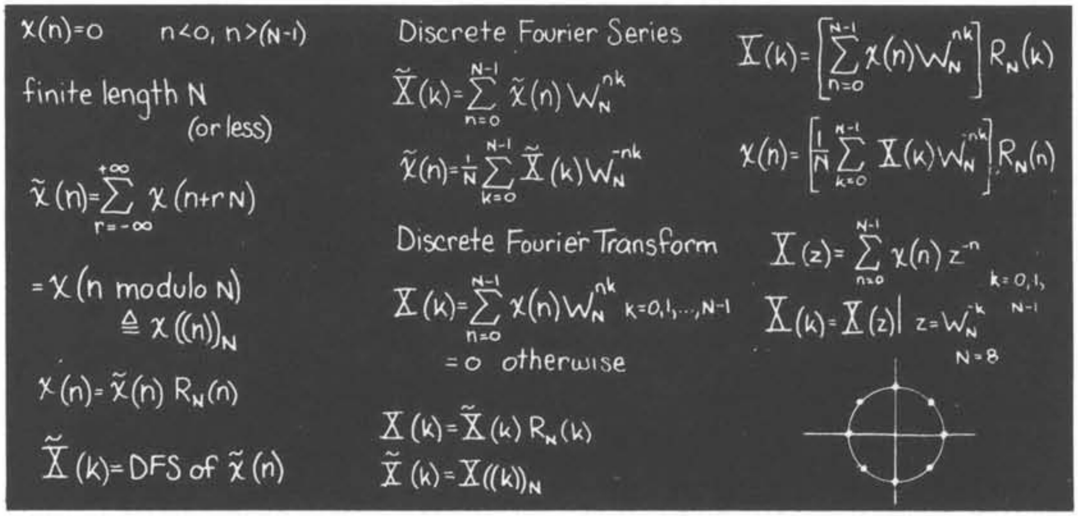
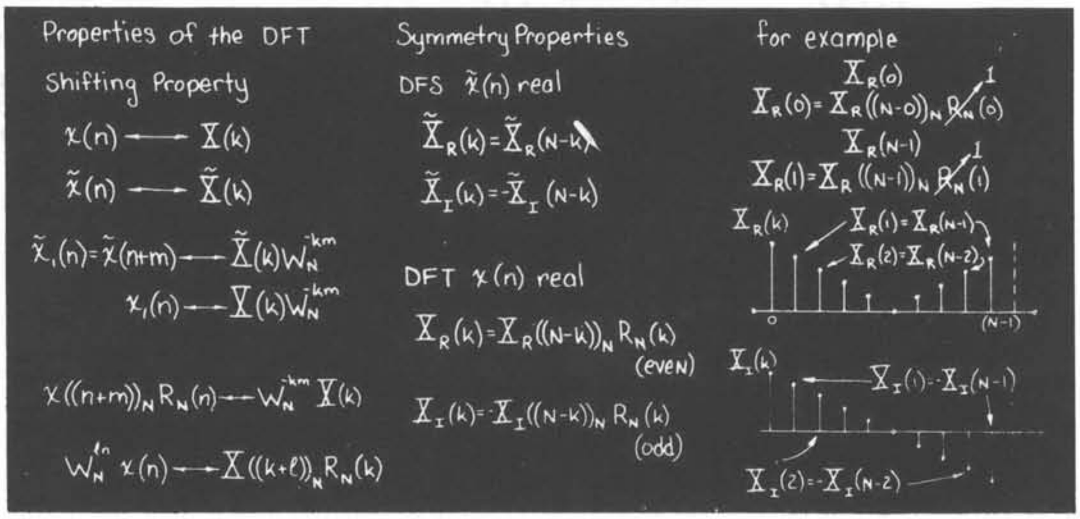
THE DISCRETE FOURIER TRANSFORM

