[MUSIC PLAYING]

PROFESSOR: We concluded the last lecture with the statement of the sampling theorem. And just as a quick reminder, the sampling theorem said that if we have a continuous-time signal and we have equally spaced samples of that signal, sampled at a sampling period, which I indicate is capital T and if x of t is band-limited-- in other words, the Fourier transform is zero outside some band where omega sub m is the highest frequency-- then under the condition that the sampling frequency, which is 2 pi divided by the period, is greater than twice the highest frequency. The original signal is uniquely recoverable from the set of samples.

And the sampling theorem essentially was derived by observing or using the notion that sampling could be done by multiplication or modulation with an impulse train. And the sampling theorem developed by examining the consequence of the modulation property in the context of the Fourier transform. In particular, if we have our signal x of t and if multiplied by an impulse train to give us a sampled signal-- another impulse train whose values or areas are samples of the original time function, as I indicate here-- then in fact, if we examine this equation or equivalently, bringing x of t inside this sum, if we examine either of these equations in the frequency domain, the Fourier transform of x of p of t is the convolution of the Fourier transform of the original signal and the Fourier transform of the impulse train.

Now the impulse train is a periodic signal. It's Fourier transform. Therefore, as we talked about with Fourier transforms is itself an impulse train. And when we do this convolution, then using the fact that the Fourier transform, the impulse train is an impulse train. The result of this convolution, then tells us that the Fourier transform of the sample signal or the impulse train, which represents the samples, is a sum of frequency-shifted replications of the Fourier transform of the original signal. So mathematically, that's the relationship. It essentially says that after sampling or modulation with an impulse train, the resulting spectrum is the original spectrum added to itself, shifted by integer multiples of the sampling frequency.

Well, let's see that as we did last time in terms of pictures. And again, to remind you of the basic picture involved, if we have an original signal with a spectrum as I indicated here-- where it's band-limited with the highest frequency omega sub m-- and if the time function is sampled so that in the frequency domain we convolve this spectrum with the spectrum shown below, which is the spectrum of the impulse train, the convolution of these two is then the Fourier transform or spectrum of the sample time function. And so that's what we end up with here. And then as you recall, to recover the original time function from this-- as long as these individual triangles don't overlap--to recover it just simply involves passing the impulse train through a low-pass filter, in effect extracting just one of these replications of the original spectrum.

So the overall system then for doing the sampling and then the reconstruction of the original signal from the samples, consists of multiplying the original time function by an impulse train. And that gives us then the sampled signal. The Fourier transform I show here of the original signal and after modulation with the impulse train, the resulting spectrum that we have is that replicated around integer multiples of the sampling frequency. And then finally, to recover the original signal or to generate a reconstructed signal, we then multiply this in the frequency domain by the frequency response of an ideal low-pass filter. And what that accomplishes for us then is recovering the original signal.

Now in this picture, an important point that I raised last time, relates to the fact that in doing the reconstruction--well we've assumed-- is that in replicating these individual versions of the original signal, those replications don't overlap and so by passing this through a low-pass filter in fact, we can recover the original signal. Well, what that requires is that this frequency, omega sub m, be less than this frequency.

And this frequency is omega sub s minus omega sub m. And so what we require is that the frequency omega sub m be less than omega sub s minus omega sub m. Or equivalently, what we require is that the sampling frequency be greater than twice the highest frequency in the original signal.

Now, if in fact that condition is violated, then we end up with a very important effect. And that effect is referred to as aliasing. In particular, if we look back at our original example--we are here-- we were able to recover our original spectrum by low-pass filtering. If in fact the sampling frequency is not high enough to avoid aliasing, then what happens in that case is that the individual replications of the Fourier transform of the original signal overlap and what we end up with is some distortion. As you can see, if we try to pass this through a low-pass filter to recover the original signal, in fact we won't recover the original signal since these individual replications have overlapped. And this is the case where omega sub s minus omega sub m is less than omega sub s. In other words, the sampling frequency is not greater in this case than twice the highest frequency.

So what happens here then is that in effect, higher frequencies get folded down into lower frequencies. What would come out of the low-pass filter is the reflection of some higher frequencies into lower frequencies. As I suggested a minute ago, that effect is referred to as aliasing. And in order to both understand that term better and to understand in fact the effect better, it's useful to examine this a little more closely for the specific example of a sinusoidal signal.

