14 Demonstration of Amplitude Modulation

Solutions to Recommended Problems

S14.1

(a) We see in Figure S14.1-1 that the modulating cosine wave has a peak amplitude of $2K = 2$, so that $K = 1$. At the point in time when the modulating cosine wave is zero, the total signal is $A = 2$, so $K/A = 0.5$. Therefore, the signal has 50% modulation. See Figure S14.1-1.

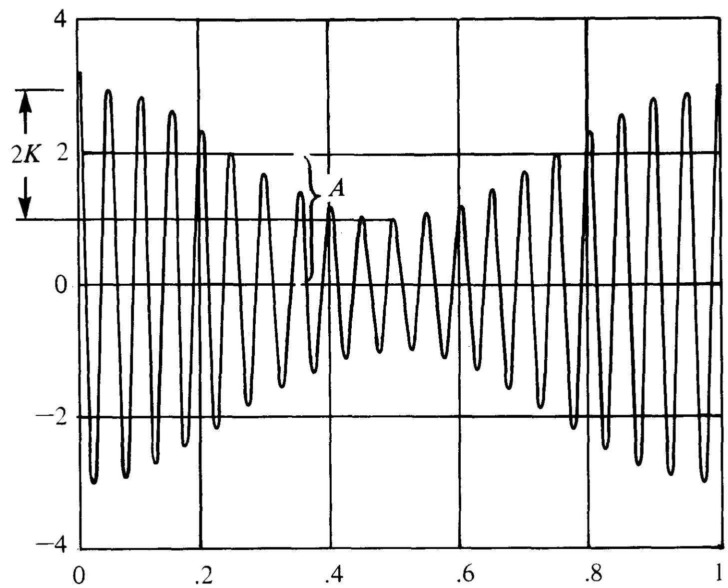

Figure S14.1-1

(b) $2K = 2$, $K = 1$, $A = 1$, so $K/A = 1$, and the signal has 100% modulation. See Figure S14.1-2.

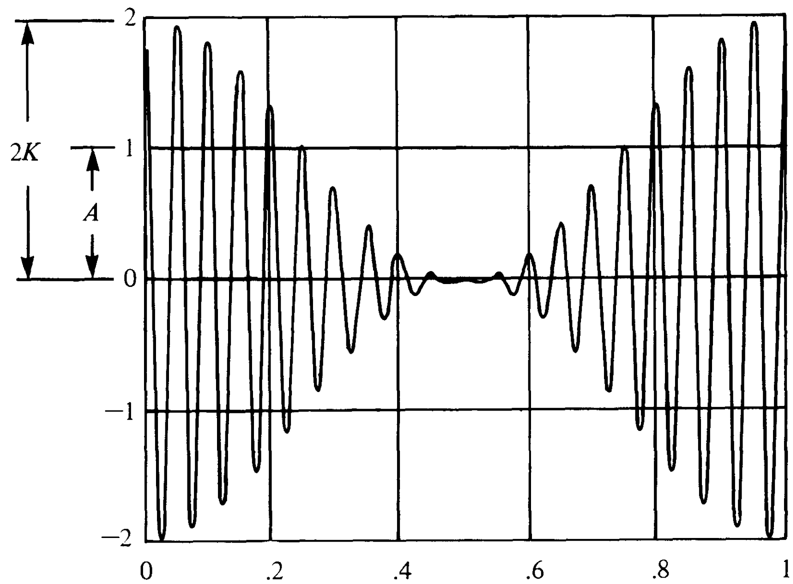

Figure S14.1-2

(c) $2K = 2$, $K = 1$, $A = 0.5$, so $K/A = 2$, and the signal has 200% modulation.

