Chapter 1

INTRODUCTION

1.0 INTRODUCTION

The human is first of all a mechanical entity who exists in a mechanical environment. The day-by-day habits of man are dictated largely by such considerations as how rapidly he can transport or feed himself. Communication with his environment is geared to such mechanical operations as the time required for his eye to scan a page or the speed with which he can speak or typewrite. Furthermore, his standard of living is very much a function of his ability to augment human muscle for better transportation and for the diverse industrial processes needed in an advanced society.

There are two major conclusions to be drawn from these thoughts. First, the unaided human faculties operate on limited time and size scales. Thus the mechanical effects of electric and magnetic forces on ponderable bodies were observed and recorded by the Greeks as early as 500 B.C., and electricity and magnetism were developed largely as classical sciences in the nineteenth century, on the basis of unaided human observations. Coulomb enunciated his inverse square law from measurements taken with an electrical torsion balance; magnetic forces, as they influenced ponderable objects such as magnetized needles, were the basis of experiments conducted by Oersted and Ampère. These electromechanical experiments constituted the origins of the modern theories of electricity and magnetism. Faraday and Maxwell unified the subjects of electrostatics and magnetostatics into a dynamical theory that predicted phenomena largely beyond the powers of direct human observation. Thus today we recognize that electromagnetic theory encompasses not only the electromechanical effects that first suggested the existence of electric and magnetic fields but also numerous radiation effects, whether they involve radio frequency waves or x-rays. Nonetheless, when man controls these phenomena, detects their existence, and puts them to good use, he most often does so by some type of electromechanical interaction—from the simple act of turning a switch to the remote operation of a computer with a teletypewriter.

The second major conclusion to be drawn from our opening remarks is that man's need for motive power for transportation and industrial processes is satisfied largely by conversion of electric energy to mechanical energy. Energy in electric form is virtually useless, yet the largest and fastest growing segment of our economy is the electric utility industry, whose source of income is the sale of electric energy. This is eloquent testimony to the fact that electric energy can be converted easily into a variety of forms to aid man in his mechanical environment. It is remarkable that the same 60-Hz power line can supply the energy requirements of a rolling mill, a television station, a digital computer, a subway train, and many other systems and devices that provide a fuller and more comfortable life. In the vast majority of these examples electromechanical energy conversion is required because of man's basic need for mechanical assistance.

As long as engineers are concerned with making the electrical sciences serve human needs, they will be involved with electromechanical phenomena.

1.0.1 Scope of Application

Because they serve so many useful functions in everyday situations, *transducers* are the most familiar illustration of applied electromechanical dynamics. These devices are essential to the operation of such diverse equipment as automatic washing machines, electric typewriters, and power circuit breakers in which they translate electrical signals into such useful functions as opening a switch. The switch can be conventional or it can open a circuit carrying 30,000 A while withstanding 400,000 V 2 msec later. The telephone receiver and high-fidelity speaker are familiar transducers; less familiar relatives are the high-power sonar antenna for underwater communication or the high-fidelity shake tables capable of vibrating an entire space vehicle in accordance with a recording of rocket noise.

Electromechanical transducers play an essential role in the automatic control of industrial processes and transportation systems, where the ultimate goal is to control a mechanical variable such as the thickness of a steel sheet or the speed of a train. Of course, a transducer can also be made to translate mechanical motion into an electrical signal. The cartridge of a phonograph pickup is an example in this category, as are such devices as telephone transmitters, microphones, accelerometers, tachometers and dynamic pressure gages.

Not all transducers are constructed to provide mechanical input or output. The (electro)mechanical filter is an example of a signal-processing device that takes advantage of the extremely high Q of mechanical circuits at relatively low frequencies. Filters, delay lines, and logic devices capable of performing even above 30 MHz are currently the object of research on electromechanical effects found in piezoelectric and piezomagnetic materials.

Primary sources of energy are often found in mechanical form in the kinetic energy of an expanding heated gas and in the potential energy of water at an elevation. Electromechanics has always played a vital role in obtaining large amounts of electric power from primary sources. This is accomplished by using large magnetic field-type devices called *rotating machines*. Today a single generator can produce 1000 MW (at a retail price of 2 cents/kWh this unit produces an income of $20,000/h), and as electric utility systems grow larger generating units (with attendant problems of an unprecedented nature) will be needed. This need is illustrated by the fact that in 1960 the national peak load in the United States was 138,000 MW, whereas it is expected that in 1980 it will be 493,000 MW, an increase of more than 250 per cent in 20 years.