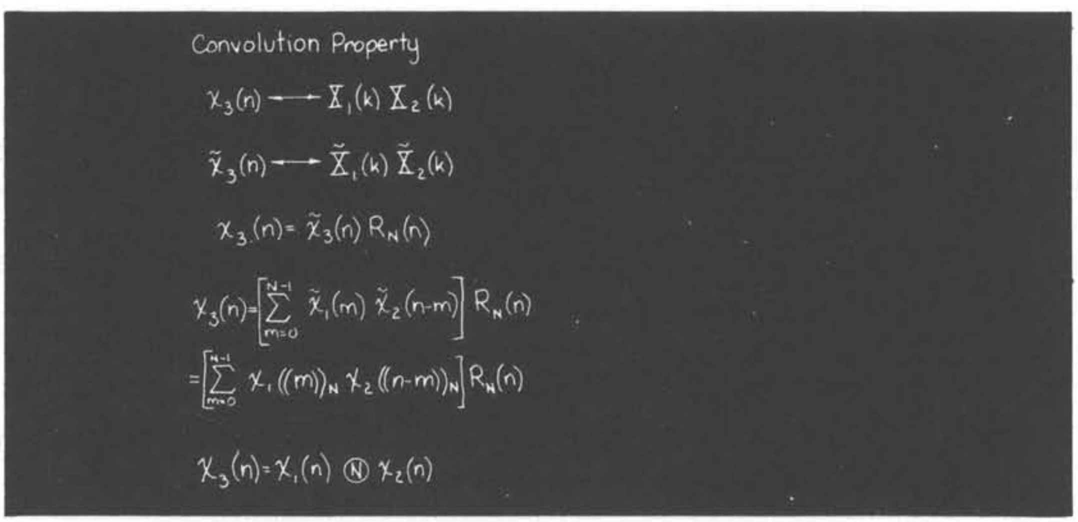
1. Lecture 9 - 42 minutes



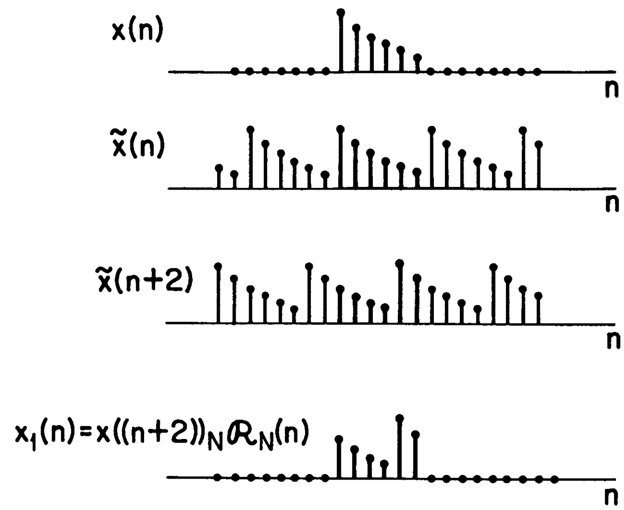
a.



b.



c.



Circular shifting of a finite length sequence.

d.

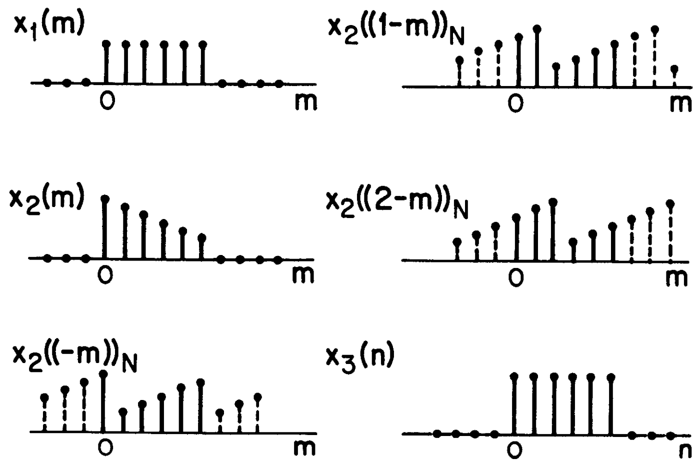


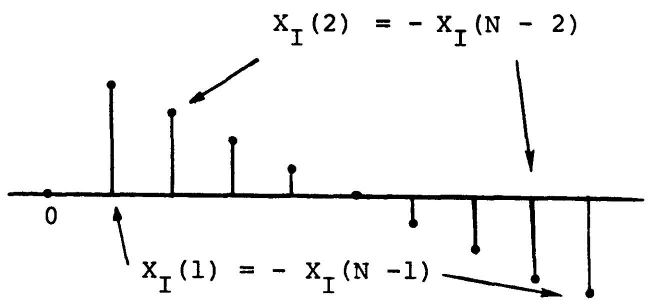
Illustration of circular convolution. (Note that $x_2((-m))_N$ is incorrectly drawn. In Problem 9.4 you are asked to correct this.)

e.

2. Corrections

Two of the figures used in this lecture require corrections. The first is the figure on board 2 which illustrates the symmetry for the imaginary part of the DFT. The value indicated as a dashed line should have a height and polarity identical to $X_I(0)$ since it represents the periodic extension of $X_I(k)$. Furthermore, because of the symmetry imposed on $X_I(k)$ the value $X_I(0)$ is constrained to be zero (see Problem 9.3 below.) A corrected version is shown at the top of page 9.3.

