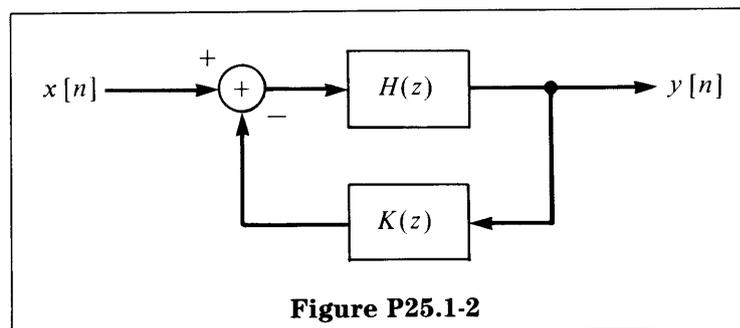
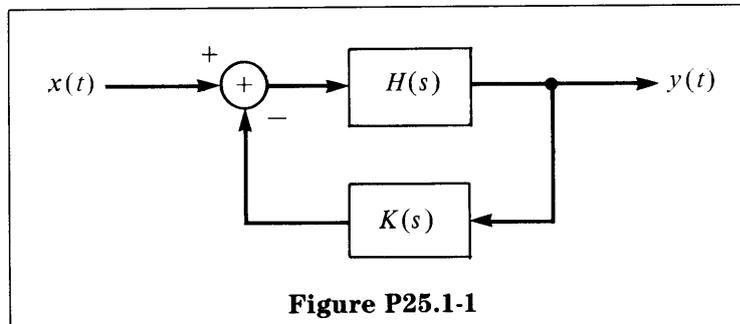


# 25 Feedback

## Recommended Problems

### P25.1

Consider the two feedback systems in Figures P25.1-1 and P25.1-2.



(a) Express the overall system functions

$$Q(s) = \frac{Y(s)}{X(s)} \quad \text{and} \quad Q(z) = \frac{Y(z)}{X(z)}$$

in terms of the system functions in the forward and feedback paths.

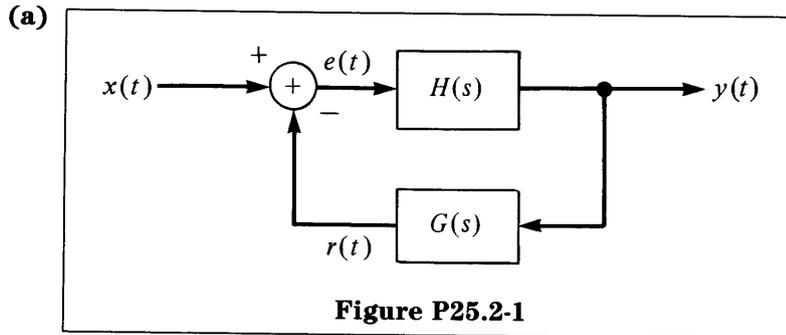
For the remainder of the problem, assume that  $K(s)$  and  $K(z)$  are both constant values  $K$  and that

$$H(s) = \frac{2}{s-2} \quad \text{and} \quad H(z) = \frac{2}{z-2} = \frac{2z^{-1}}{1-2z^{-1}}$$

- (b) Plot the pole-zero pattern of  $Q(s)$  and  $Q(z)$  for  $K = 0, -1$ , and  $1$ .
- (c) Indicate the locus in the  $s$  plane of the poles of  $Q(s)$  and in the  $z$  plane the poles of  $Q(z)$  as  $K$  increases from  $K = 0$  toward large positive values and as  $K$  decreases from  $K = 0$  toward large negative values.
- (d) Specify the range of values of  $K$  for which each of the systems is stable.

**P25.2**

For the systems in Figures P25.2-1 to P25.2-3, derive the transfer functions indicated.