So let's concentrate on that. And what we want to look at is the effect of aliasing when our input signal is a sinusoidal signal. Now to do that, what I want to show shortly is a computer-generated movie that we've made. And let's first walk through a few frames of it to give you-- first of all, to set up our notation and to suggest what it is that we're trying to demonstrate.

Well, what we have is an input signal-- is a sinusoidal signal. And the spectrum or Fourier transform of that is an impulse in the frequency domain at the frequency of the sinusoid. We then have samples of that and when we sample that-- and for this particular example, it's sampled at 10 kilohertz-- this spectrum is then replicated at multiples of the sampling frequency. And I haven't shown negative frequencies here, but the contribution due to the negative frequency is at 10 kilohertz minus the input sinusoid.

We then carry out a reconstruction with an ideal low-pass filter. And the ideal low-pass filter is set at half the sampling frequency or 5 kilohertz. So what we have then is the input signal x of t and the impulse train x of p of t. And then the reconstructed signal is the output from the low-pass filter which I denote as x of r of t. Now as the input frequency x of t increases, this impulse moves up in frequency, but this impulse moves down in frequency. And so let's just look at a few frames as the input frequency increases.

So we have here a case where the input frequency has moved up close to 5 kilohertz. As we continue further, these two impulses will cross and what we'll end up with, as I indicated, is aliasing. So here now is a case where we have aliasing. The replication of the negative frequency has crossed into the passband of the filter and the reconstructed sinusoid will now be the frequency associated with this impulse rather than the frequency associated with the original sinusoid. And to dramatize that even further, here is the example where now the input frequency has moved up close to 10 kilohertz, but what comes out of the low-pass filter is a much lower frequency. And in fact, you can see that here is the reconstructed sinusoid, whereas here we have the input sinusoid.

Well, now what I'm going to want to do is demonstrate this as I indicated with a computer-generated movie. And what we'll see is the effect of reconstructing from the samples using a low-pass filter for an input which changes in frequency and with a sampling rate of 10 kilohertz. And what we'll see in the first part of this movie is the input x of t and the reconstructed signal x of r of t without explicitly showing the samples. And then, at a later point, we'll also show this and indicate that in fact the samples of those two are equal, even though they themselves are not.

So at the top, we'll have the input sinusoid without showing the samples. And its

Fourier transform is an impulse in the frequency domain as we've indicated. And if we sample it, that impulse then gets replicated. And so its samples, in particular, will have a Fourier transform not only with an impulse at the input sinusoidal frequency, but also at 10 kilohertz minus that frequency.

Now for the reconstruction, we passed the samples through an ideal low-pass filter. I picked the cutoff frequency of the low-pass filter at half the sampling frequency, namely 5 kilohertz. And here, what we see is that the output reconstructed signal in fact matches in frequency the input signal.

Now as we change the input frequency, the reconstructed sinusoid is identical until we get to an input frequency, which exceeds half the sampling frequency. At that point we have aliasing and while the input frequency is increasing, the output frequency in fact is decreasing because that's what's inside the passband of the filter. Now let's sweep it back. And as the input frequency decreases, the output frequency increases until there's no aliasing and now the output reconstructed signal is equal to the input.

So we've sampled a signal and then reconstructed the signal from the samples. And keep in mind, that given a set of samples, there are lots of continuous curves that we can thread through the set of samples. The one that we picked, of course, is the one consistent with the assumption about the signal bandwidth. In particular, we've reconstructed the signal whose spectrum falls within the passband of the filter.

Now what I'd like to show is the same reconstruction and input as I showed before, but now let's look at the samples and what we'll see is that when there's aliasing, even though the output-- the reconstructed signal-- is not identical to the input. In fact it's consistent with the input samples that is sampling the reconstructed signal. It gives a set of samples identical to the samples of the input and it's just that the interpolation in between those samples is an interpolation consistent with the assumed bandwidth of the input based on the sampling theorem.

So let's now look at that with the samples also shown along with the sinusoid. So at the top, we have the input sinusoid together with its samples. The bottom trace is the Fourier transform of the sampled waveform. The middle trace is the reconstructed sinusoid together with its samples. And notice, of course, that the samples of the input or reconstructed signal are identical. And also the input sinusoidal frequency and the output sinusoidal frequency are identical. And we now increase the frequency at the input. The reconstructed sinusoid tracks the input in frequency and, of course, the samples of the two are identical. The interpolation in between the samples is identical because of the fact that the input frequency is still less than half the sampling frequency.