The second figure requiring correction is the viewgraph used to illustrate circular convolution. On that figure $x_2((-m))_N$ is incorrectly drawn. In problem 9.4 below you are asked to correct this figure.



3. Comments

In this lecture the representation of finite length sequences by means of the Discrete Fourier Transform is introduced. This representation corresponds essentially to the Discrete Fourier Series representation of the periodic counterpart of the finite length sequence.

The relationship between the DFT and the z-transform is also stressed, deriving in particular the fact that the DFT corresponds to samples of the z-transform equally spaced in angle around the unit circle. This also implies, of course, that the Fourier series coefficients of a periodic sequence correspond to samples on the unit circle of the z-transform of one period of the periodic sequence.

The lecture concludes with a discussion of properties of the DFT. All of the properties of the DFT are direct consequences of the properties of Fourier series and are generally different than the properties of the Fourier transform as discussed in lecture 4. The shifting property, for example relates to a circular shift of a sequence rather than a linear shift, and it is a circular convolution that results in a product of DFT's.

4. Reading

Text: Sections 8.5 (page 527), 8.6 and 8.7. Section 8.7.5 on circular convolution should be read only lightly for this lesson. It will be a main focus for lesson 10.

5. Problems

Problem 9.1

Compute the DFT of each of the following finite-length sequences considered to be of length N.

(a) $x(n) = \delta(n)$.

(b) $x(n) = \delta(n - n_0)$, where $0 < n_0 < N$.

(c) $x(n) = a^n$, $0 \leq n \leq N - 1$.

Problem 9.2

In figure P9.2-1 below is shown a finite length sequence x(n). Sketch the sequences $x_1(n)$ and $x_2(n)$ specified as

$$x_1(n) = x((n - 2))_4 \; R_4(n)$$

$$x_2(n) = x((-n))_4 \; R_4(n)$$

(Note that $x_1(n)$ is x(n) circularly shifted by two points).

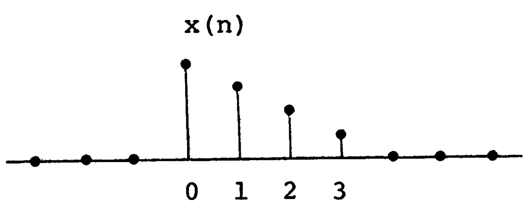


Figure P9.2-1

Problem 9.3

In the lecture we showed that X(k), the DFT of a real finite length sequence is conjugate symmetric, i.e.

$$X_R(k) = X_R((N - k))_N \; R_N(k)$$

and

$$X_I(k) = - X_I((N - k))_N \; R_N(k)$$

From this symmetry property, show that $X_I(0)$ must be zero. Also show that if N is even then $X_I(N/2)$ must be zero.

Problem 9.4

In the viewgraph used to illustrate circular convolution, the sequence $x_2((-m))_N$ is incorrectly drawn. Sketch the correct sequence $x_2((-m))_N$.

Problem 9.5

In Figure P9.5-1 below are shown two finite length sequences. Sketch their 6-point circular convolution.

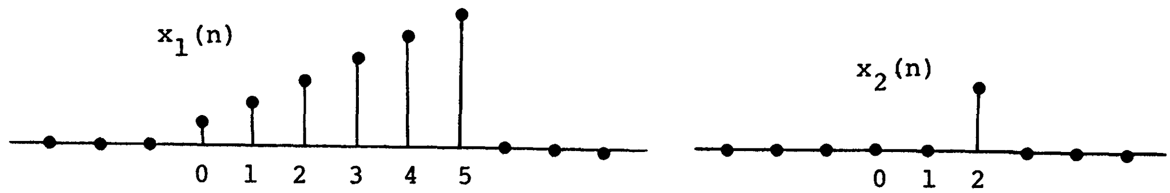


Figure P9.5-1

Problem 9.6*

The DFT of a finite-duration sequence corresponds to samples of its z-transform on the unit circle. For example, the DFT of a 10-point sequence $x(n)$ corresponds to samples of $X(z)$ at the 10 equally spaced points indicated in figure P9.6-1. We wish to find the equally spaced samples of $X(z)$ on the contour shown in figure P9.6-2; i.e., $X(z)\big|_{z=0.5e^{j[(2\pi k/10)+(\pi/10)]}}$. Show how to modify $x(n)$ to obtain a sequence $x_1(n)$ such that the DFT of $x_1(n)$ corresponds to the desired samples of $X(z)$.