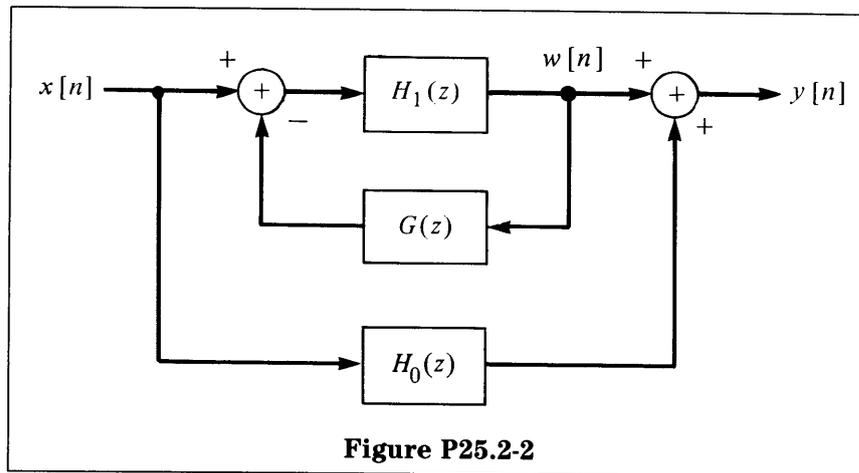
(i)  $\frac{Y(s)}{X(s)}$

(ii)  $\frac{E(s)}{X(s)}$

(iii)  $\frac{Y(s)}{E(s)}$

(iv)  $\frac{Y(s)}{R(s)}$

(b)  $\frac{Y(z)}{X(z)}$



(c)  $\frac{Y(s)}{X(s)}$

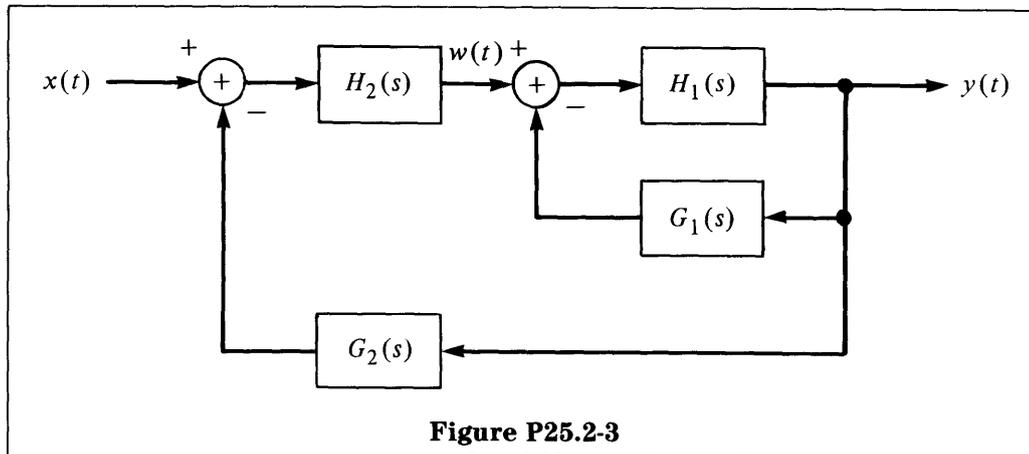


Figure P25.2-3

**P25.3**

Consider the system  $G(s)$  with frequency response  $G(j\omega)$  indicated in Figure P25.3-1.

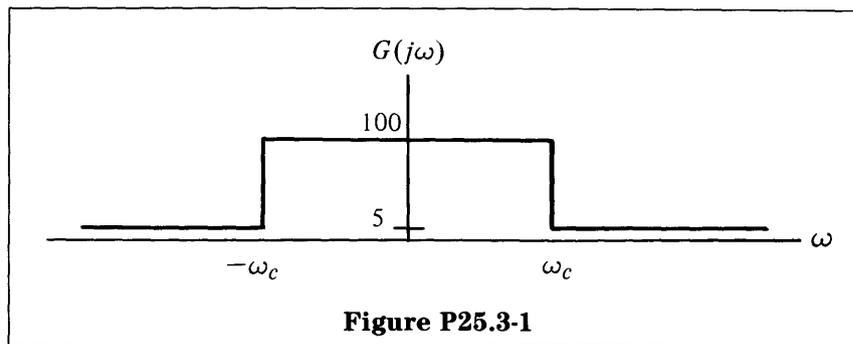


Figure P25.3-1

(a) Sketch the approximate overall frequency response of each of the feedback systems in Figure P25.3-2.

