{aligned}$$

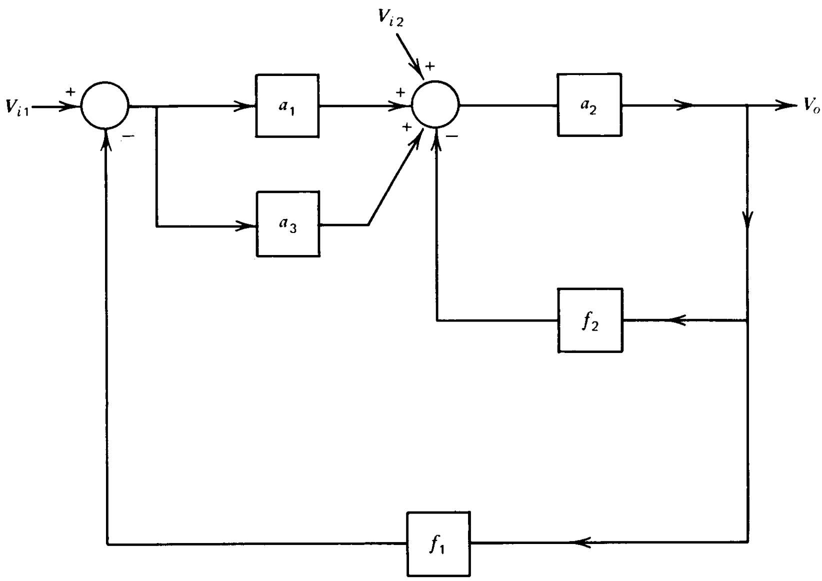



**Figure 2.25** Linear block diagram.

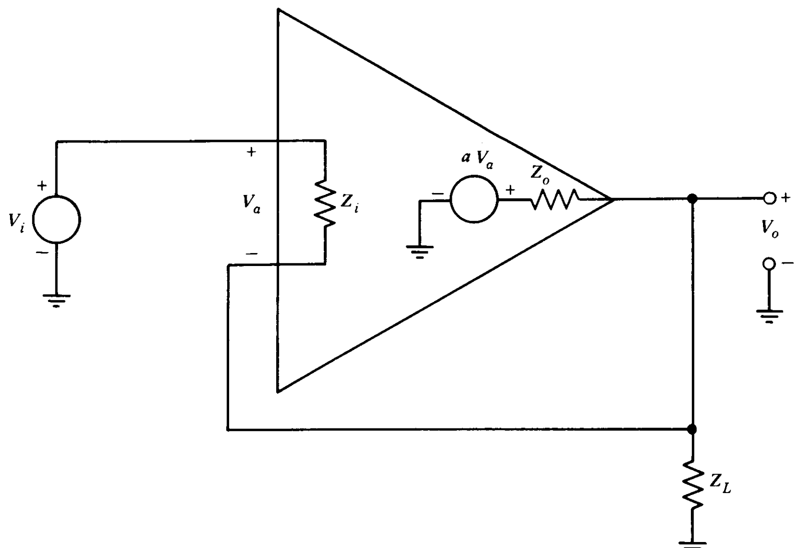



**Figure 2.26** Noninverting amplifier.

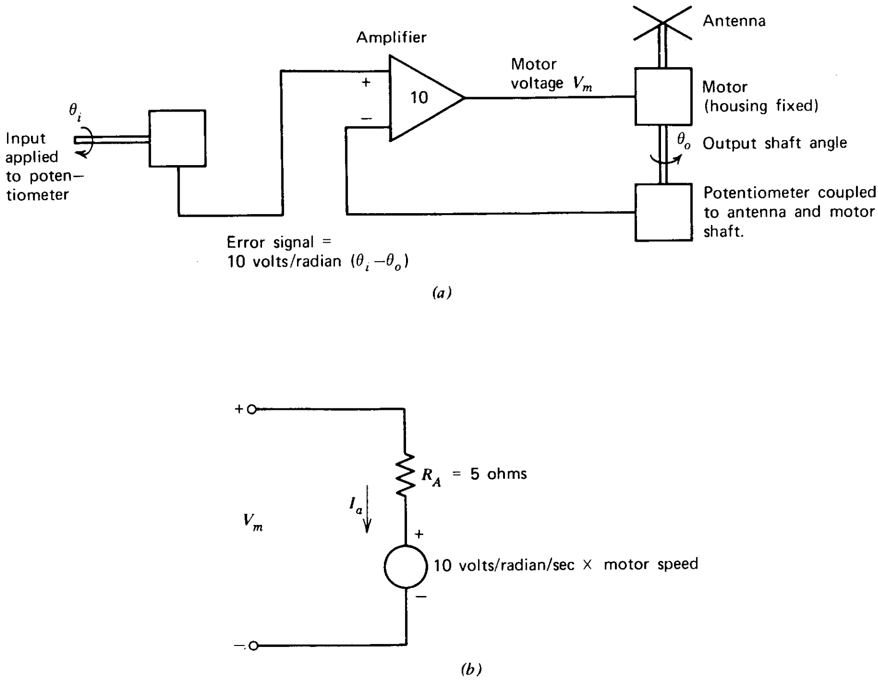



**Figure 2.27** Antenna rotator System. (*a*) System configuration. (*b*) Model for motor.

Use the block-diagram reduction equation (Eqn. 2.22) to determine the values of the four dependent variables.

**P2.11**

The connection shown in Fig. 2.28 feeds back information about both load current and load voltage to the amplifier input. Draw a block diagram that allows you to calculate the output resistance $V_o/I_d$.