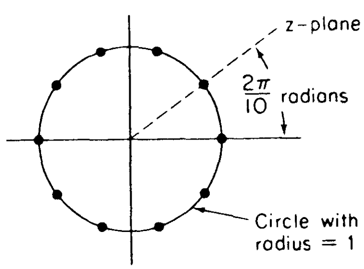


Figure P9.6-1

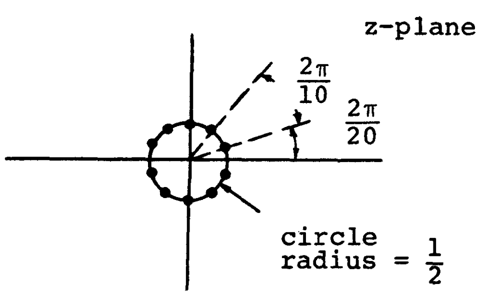
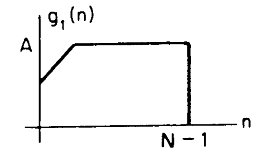


Figure P9.6-2

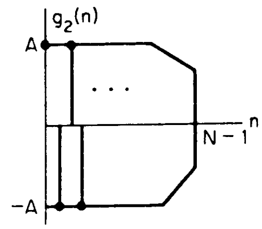
Problem 9.7*

Consider a finite-duration sequence x(n), which is zero for n < 0 and n ≥ N, where N is even. Let the z-transform of x(n) be denoted by X(z). Listed here are two tables. In Table P9.7-1 are seven sequences obtained from x(n). In Table P9.7-2 are nine sequences obtained from X(z). For each sequence in Table P9.7-1, find its DFT in Table P9.7-2. The size of the transform considered must be greater than or equal to the length of the sequence $g_k(n)$. For purposes of illustration only assume that x(n) can be represented by the envelope shown in figure P9.7-1

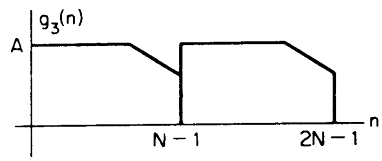
$$g_1(n) = x(N-1-n).$$



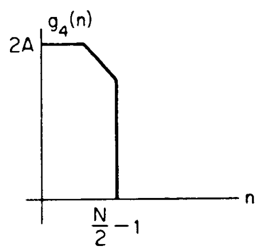
$$g_2(n) = (-1)^n x(n)$$



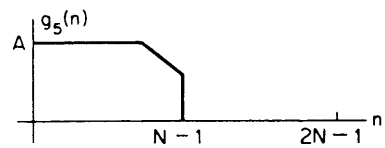
$$g_3(n) = \begin{cases} x(n), & 0 \le n \le N-1 \\ x(n-N), & N \le n \le 2N-1 \\ 0, & \text{otherwise} \end{cases}$$



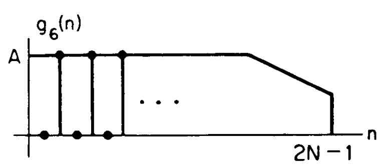
$$g_4(n) = \begin{cases} x(n) + x(n+N/2), & 0 \le n \le N/2-1 \\ 0, & \text{otherwise} \end{cases}$$



$$g_5(n) = \begin{cases} x(n), & 0 \le n \le N-1 \\ 0, & N \le n \le 2N-1 \\ 0, & \text{otherwise} \end{cases}$$



$$g_6(n) = \begin{cases} x\left(\frac{n}{2}\right), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$



$$g_7(n) = x(2n)$$

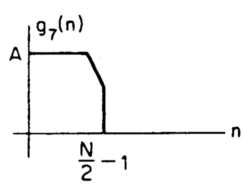


Table P9.7-1

$H_1(k) = X(e^{j2\pi k/N})$

$H_2(k) = X(e^{j2\pi k/2N})$

$$H_3(k) = \begin{cases} 2X(e^{j2\pi k/2N}), & k \text{ even} \\ 0, & k \text{ odd} \end{cases}$$

$H_4(k) = X(e^{j2\pi k/(2N-1)})$

$H_5(k) = 0.5[X(e^{j2\pi k/N}) + X(e^{j2\pi(k+N/2)/N})]$

$H_6(k) = X(e^{j4\pi k/N})$

$H_7(k) = e^{j2\pi k/N}X(e^{-j2\pi k/N})$

$H_8(k) = X(e^{j(2\pi/N)(k+N/2)})$

$H_9(k) = X(e^{-j2\pi k/N})$

Table P9.7-2

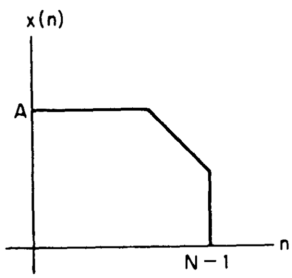


Figure P9.7-1