And so, as long as the input is frequency is less than half the sampling frequency, not only will the samples be identical, but also the reconstructed continuous waveform will match the input waveform. Now when we get to half the sampling frequency, we're just on the verge of aliasing. This isn't aliasing quite yet, but any increase in the input frequency will now generate aliasing.

We now have aliasing, the output frequency is lower than the input frequency, but notice that the samples are identical. Now the low-pass filter is interpolating in between those samples with a sinusoid that falls within the passband of the low-pass filter, which no longer matches the frequency of the input sinusoid. But the important point is that even when we have aliasing, the samples of the reconstructed waveform are identical to the samples of the original waveform. And notice that as the input frequency increases, in fact the interpolated output, the reconstructed output has decreased in frequency.

Now as the input frequency begins to get closer to 10 kilohertz-- in fact your eye tends to also interpolate between the samples with a frequency that is lower than the input frequency. And that's particularly evident here. Notice that the input samples in fact look like they would be associated with a much lower frequency sinusoid, than in fact was the sinusoid that generated them. The lower-frequency sinusoid in fact corresponds to the reconstructed one. Now as we sweep back down, the aliasing eventually disappears and the output sinusoid tracks the input sinusoid in frequency.

So we've seen the effect of aliasing for sinusoidal signals in terms of waveforms. Now let's hear how it sounds. Now what we have for this demonstration is an oscillator and a sampler. And the output of the sampler goes into a low-pass filter. So the input from the oscillator goes into the sampler and the output of the sampler goes into the low-pass filter. The sampler frequency is 10 kilohertz. And so the low-pass filter has a cutoff frequency as I indicate here, of 5 kilohertz. And what we'll listen to is the reconstructed output as the oscillator input frequency varies.

And recall that what should happen is that when the oscillator input frequency gets past half the sampling frequency, we should hear aliasing. So we'll start the oscillator at 2 kilohertz.

[OSCILLATOR SOUND IN BACKGROUND]

PROFESSOR: And keep in mind that what you see on the dial is the input frequency, what you hear is the output frequency. As long as the input frequency is less than half the sampling frequency-- in other words, 5 kilohertz -- the reconstructed signal sounds identical to the input.

Now at 5 kilohertz, we're right on the verge of aliasing, and when we increase the input frequency past 5 kilohertz, the reconstructed frequency in fact will decrease. So as we move, for example, from 5 kilohertz up to let's say, 6 kilohertz. 6 kilohertz in fact gets aliased down to, what? It gets aliased down to 4 kilohertz. So 6 kilohertz at the input is 4 kilohertz at the output. Now, if we move up even further, 7 kilohertz at the input gets aliased down to 3 kilohertz at the output. So that, then is an audio demonstration of aliasing.

So to summarize, if we sample a signal and then reconstruct from the samples using a low-pass filter, as long as the sampling frequency is greater than twice the highest frequency in the signal we reconstruct exactly. If on the other hand, the sampling frequency is too low, less than twice the highest frequency, then we get aliasing. In other words, higher frequencies get folded or reflected down into lower frequencies as they come through the low-pass filter.

Now, one of the common applications of the whole concept of sampling is the use of sampling to convert a continuous-time signal into a discrete-time signal to carry out what's often referred to as discrete-time processing of continuous-time signals. And this in fact is something that we'll be talking about in a fair amount of detail, beginning with the next lecture. But let me indicate that for that kind of processing, essentially what happens, is that we begin with the continuous-time signal and convert it to a discrete-time signal, carry out the discrete-time processing, and then convert back to continuous-time.

And the conversion from a continuous-time signal to a discrete-time signal in fact, is done by exploiting sampling, specifically by sampling the continuous-time signal with an impulse train and then converting the impulse train into a sequence in a matter that I'll talk about in more detail next time. Now in doing that-- of course, as you can imagine-- it's important since we want an accurate representation of the original continuous-time signal, to choose the sampling frequency, to very carefully avoid aliasing. And so in fact, in that context and in many other contexts, aliasing is something that we're very eager to avoid. However, it's also important to understand that aliasing isn't all bad. And there are some very specific contexts in which aliasing is very useful and very heavily exploited.

One example of a very useful context of aliasing is when you want to look at things that happen at frequencies that you can't look at, for one reason or another. And sampling and aliasing is used to map those into lower frequencies. One very common example of that is the use of the stroboscope which was invented by Dr. Harold Edgerton at MIT. And sometime earlier, in fact we had the opportunity to visit Dr. Edgerton's laboratory at MIT and see some examples of this. So I'd like to-- as a conclusion to this lecture-- take you on a visit to the strobe lab at MIT.