You may assume that $R \gg R_S$ and that the load can be modeled as a resistor $R_L$. What is the output resistance for very large $a$?

**P2.12**

An operational amplifier connected to provide an adjustable output resistance is shown in Fig. 2.29. Find a Thévenin-equivalent circuit facing the load as a function of the potentiometer setting $\alpha$. You may assume that the resistance $R$ is very large and that the operational amplifier has ideal characteristics.

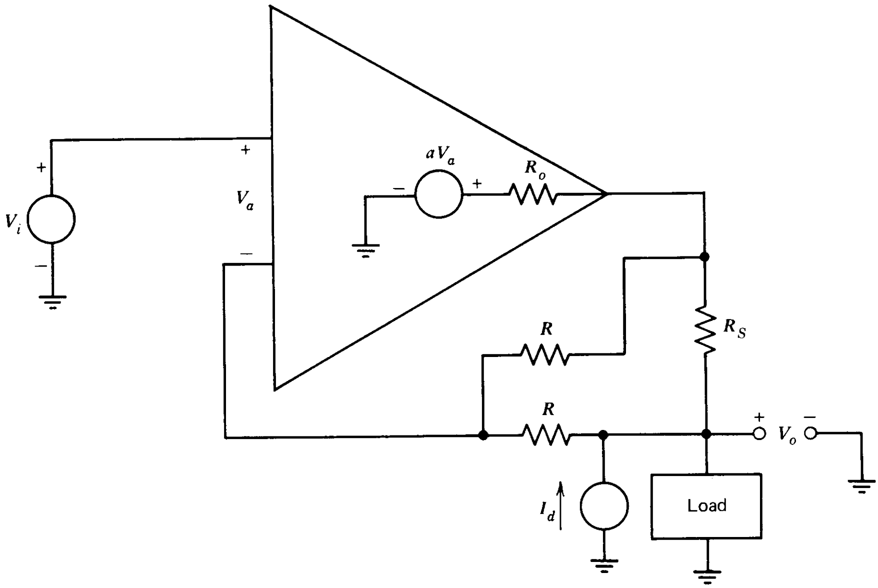



**Figure 2.28** Operational-amplifier connection with controlled output resistance.

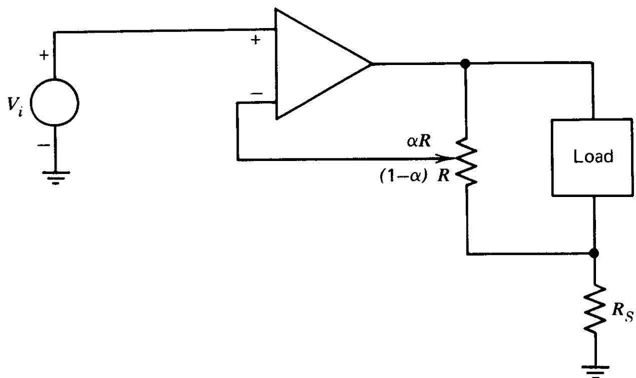



**Figure 2.29** Circuit with adjustable output resistance.

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