A large part of this electric power will be used to drive electric motors of immense variety to do a multitude of useful tasks, from moving the hands of an electric clock at a fraction of a watt to operating a steel rolling mill at 20 MW.

Because of our need for great amounts of energy, it is in the national interest to seek ways of producing it more efficiently (to conserve natural resources) and with less costly equipment (to conserve capital). The *magnetohydrodynamic* generator, which employs an expanding heated gas as the moving conductor, shows some promise of meeting one or both of these objectives. Another possibility is the use of the interaction between charged particles and a flowing, nonconducting gas to achieve *electrohydrodynamic* power generation. Versions of this machine are similar in principle to the *Van de Graaff* generator which is currently producing extremely high voltages (20 million volts) for a variety of purposes, including medical treatment, physical research, and irradiation of various substances.

The efficient and economical conversion of mechanical energy to electrical form is not only of great interest to the rapidly expanding utility industry but is also of extreme importance to the space program, in which sources of electric power must satisfy new engineering requirements imposed by the environment, with obvious limitations on weight and size and with stringent requirements on reliability.

Electromechanical devices provide *power amplification* of signals for purposes similar to those involving electronic amplifiers; for example, in control systems in which large amounts of power (up to about 20 MW) must be produced with high fidelity over a bandwidth from zero to a few Hertz *dc rotating machines* are used. From this the impression is obtained that electromechanical amplifiers function only at low frequencies; but there are electromechanical devices that provide amplification in the gigacycle-per-second range—*electron beam devices* which, like other physical electronic devices, depend on the small mass of the electron for high-frequency operation.

In current research concerned with controlled thermonuclear fusion the plasma can be regarded for some purposes as a highly conducting gas elevated to such a high temperature that it cannot be contained by solid boundaries. Thus proposed thermonuclear devices attempt to contain the plasma in a magnetic bottle. This illustrates another important application of electromechanical dynamics—the *orientation*, *levitation*, or *confinement* of mechanical media. More conventional examples in this category are those that use magnetic or electric fields to levitate the rotor of a gyroscope, to suspend the moving member of an accelerometer, or to position a model in a wind tunnel. Metallurgists employ ac magnetic fields to form a crucible for molten metals that must be free of contamination, and electric fields are proposed for orienting cyrogenic propellants in the zero-gravity environment of space. The use of electric and magnetic fields in shaping malleable metals and solidifying liquids has just begun.

The *propulsion* of vehicles represents still another application of electromechanics. Even when the primary source of energy is a rotating shaft from a reciprocating engine or a turbine, as in a locomotive or ship, the problem of transmitting and controlling the power to the wheels or propeller is simplified by converting the power to electrical form with a generator and installing electric motors to propel the vehicle. An important addition to this class of vehicles would be the electric car in which energy is stored in batteries and the wheels are driven by electric motors. Less familiar electromechanical propulsion schemes are being developed, largely for space applications, which make use of magnetohydrodynamic or electrohydrodynamic acceleration of matter to provide thrust. In this regard the particle accelerators required in high-energy physics research should be recognized as electromechanical devices.

1.0.2 Objectives

It should be apparent from the discussion of the preceding section that electromechanical dynamics covers a broad range of applications, many of which represent highly developed technologies, whereas others are the subject of research or development. In either case a single application could be the subject of an entire book and in many cases books already exist. Our objective here is to lay a cohesive and unified foundation by treating those concepts and techniques that are fundamental to an understanding of a wide range of electromechanical phenomena. As a consequence, we do not dwell at length on any area of application.

With our basic unified approach it is often difficult to distinguish between those aspects of electromechanics that may be considered research in the scientific sense and those that represent engineering applications. For example, there are many practical uses for a magnetohydrodynamic flow meter, yet the type of theoretical model needed in its study is also pertinent to an understanding of the origin of the earth's magnetic field as it is generated by motion of the molten interior of the earth. In fact, a study of magnetohydrodynamics involves models that are germane to an engineering problem such as the levitation of a molten metal, an applied physics problem such as plasma confinement, or a problem of astrophysical interest such as the dynamics of stellar structures.