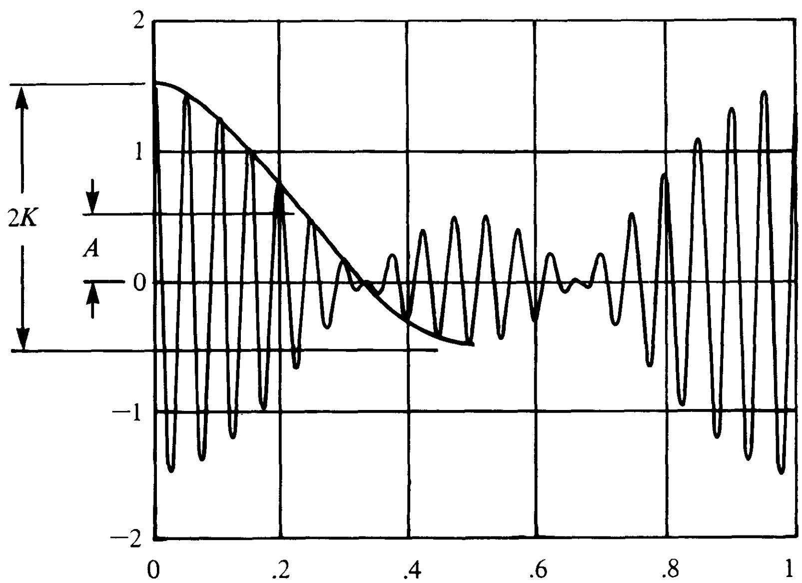

Figure S14.1-3

S14.2

(a) (i)

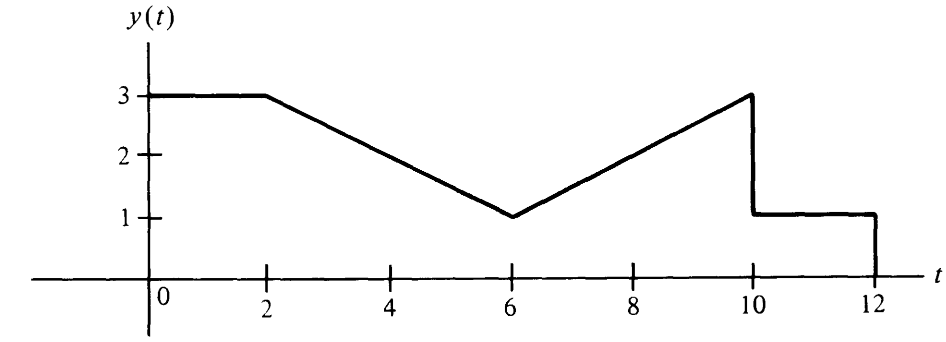

Figure S14.2-1

(ii)

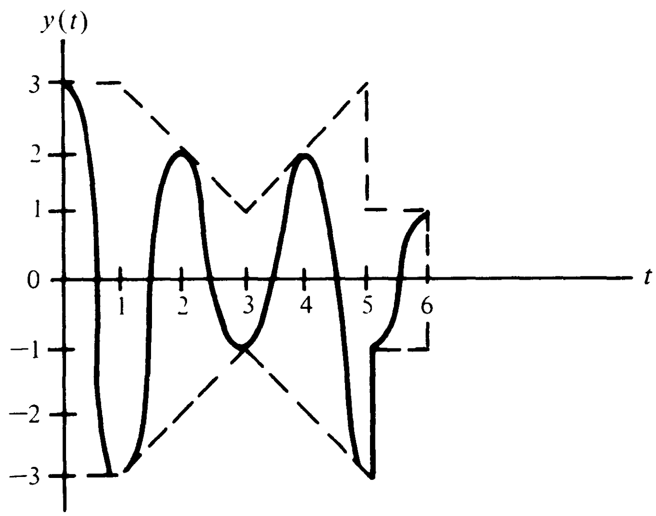

Figure S14.2-2

(iii)