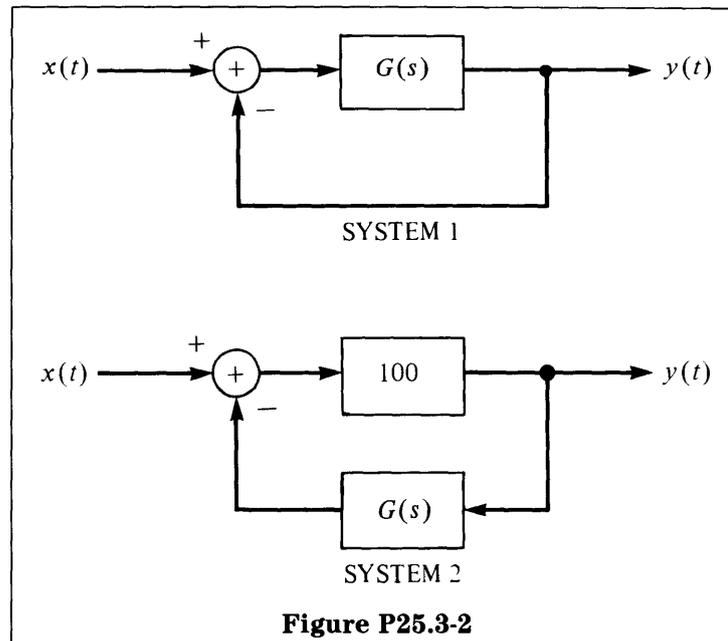
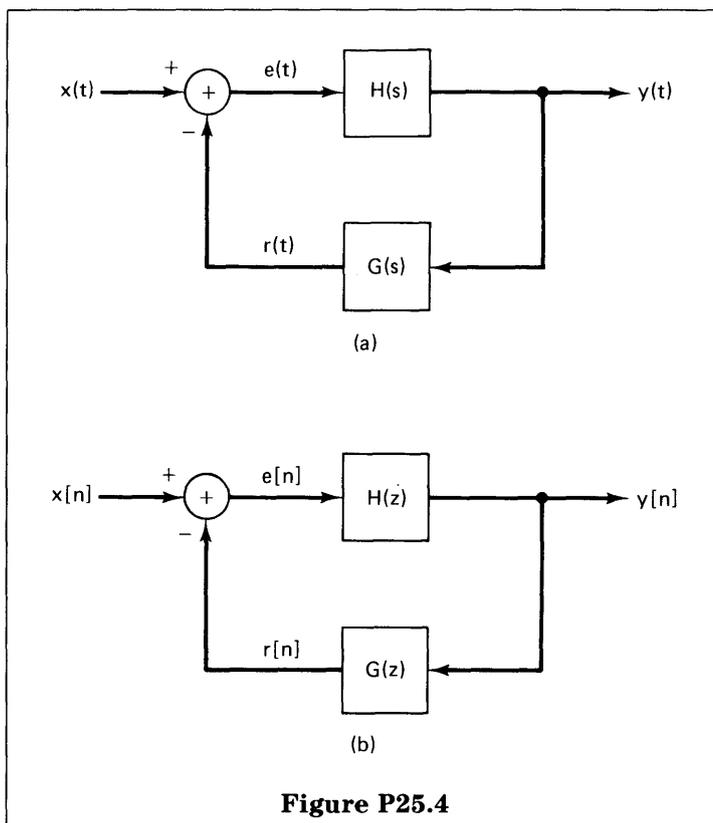


Figure P25.3-2

- (b) Determine which of the two systems in part (a) attempts to equalize the gain  $G(j\omega)$ , i.e., to make it more constant over all frequencies, and which system is intended as an approximation to a system that is the inverse of  $G(j\omega)$ .

**P25.4**

Consider the basic feedback systems of Figure P25.4. Determine the closed-loop system impulse response for each of the following specifications of the system functions in the forward and feedback paths.



**Figure P25.4**

(a)  $H(s) = \frac{1}{(s + 1)(s + 3)}$ ,  $G(s) = 1$

(b)  $H(s) = \frac{1}{s + 3}$ ,  $G(s) = s + 1$

(c)  $H(s) = \frac{1}{2}$ ,  $G(s) = e^{-s/3}$

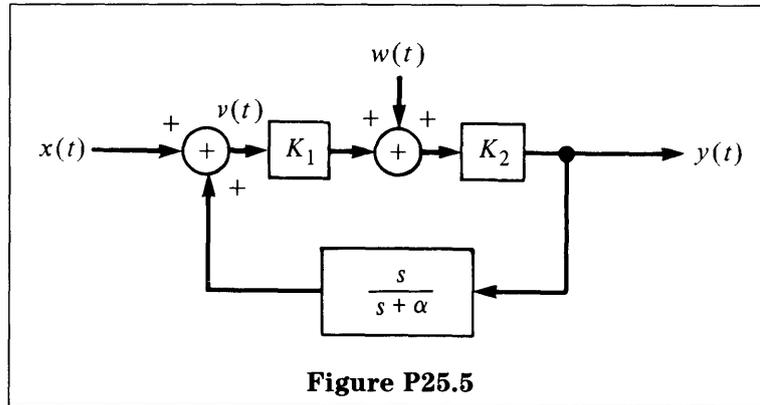
(d)  $H(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}}$ ,  $G(z) = \frac{2}{3} - \frac{1}{6}z^{-1}$