The subject of electromechanical dynamics, as we approach it in the following chapters, provides a foundation for a range of interests that extends from the purely scientific to engineering applications and from interactions that occur in systems that can be represented by lumped parameters to those that need continuum representations.

The selection of appropriate mathematical models for electromechanical systems is a process that requires the maturity and insight that can result only from experience with electromechanical phenomena. Of course, the model chosen depends on the nature of the system being studied and the accuracy required. We shall not try to develop a formalism for the largely intuitive process of modeling but rather shall study representative systems with a variety of mathematical models to illustrate the principal phenomena that result from electromechanical interactions. In the course of this study the student should develop facility with the basic models and the mathematical tools used in their analysis and should acquire the insight into the interrelations among the physical phenomena that is necessary for him to be able to develop mathematical models on his own.

1.1 ELECTROMAGNETIC THEORY

The mathematical description of the electrical part of any electromechanical system is based on electromagnetic theory. We therefore assume that the reader is familiar with the basic theory and in particular with magnetostatics and electrostatics.

The subject of electromechanics necessarily includes the behavior of electromagnetic fields in the presence of moving media. In this introductory chapter it therefore seems appropriate to review the laws of electricity and magnetism and to include a discussion of those extensions of the theory required to account for the effects of moving media. This review, however, would represent a digression from our main purpose—the study of electromechanical dynamics. Consequently a discussion is presented in Appendix B for completeness. We can get well into the study of electromechanical dynamics with a few simple extensions of magnetostatic and electrostatic theory. Therefore we cite the electromagnetic equations that form the basis for our study and start to use them immediately. The equations can be accepted as postulates, justified by their relation to ordinary magnetostatic and electrostatic theory and by the fact that they give adequate representation of the electromechanical systems we shall study. As our work progresses from the lumped-parameter models in Chapters 2 to 5 to situations requiring continuum models, the physical significance of the field equations in electromechanical interactions will be more apparent. It is at that point that a meaningful discussion can be made of the most significant effects of moving media on electromagnetic fields, and the reader may find that a study of Appendix B will be most helpful at that time.

1.1.1 Differential Equations

The symbols and units of electromagnetic quantities are defined in Table 1.1. At the outset, we consider two limiting cases of the electromagnetic field equations, which define the dynamics of quasi-static (almost static) magnetic and electric field systems. In spite of the restrictions implied by these limits, our models are adequate for virtually all electromechanical systems of technical importance. A discussion of the quasi-static approximations, which shows how both limiting cases come from the more general electromagnetic theory, is given in Appendix B.

1.1.1a Magnetic Field Systems

The electromagnetic field and source quantities in a magnetic field system are related by the following partial differential equations:

$$\nabla \times \mathbf{H} = \mathbf{J}_f, \tag{1.1.1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.1.2}$$

$$\nabla \cdot \mathbf{J}_f = 0, \tag{1.1.3}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \tag{1.1.4}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{1.1.5}$$

Thus in our magnetic field system, even with time-varying sources and deforming media, the magnetic field intensity **H** and flux density **B** are determined as if the system were magnetostatic. Then the electric field intensity **E** is found from the resulting flux density by using (1.1.5). This is the origin of the term quasi-static magnetic field system. In addition to these equations, we need constituent relations that describe how the physical

Table 1.1 Symbols and Units of Electromagnetic Quantities

Symbol	Field Variable Name	MKS Rationalized Units
$\mathbf{H}$	Magnetic field intensity	A/m
$\mathbf{J}_f$	Free current density	A/m^2
$\mathbf{K}_f$	Free surface current density	A/m
$\mathbf{B}$	Magnetic flux density	Wb/m^2
$\mathbf{M}$	Magnetization density	A/m
$\mathbf{E}$	Electric field intensity	V/m
$\mathbf{D}$	Electric displacement	C/m^2
ρ_f	Free charge density	C/m^3
σ_f	Free surface charge density	C/m^2
$\mathbf{P}$	Polarization density	C/m^2
$\mathbf{F}$	Force density	N/m^3
μ_0	Permeability of free space	$4\pi \times 10^{-7}$ H/m
ϵ_0	Permittivity of free space	8.854×10^{-12} F/m

properties of the materials affect the field and source quantities. The magnetization density $\mathbf{M}$ is introduced to account for the effects of magnetizable materials. The most common constitutive law for $\mathbf{M}$ takes the form

$$\mathbf{M} = \chi_m \mathbf{H}, \tag{1.1.6}$$

where χ_m is the magnetic susceptibility. An alternative way of expressing this relation is to define the permeability $\mu = \mu_0(1 + \chi_m)$, where μ_0 is the permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}, \tag{1.1.7}$$

in which case it follows from (1.1.4) that the constitutive law of (1.1.6) can also be written as

$$\mathbf{B} = \mu \mathbf{H}. \tag{1.1.8}$$

We shall make considerable use of this simple linear model for magnetizable materials.