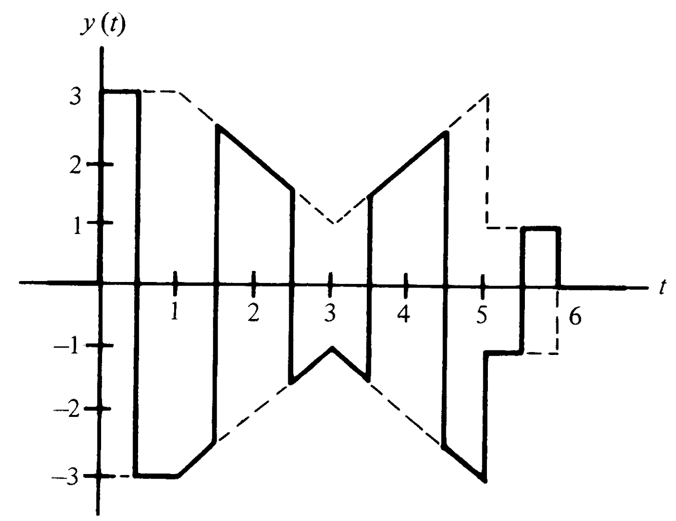

Figure S14.2-3

(b) (i)

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} x\left(\frac{t}{2}\right) e^{-j\omega t} dt, \qquad t' = \frac{t}{2}, \qquad dt' = \frac{1}{2}\,dt$$

$$= \int_{-\infty}^{\infty} x(t')e^{-j\omega 2t'} 2\,dt'$$

$$= 2X(2\omega)$$

Therefore, $Y(\omega)$ is a compressed version of $X(\omega)$. See Figure S14.2-4.

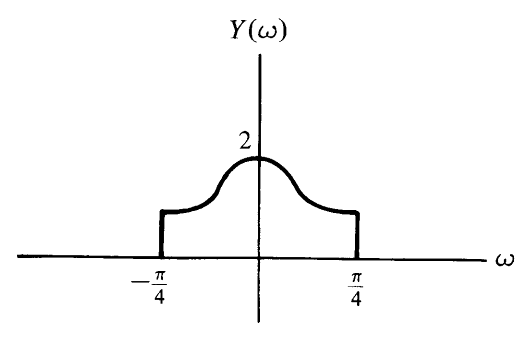

Figure S14.2-4

(ii) From the convolution theorem,

$$Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)H(\omega - \Omega)\,d\Omega,$$

where $\cos \pi t \overset{\mathcal{F}}{\longleftrightarrow} H(\omega)$, and $H(\omega)$ is as shown in Figure S14.2-5. Therefore, $Y(\omega)$ is as given in Figure S14.2-6.

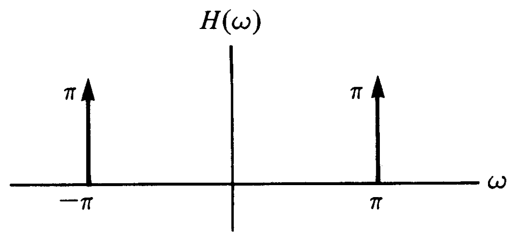

Figure S14.2-5

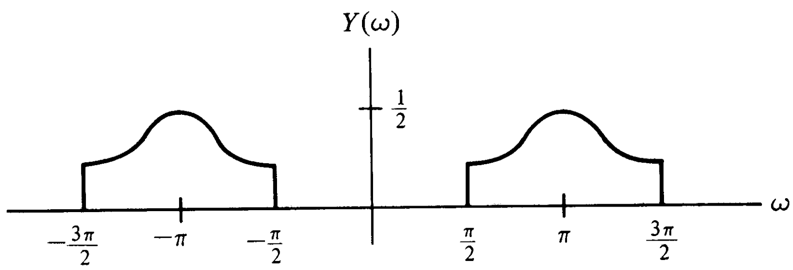

Figure S14.2-6

(iii) $Y(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\Omega)P(\omega - \Omega)\, d\Omega$

$P(\omega)$ is an impulsive spectrum, as shown in Figure S14.2-7, because the corresponding $p(t)$ is periodic. (Note that only odd harmonics are present.)

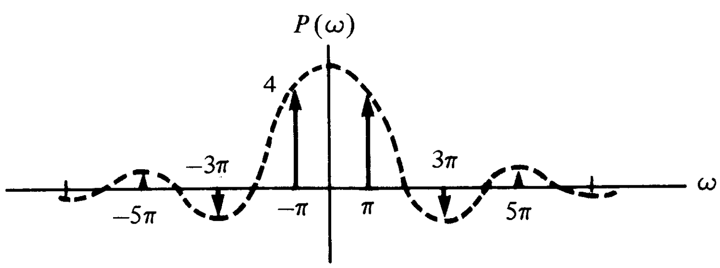

Figure S14.2-7

Therefore $Y(\omega)$ is as shown in Figure S14.2-8.