(e)  $H(z) = \frac{2}{3} - \frac{1}{6}z^{-1}$ ,  $G(z) = \frac{z^{-1}}{1 - \frac{1}{2}z^{-1}}$

# Optional Problems

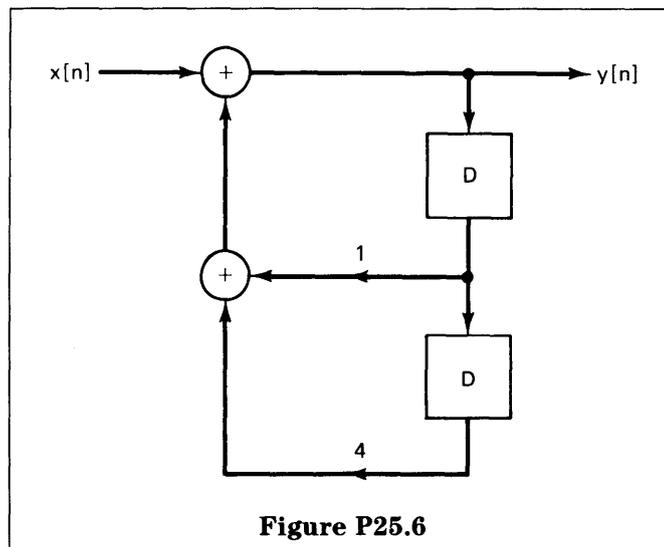
## P25.5

For the system given in Figure P25.5, find  $Y(s)$  in terms of  $X(s)$  and  $W(s)$ .



## P25.6

Consider the causal discrete-time system depicted in Figure P25.6.



- (a) Show that this is not a stable system.
- (b) Suppose that we allow feedback with one unit of delay. That is, suppose that

$$x[n] = x_e[n] - Ky[n - 1],$$

where  $x_e[n]$  is an externally applied signal now regarded as the input to the overall closed-loop system. Is it possible to stabilize the system with feedback of this form? If so, find the maximum range of values of  $K$  for which the system is stable.

- (c) Suppose that instead of using feedback with one unit of delay as in part (b), we allow feedback with two units of delay, so that

$$x[n] = x_e[n] - Ky[n - 2]$$

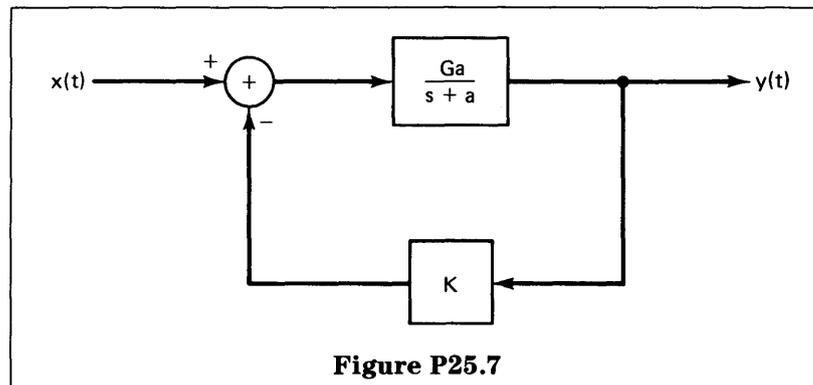
Specify the full range of values of  $K$  (if any such values exist) for which this system is stable.

**P25.7**

In this problem we illustrate how feedback can be used to increase the bandwidth of an amplifier. Consider an amplifier whose gain falls off at high frequencies. Specifically, suppose that the system function of this amplifier is

$$H(s) = \frac{Ga}{s + a}$$

- (a) What is the dc gain of the amplifier (i.e., the magnitude of its frequency response to zero frequency)?
- (b) What is the system time constant?
- (c) Suppose that we define bandwidth as the frequency at which the magnitude of the amplifier frequency response is  $1/\sqrt{2}$  times its magnitude of dc. What is the bandwidth of this amplifier?
- (d) Suppose that we place this amplifier in a feedback loop as depicted in Figure P25.7. What is the dc gain of the closed-loop system? What are the time constant and bandwidth of the closed-loop system?



- (e) Find the value of  $K$  that leads to a closed-loop bandwidth that is exactly double the bandwidth of the open-loop amplifier. What are the corresponding closed-loop system time constant and dc gain?

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