Free currents in a stationary material most often arise from conduction induced by the electric field according to Ohm's law:

$$\mathbf{J}_f = \sigma \mathbf{E}, \tag{1.1.9}$$

where σ is the conductivity (mhos/m). A similar constitutive law relates the surface current density $\mathbf{K}_f$ to the electric field intensity $\mathbf{E}_t$ tangential to the surface

$$\mathbf{K}_f = \sigma_s \mathbf{E}_t, \tag{1.1.10}$$

where σ_s is the surface conductivity (mhos). These constitutive laws for the conduction process represent macroscopic models for the migration of charges in materials under the influence of an electric field.

Ideally, quasi-static magnetic field systems are characterized by perfectly conducting ($\sigma \to \infty$) current loops, in which case static conditions ($\partial/\partial t = 0$) result in zero electric field intensity. All practical conductors (except superconductors) have finite conductivity; consequently, a system is modeled as a magnetic field system when the electrical conductivity σ for a current loop is high enough to cause only small departures from the ideal. Thus in Chapter 2 iron structures with coils of wire wound around them are represented as ideal (electrically lossless) magnetic field systems in which the winding resistance is included as an external resistance in series with the winding terminals.

1.1.1b Electric Field Systems

The electromagnetic field and source quantities in an electric field system are related by the following partial differential equations:

$$\nabla \times \mathbf{E} = 0, \tag{1.1.11}$$

$$\nabla \cdot \mathbf{D} = \rho_f, \tag{1.1.12}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \tag{1.1.13}$$

$$\nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}, \tag{1.1.14}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}. \tag{1.1.15}$$

Equations 1.1.11 to 1.1.13 describe the fields in an electrostatic system. Hence in our electric field system, even with time-varying sources and geometry, the electric field intensity **E** and electric displacement **D** are determined as though the system were static. Then the current density $\mathbf{J}_f$ is determined by (1.1.14), which expresses conservation of charge. In turn, the magnetic field intensity **H** (if it is of interest) is found from (1.1.15). It is because of the basically electrostatic relationship between the electric field intensity and the free charge density that these equations define the dynamics of a quasi-static electric field system.

Ideally, a quasi-static electric field system is characterized by a set of perfectly conducting ($\sigma \to \infty$) equipotentials separated by perfectly insulating ($\sigma \to 0$) dielectrics, in which case static conditions ($\partial/\partial t = 0$) result in no current density $\mathbf{J}_f$, hence no magnetic field intensity **H**. Of course, real dielectrics have finite conductivity; thus a system is representable as an electric field system when the electrical conductivity is low enough to cause only a small departure from the ideal. In terms of the lumped-parameter representation to be introduced in Chapter 2, an electric field system is modeled as an ideal circuit consisting of equipotentials separated by perfect insulators with resistances connected externally between terminals to account for the finite conductivity of the dielectric.

In this book the constituent relation for the conduction process usually takes the form of (1.1.9) or (1.1.10). In electric field systems, however, there can be appreciable net charge density, and we must be careful to distinguish between a *net flow of charge*, which occurs in electrically neutral conductors such as metals, and a *flow of net charge*, which occurs in situations such as the drift of negative charge in a vacuum tube. To allow for this differentiation when it is needed a more general form of the conduction constituent relation is used:

$$\mathbf{J}_f = (\rho_{f+}\mu_+ + \rho_{f-}\mu_-)\mathbf{E}, \tag{1.1.16}$$

where ρ_{f+} and ρ_{f-} are the densities of the two species of moving charges and μ_+ and μ_- are the respective mobilities in the field intensity $\mathbf{E}$. When the charge densities and mobilities are constants, (1.1.16) reduces to (1.1.9). In some electric field systems ρ_{f+} and ρ_{f-} are not constant, and (1.1.16) allows us to include the variable charge densities in our conduction model. As questions appear in this regard, it will be helpful to refer to Sections B.1.2 and B.3.3.