In the lecture-- in discussing aliasing-- we've stressed the fact that in most situations, it's something that we'd like to avoid. However, right now we're at MIT, in Strobe Alley as it's called, on the way to visit the laboratory of my MIT colleague, Professor Harold Edgerton, where in fact aliasing is an everyday occurrence. Basically, the idea is the following-- that if in fact you want to make measurements at frequencies that, for one reason or another, you can't measure, then sampling and, consequently, aliasing can be used to bring those frequencies down into a frequency range that you can measure.

Well, Professor Edgerton alias Doc Edgerton invented the stroboscope for exactly that reason. And, kind of, the idea is the following. The eye, essentially, is a low-pass filter and so there are things that happen at frequencies above which your eye can track. And by sampling with light pulses, sampling in time, what in effect you're able to do is sample in such a way that higher frequencies get aliased down to lower frequencies so that, in fact, your eye can track them. So let's take a look inside the lab and in fact see an illustration of this strobe and some of its effects.

Let me introduce you to my MIT colleague, Doc Edgerton. Also by the way, this is a great place for kids of all ages and so my daughter, Justine, insisted on coming along to also help out. Doc, maybe we could begin with you just saying a little bit about what the strobe is and what some of the history is?

DR. HAROLD EDGERTON: Sure, it's a very simple application of intermittent light. And this is a xenon lamp that flashes in a controlled rate depending on this knob which Justine's going to turn. And we're going to look at a motor that's driving an unbalanced weight to set up some [INAUDIBLE] oscillations in the spring. I'll turn on the motor. I'll turn on the strobe.

[STROBOSCOPE SOUND IN BACKGROUND]

DR. HAROLD EDGERTON: Just get the right range. All right, Justine, turn that now, until it stops. See that, Justine, the frequency is that the light, which corresponds to the frequency of the motor. And it's a little less or a little more, when you lean to go forward to backwards.

PROFESSOR: Doc, maybe we could turn this strobe off for minute. And let me point out, by the way, the fact that when we're looking at this without the strobe on, what we're seeing essentially are frequencies that our eye can't track. So we can't see the motor turning and we can't really see other than with a blur. We can't see the movement of the spring. And so I guess, your point is that when we put the strobe on, we're essentially sampling this. And now we brought this down to a frequency that our eye is able to track.

In fact, I guess if we turn the incandescent light off, what we'll be able to really bring out are the alias frequencies. So now, what we're looking at in fact are the alias frequencies. The spring, of course, is moving a lot faster than we see it, isn't that right?

DR. HAROLD EDGERTON: Yes, it's going approximately 30 times a second. The motor is going far from 30 times a second. I will speed this up while I hit the next mode, where I get a figure 8 out of this thing. You want to see that now?

PROFESSOR: Yeah, great.

DR. HAROLD EDGERTON: [INAUDIBLE]

[MACHINE NOISE GETS LOUDER]

PROFESSOR: So what we'll be seeing now is essentially a second harmonic, is it?

DR. HAROLD EDGERTON: Yes, that's the second harmonic.

PROFESSOR: Justine, you think you could make that spring dance around a little bit by changing the strobe frequency?

DR. HAROLD EDGERTON: Yeah, you need to go around that way. You go around this way.

PROFESSOR: Hey, that's really neat. Let's turn the lights back on if we can.

DR. HAROLD EDGERTON: Tomorrow [INAUDIBLE] it's periodic, it has to be periodic.

PROFESSOR: And what's interesting now, if we look at this in a-- let's see, can you flip this strobe off again?

DR. HAROLD EDGERTON: Sure.

DR. HAROLD EDGERTON: Notice, Justine, when we look at the spring now, all that we can see is a blur. And you really can't see-- because your eye can't track it, you can't see things happening spatially in frequency.

You said, by the way, that this was originally demonstrated at the World's Fair.

DR. HAROLD EDGERTON: This particular instrument was made the World's Fair in Chicago-- not the last one, but the one before that.

PROFESSOR: Wow.

DR. HAROLD EDGERTON: It was a--you see it all scratched up because it's a-- the [INAUDIBLE] use this thing is to break the springs. Because of the uses, you try to find the parts that fail.