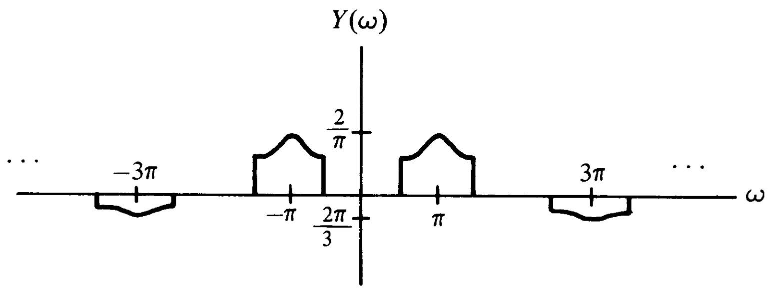

Figure S14.2-8

S14.3

(a) ii
(b) i
(c) iii
(d) vi
(e) v
(f) iv
(g) vii
(h) x
(i) ix
(j) viii

S14.4

(a) We are considering

$$X(\Omega) = \sum_{n=0}^{N-1} x[n]e^{-j\Omega n},$$

which is effectively the Fourier transform of a signal of infinite duration multiplied by a window of length N:

$$X(\Omega) = \sum_{n=-\infty}^{\infty} \cos \omega_0 nT(u[n] - u[n - N])e^{-j\Omega n}$$

From the convolution theorem we can compute the Fourier transform of the product of these two sequences:

$$\cos \omega_0 nT \overset{\mathcal{F}}{\longleftrightarrow} \pi[\delta(\Omega - \omega_0 T) + \delta(\Omega + \omega_0 T)], \quad -\pi < \Omega < \pi$$

$$u[n] - u[n - N] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} = e^{-j\Omega(N-1)/2} \frac{\sin N\Omega/2}{\sin \Omega/2}$$

Therefore,

$$X(\Omega) = \frac{1}{2} e^{-j(\Omega-\omega_0 T)(N-1)/2} \frac{\sin[N(\Omega - \omega_0 T)/2]}{\sin[(\Omega - \omega_0 T)/2]} + \frac{1}{2} e^{-j(\Omega+\omega_0 T)(N-1)/2} \frac{\sin[N(\Omega + \omega_0 T)/2]}{\sin[(\Omega + \omega_0 T)/2]},$$

as shown in Figure S14.4-1. (Note that the spectrum is periodic with period 2π.)

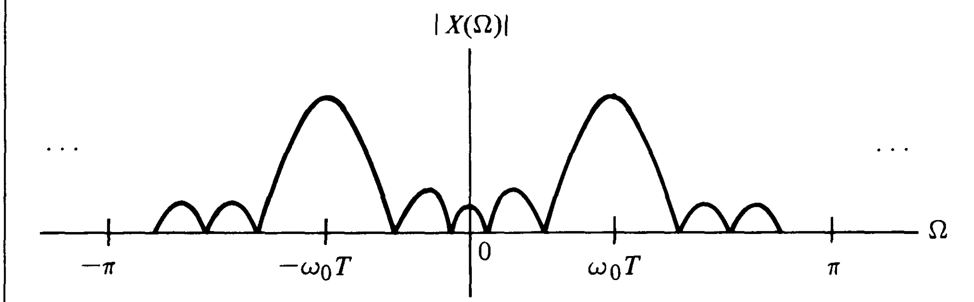

Figure S14.4-1

(b)

$$X(\Omega_k) = \sum_{n=0}^{N-1} x[n] e^{-j\Omega_k n}$$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{N-1} \cos \omega_0 n T e^{-j(2\pi k/N)n}$$

$$= \sum_{n=0}^{N-1} \frac{1}{2} e^{j\omega_0 n T} e^{-j(2\pi k/N)n} + \sum_{n=0}^{N-1} \frac{1}{2} e^{-j\omega_0 n T} e^{-j(2\pi k/N)n}$$

$$= \frac{1}{2}\left(\frac{1 - e^{j(\omega_0 T - 2\pi k/N)N}}{1 - e^{j(\omega_0 T - 2\pi k/N)}}\right) + \frac{1}{2} \frac{1 - e^{j(-\omega_0 T - 2\pi k/N)N}}{1 - e^{j(-\omega_0 T - 2\pi k/N)}}$$