To account for the polarization density $\mathbf{P}$ of a dielectric material, we most often use the linear relation

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E}, \tag{1.1.17}$$

where ϵ_0 is the permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12}\,\text{F/m} \tag{1.1.18}$$

and χ_e is the electric susceptibility. In terms of the material permittivity, $\epsilon = \epsilon_0(1 + \chi_e)$ (1.1.17) can also be written

$$\mathbf{D} = \epsilon\mathbf{E}, \tag{1.1.19}$$

where (1.1.13) has been used.

1.1.2 Integral Equations

It is often necessary to have the electromagnetic equations in integral form; for example, boundary conditions are found from integral equations and terminal quantities—voltage and current—are found by integrating field quantities.

In stationary systems the contours, surfaces, and volumes are all fixed in space and the transition from differential to integral equations is simply a matter of using the appropriate integral theorems. In electromechanical dynamics we need integral equations for contours, surfaces, and volumes that are deforming, and the resulting integral equations are different from those found in stationary systems. The formalism of integrating differential equations in the presence of motion is presented in Section B.4. The results are presented here essentially as postulates.

1.1.2a Magnetic Field Systems

The integral forms of (1.1.1) to (1.1.3) and (1.1.5) are

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J}_f \cdot \mathbf{n}\, da, \tag{1.1.20}$$

$$\oint_S \mathbf{B} \cdot \mathbf{n}\, da = 0, \tag{1.1.21}$$

$$\oint_S \mathbf{J}_f \cdot \mathbf{n}\, da = 0, \tag{1.1.22}$$

$$\oint_C \mathbf{E}' \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot \mathbf{n}\, da. \tag{1.1.23}$$

The contours C, surfaces S, and unit normal vectors $\mathbf{n}$ are defined in the conventional manner, as shown in Fig. 1.1.1. The surfaces of integration S

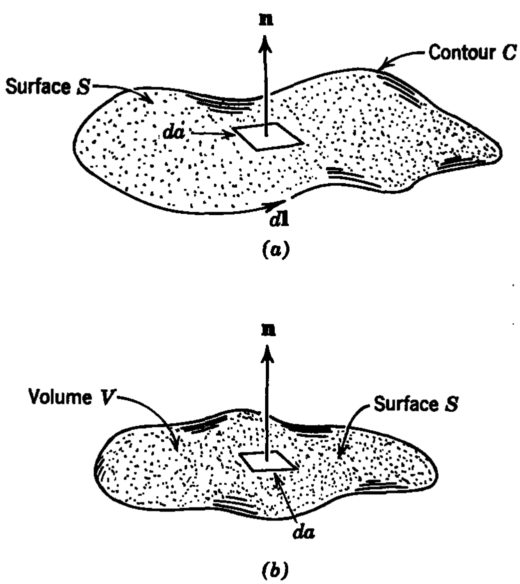

Fig. 1.1.1 (*a*) Surface S enclosed by the contour C, showing the right-handed relationship between the normal vector $\mathbf{n}$ and the line element $d\mathbf{l}$; (*b*) surface S enclosing a volume V. The normal vector $\mathbf{n}$ is directed outward, as shown.

for (1.1.21) and (1.1.22) enclose a volume V, whereas those of (1.1.20) and (1.1.23) are enclosed by a contour C.

Equations 1.1.20 to 1.1.23 are valid even when the contours and surfaces are deforming, as demonstrated in Appendix B. Note that in (1.1.23) the electric field intensity is written as $\mathbf{E}'$, and it is this value that would be measured by an observer attached to the deforming contour at the point in question. As demonstrated in Section B.4.1, when $\mathbf{E}' = \mathbf{E} \times (\mathbf{v} \times \mathbf{B})$, where $\mathbf{v}$ is the local velocity of the contour, (1.1.23) results from (1.1.5). More is said about the relation between quantities measured by observers in relative motion in Chapter 6.

In describing magnetic field systems, in addition to (1.1.20) to (1.1.23), we need constituent relations such as (1.1.8) and (1.1.9). We must keep in mind that these constituent relations are defined for stationary media. When there is motion, these equations still hold, but only for an observer moving with the medium. Thus we know that a perfect conductor can support no electric field intensity $\mathbf{E}'$. When the contour of (1.1.23) is fixed to a perfect conductor, the contribution to the contour integral from that portion in the conductor is zero, whether the conductor is moving or not. This is because $\mathbf{E}'$ is the quantity measured by an observer moving with the contour (conductor).