PROFESSOR: I see. You put them under stress and fatigue and--

DR. HAROLD EDGERTON: If I run this for half an hour and so, the spring will break. And they work on automobiles, they run them until something vibrates. Then they find out what the part is and what frequency it is.

PROFESSOR: Well let's--by the way, I bet you run this for a lot more than half an hour in this state.

DR. HAROLD EDGERTON: Oh, yeah, we've broken many, many springs in this thing-- and it's continuous. We experiment, try new things on it.

PROFESSOR: Maybe we could look at a couple of other things. How about the fan? Maybe--

DR. HAROLD EDGERTON: Sure, I'll plug this fan and this is a classic experiment for the strobe.

[MACHINE NOISE STOPS]

DR. HAROLD That's a good idea. Get that thing off. Makes too much noise.

EDGERTON:

PROFESSOR: Guess, we move that over here.

DR. HAROLD EDGERTON: This is just an ordinary electric fan, but it has a mark on one blade, so that you can identify it. We'll plug it in, get it up to speed.

PROFESSOR: This looks like a fan that was also demonstrated in the World's Fair, a few years ago.

DR. HAROLD EDGERTON: Yeah, could've been. There was a movie *Quicker'n a Wink* had this thing in there and--

PROFESSOR: With this very fan?

DR. HAROLD EDGERTON: Well, one like it. It was loaned to MGM. And Pete Smith, he said he wanted me to throw out a custard pie into it. I said, no, I'm a serious scientist. So he says, let's compromise on the egg. So we dropped an egg into it and you would see a high-speed movie of the egg dropping. No, not with the strobe, but with this [INAUDIBLE]

PROFESSOR: That was with the high-speed photography.

DR. HAROLD EDGERTON: High-speed movies, yeah.

PROFESSOR: So again, I guess, without the strobe, when we look at it, what we're looking at are frequencies that are much higher than the eye can follow. And now, with the strobe on, you can see both the alias frequency and you can also see the original frequency because we had the incandescent light on.

Let's turn down the background light again. And then, really all that we're able to see are the aliasing frequencies. And I guess when we see more than one mark, that means that we're actually running it at--

DR. HAROLD EDGERTON: Four times the speed of the fan.

PROFESSOR: --four times the speed the fan, yeah.

DR. HAROLD EDGERTON: You see a little variation in the--

PROFESSOR: Oh, yeah. Right.

DR. HAROLD EDGERTON: It's because the blades aren't exactly the same.

PROFESSOR: Actually, this gives me a chance to illustrate another important point related to the lecture. Let's see, can we bring it down to a frequency so that we only get one mark?

DR. HAROLD EDGERTON: Sure. You may miss this because it's too lowered to just one blade there now.

PROFESSOR: So the way we have it now, we've essentially aliased the fan's speed down so that it's just a little higher than DC. And now, I'm right at DC. And now, if I go down just a little further, in fact it looks like the fan is turning backwards. And if you think of this in the context of aliasing, it's like the two impulses in the frequency domain have crossed over. And what you get in effect, if you analyze it mathematically, is you get a phase reversal. And it wasn't until I first understood about aliasing, by the way, Doc, that I understood why when I went to Western movies, every once in a while you'd see the wagon wheels turning backwards. Then there's the wagon wheels of the Western movie going backwards, I guess.

And, Justine, why don't you see if you can--

DR. HAROLD EDGERTON: Too much flicker there. Why don't you bring it up so you get two marks.

PROFESSOR: See if you can bring the frequency up so that you get two marks.

DR. HAROLD EDGERTON: You turn that, Justine. Grab right ahold of that and give it a big twist. You went past it. They're not regular there now. Here we are. Now hold it right there. Put your finger on there, hold the dial. It's flashing twice per revolution now, Al.

PROFESSOR: I guess another thing that this demonstrates is something that I've heard a long time ago, which is that you should never use a power saw with a fluorescent light because the fluorescent light gives you a little bit of a strobe effect and you could actually convince yourself that that's standing still and make the mistake of trying to put your finger between the blades.

DR. HAROLD EDGERTON: You want to stick your finger in there?

PROFESSOR: No, I don't think I want to try it. How about you, Justine? What do you think? Is that standing still or is that moving?

DR. HAROLD EDGERTON: She knows it's going. We won't let her get close to that fan.

PROFESSOR: Actually if we turn the lights back on again, what that will let us see once again is that we can see both the alias frequencies when we do that and we can also see the higher frequencies because of the incandescent lighting. Maybe what we can do now is take a look at some other fun things. And one I guess I'm curious about is the disk that you have over there.