(i) For $\omega_0 T = 2\pi(\frac{2}{5})$ and $N = 5$, the first term is zero for

$$k = \ldots -3, 2, 7, \ldots$$

However, when $k = 2$ we have the ratio of

$$\frac{1}{2}\left(\frac{1 - e^{j2\pi(2/5-k/5)5}}{1 - e^{j2\pi(2/5-k/5)}}\right) = \frac{0}{0}$$

and we treat the limit as $k \to 0$. Using L'Hôpital's rule, we have $\frac{1}{2}(5) = 2.5$. Similarly, the second term is zero except when $k = \ldots -2, 3, 8, \ldots$. Taking the limit yields 2.5. So $X(2\pi k/5)$ is as shown in Figure S14.4-2.

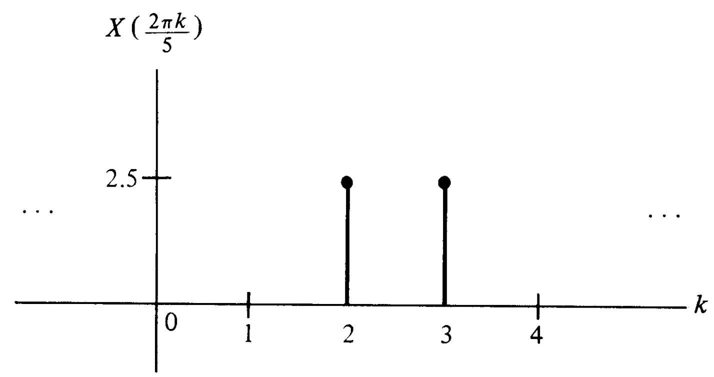

Figure S14.4-2

Note that $X(2\pi k/5)$ is periodic in k with period 5 since $X(\Omega)$ is periodic in Ω with period 2π.

(ii) $$X\left(\frac{2\pi k}{N}\right) = \frac{1}{2}\left(\frac{1 - e^{j(\omega_0 T - 2\pi k/N)N}}{1 - e^{j(\omega_0 T - 2\pi k/N)}}\right) + \frac{1}{2}\left(\frac{1 - e^{j(-\omega_0 T - 2\pi k/N)N}}{1 - e^{j(-\omega_0 T - 2\pi k/N)}}\right)$$

Now $\omega_0 T = 2\pi\frac{3}{10}$, and the numerator and denominator are nonzero for all k. Evaluating the preceding expression yields $X(k)$ as shown in Figure S14.4-3.

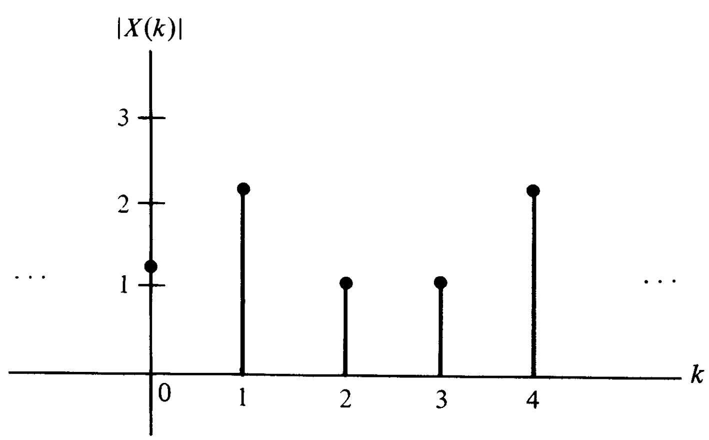

Figure S14.4-3