Doc, maybe you can tell us what we have here?

DR. HAROLD EDGERTON: Sure, Al. This is a disk to show how you can get motion pictures out of a series of still pictures. This circle is repeated 12 times. The white dot goes from the outer part of it on this side to the inner part on the other. If I flash one time per revolution on this, you'll see it exactly as it is. But if I skip one picture each time, then you get the relative motion of this ball. Well, the object is to show the ball rotating either this way or that way depending on whether the strobe was going faster or slower than the other.

This way motion pictures were developed hundred years ago, long before photography. They drew pictures of people in different poses, animated pictures.

Like to see it run?

PROFESSOR: Yeah, great. It's actually, the title, kind of, is "Aliasing Can Be Fun."

DR. HAROLD EDGERTON: That's right. Let me get it up to speed. On the way up, you get a lot of different, sort of, patterns as it goes through. When it eventually reaches its speed, which is about 1,100 per minute, you'll see it stop.

PROFESSOR: And the background blur, basically at the high frequencies that the eye can't follow and then, kind of, superimposed on that again, we can see the frequencies that are aliased down. And that's what the eye can follow.

DR. HAROLD EDGERTON: Right now, we have one flash per revolution, so you can see the part of the disk that's illuminated with the strobe exactly as if it was standing still. Now if I increase the frequency, so they skip one circle, then you get the illusion that, that dot is moving.

PROFESSOR: In fact, let's really enhance the revolution, let's turn the incandescent lights off again. And now, now what we see really are the alias frequencies. What do you think of this Justine?

JUSTINE: Neat.

DR. HAROLD EDGERTON: It looks like magic. I still have great joy in watching this thing, though it's so simple.

PROFESSOR: Now, while we're watching this, something also I might point out for the lecture--for the course-- is that actually there really are two sampling frequencies that we're seeing. One is the strobe, which is the strobe that you're running. The other is the inherent frame rate for the TV, that's running at 30 frames a second. And that's one of the reasons, by the way, that people watching this on the video course are in fact seeing a flicker or a beating or modulation between the two unsynchronized frame rates.

DR. HAROLD EDGERTON: I'll run the frequencies of the strobe up, so we get two of them in there. You keep watching, we had all these other interesting patterns. There's two now. And I'll make the two bounce on each other. You get all these patterns for free. You design a disk to show one thing and then when you run it, you find all the other patterns.

PROFESSOR: I think it would be a terrific homework problem for the video course, to have them all sit down and analyze all the frequencies that they're seeing and what they're being aliased to. What do you think of that?

DR. HAROLD EDGERTON: That's a good idea. As a teacher, I love to give quizzes. Find out whether the students are listening.

PROFESSOR: I think that'll chase a few people away from the course, that's what I like--

DR. HAROLD EDGERTON: No, it attracts them because you get involved in these optical things, there's no limit on what you can do.

PROFESSOR: Let's bring the incandescent lights back up again, just to remind everybody that in back of all these are some frequencies that are a lot higher than the ones that we begin to get the impression that we're watching.

DR. HAROLD EDGERTON: It's just a motor running at constant speed with a pattern on it.

PROFESSOR: Doc, I have to say that there aren't many people I know that have as much fun in their work as you do.

DR. HAROLD EDGERTON: Well, I'm a lucky man.

PROFESSOR: Well, what I'd like to do now, maybe, is take a look at one last experiment, if you could.

DR. HAROLD EDGERTON: Sure.

PROFESSOR: And what I'd like to do is go take a look at, I guess, what sometimes is called the-- well, not the water drop experiment-- what's the name of the--

DR. HAROLD EDGERTON: You mean the Double Piddler Hydraulic Happening Machine?

PROFESSOR: That's the one I was thinking of. Let's take a look over there.

DR. HAROLD EDGERTON: Come on, Justine, let's go and turn on the water.

PROFESSOR: So, Doc, this is the--what did you call it DPHHM for Double Piddler Hydraulic Happening Machine? Got it.

DR. HAROLD EDGERTON: It looks like a continuous stream, but it's not. It's a pump over there. It's pumping 60 pulses a second. The water is coming out in spurts.

PROFESSOR: So actually, again it's the 60 pulses a second your eye can't follow.

DR. HAROLD EDGERTON: Your eye's no good at 60 a second.

PROFESSOR: Basically looks like a blur.

DR. HAROLD EDGERTON: It is a blur, a nice juicy blur. Now we put the strobe on.

PROFESSOR: So again, I guess we have this essentially aliased down. And again with the incandescent light, you can see both the high frequency and the alias frequency. And let's see, I guess that's what the frequency close to DC and we can adjust it so that it's stopped.

DR. HAROLD EDGERTON: All right, make the water go up.

PROFESSOR: And then we can actually make it go up.

DR. HAROLD EDGERTON: Of course, nobody believes that.

PROFESSOR: Yeah, in fact, let me just again, to stress this point to the class. The idea here of the phase reversal-- of course, you can see it in the time domain-- you just think about when the flashes of light come. But if you think of these impulses that we have in the frequency domain and we're aliasing as we change the sampling frequency, what happens is that these impulses cross over and what that means is that we get a phase reversal depending on which phases are associated with which side of DC so that's kind of the idea of the phase reversal. Let's turn the--

DR. HAROLD EDGERTON: Well, we tried to have Justine put her finger in between those two drops.

PROFESSOR: Yeah, let's turn the incandescent light off first. And--

DR. HAROLD EDGERTON: Take one finger out now. Put it right in between those two drops.

PROFESSOR: Justine,you think you can do that?

DR. HAROLD EDGERTON: Better get on the other side. Use your other hand, so they can see with it. You can--

PROFESSOR: Think you can get your finger in there?

PROFESSOR: Whoop, there's water there all the time.

PROFESSOR: Well I don't know, Doc. It seems to me if we-- can't we just adjust this so that the dots just go through each other?

DR. HAROLD EDGERTON: Sure.

PROFESSOR: Now if the dots can do it, Justine, how come you can't get your finger in there?

JUSTINE: I don't know.

PROFESSOR: Why don't you try that once more? I guess not.

DR. HAROLD EDGERTON: No, that's one thing you can't do with it.

PROFESSOR: Well let's bring the lights back up and again, just to stress the point, here we are at DC, here we are at a frequency that's just a little above DC, and we can go back down to DC and we can actually get a phase reversal. And I guess, if we do this long enough, we can empty out the whole ocean and put it back in wherever it comes from. Isn't that right?

DR. HAROLD EDGERTON: And we caution the students when they run this, not to run it too long--

PROFESSOR: That's right.

DR. HAROLD EDGERTON: We've got the bucket here.

PROFESSOR: You have to be careful--

DR. HAROLD EDGERTON: --and it's been a while since somebody believes me.

PROFESSOR: Well, I don't know about them, but I guess I believe you, Doc.

DR. HAROLD EDGERTON: I'll put a little more pressure on so we get little more interesting patterns. Little patterns or surface tension that's pulling in things together. We have these machines, they're all over the place. They're a lot of fun.

PROFESSOR: Well, Doc, this is really terrific. I think that this whole idea of using aliasing and strobes and the kinds of things that you do with them are just fantastic. And we really appreciate the chance to come in here and see the demonstration.

DR. HAROLD EDGERTON: Well, that's the whole game. We've [? been happy to use ?] them for years and probably will for many years to come.

PROFESSOR: So as I emphasized at the beginning, in lots of situations aliasing can, in fact, be very useful. Also what this demonstrates is that particularly when you have a colleague, like Doc Edgerton, aliasing and for that matter science, in general, can be an awful lot of fun.

DR. HAROLD EDGERTON: Thanks for coming in.

PROFESSOR: Thanks a lot, Doc.

DR. HAROLD EDGERTON: See you again.

PROFESSOR: And thank you, Justine.

JUSTINE: You're welcome.

PROFESSOR: Well, I have to say that visit was an awful lot of fun for me and for Justine and in fact, for the whole camera crew that was there. And hopefully, all of you at some point will also have a chance to visit at Strobe Alley.

Well, hopefully what we've gone through today gives you a good feeling for the concepts of sampling and aliasing and both, why it might be useful and why we might want to avoid it.

In the next lecture, we'll continue on the discussion of sampling. And in particular, what I'll be talking about is the interpretation of the reconstruction process not in the frequency domain, but in a time domain and interpretation specifically associated with the concept of interpolating between the samples. We'll then proceed from there to a discussion as I've alluded to in several lectures of what I've referred to as discrete-time processing of continuous-time signals, very heavily exploiting the concept and issues associated with sampling. Thank you.