1.1.2b Electric Field Systems

The integral forms of (1.1.11) to (1.1.15) are

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0, \tag{1.1.24}$$

$$\oint_S \mathbf{D} \cdot \mathbf{n}\, da = \int_V \rho_f\, dV, \tag{1.1.25}$$

$$\oint_S \mathbf{J}_f' \cdot \mathbf{n}\, da = -\frac{d}{dt}\int_V \rho_f\, dV, \tag{1.1.26}$$

$$\oint_C \mathbf{H}' \cdot d\mathbf{l} = \int_S \mathbf{J}_f' \cdot \mathbf{n}\, da + \frac{d}{dt}\int_S \mathbf{D} \cdot \mathbf{n}\, da. \tag{1.1.27}$$

These equations are valid for moving and deforming contours C, surfaces S, and volumes V (see Fig. 1.1.1).

Equations 1.1.24 and 1.1.25 are the same as those used to find $\mathbf{E}$ and $\mathbf{D}$ in an electrostatics problem. The current density and magnetic field intensity have been written in (1.1.26) and (1.1.27) as $\mathbf{J}_f'$ and $\mathbf{H}'$ to indicate that they are the values that would be measured by an observer moving with the contour or surface at the point in question. It is shown in Section B.4.2 that

(1.1.26) and (1.1.27) result from integrating (1.1.14) and (1.1.15) when $\mathbf{J}'_f = \mathbf{J}_f - \rho_f\mathbf{v}$ and $\mathbf{H}' = \mathbf{H} - \mathbf{v} \times \mathbf{D}$, where $\mathbf{v}$ is the local velocity of the contour or surface.

1.1.3 Electromagnetic Forces

The force experienced by a test charge q moving with velocity $\mathbf{v}$ is

$$\mathbf{f} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}. \tag{1.1.28}$$

This is referred to as the Lorentz force and provides a definition of the fields $\mathbf{E}$ and $\mathbf{B}$. For this case of a single moving charge the quantity $q\mathbf{v}$ constitutes a current. Hence the first term in (1.1.28) is the force on a static charge, whereas the second is the force on a current.

In a continuum theory in which we are concerned with a charge density ρ_f and a current density $\mathbf{J}_f$ forces are stated in terms of a force density

$$\mathbf{F} = \rho_f\mathbf{E} + \mathbf{J}_f \times \mathbf{B}. \tag{1.1.29}$$

Free charge and free current densities are used in (1.1.29) to make it clear that this expression does not account for forces due to polarization and magnetization. The terms in (1.1.29) provide a continuum representation of the terms in (1.1.28). The averaging process required to relate the force density of (1.1.29) to the Lorentz force is discussed in Sections B.1.1 and B.1.3. For our present purposes we accept these relations as equivalent and reserve discussion of the conditions under which this assumption is valid for Chapter 8.

In the class of problems undertaken in this book one or the other of the force densities in (1.1.29) is negligible. Hence in the magnetic field systems to be considered the force density is

$$\mathbf{F} = \mathbf{J}_f \times \mathbf{B}, \tag{1.1.30}$$

whereas in the electric field systems

$$\mathbf{F} = \rho_f\mathbf{E}. \tag{1.1.31}$$

In any particular example the validity of these approximations can be tested after the analysis has been completed by evaluating the force that has been ignored and comparing it with the force used in the model.

1.2 DISCUSSION

The equations summarized in Table 1.2 are those needed to describe the electrical side of electromechanical dynamics as presented here. We find that they are of far-reaching physical significance. Nonetheless, they are approximate and their regions of validity should be understood. Furthermore, their

Table 1.2 Summary of Quasi-Static Electromagnetic Equations

	Differential Equations		Integral Equations	
Magnetic field system	$\nabla \times \mathbf{H} = \mathbf{J}_f$	(1.1.1)	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J}_f \cdot \mathbf{n}\, da$	(1.1.20)
	$\nabla \cdot \mathbf{B} = 0$	(1.1.2)	$\oint_S \mathbf{B} \cdot \mathbf{n}\, da = 0$	(1.1.21)
	$\nabla \cdot \mathbf{J}_f = 0$	(1.1.3)	$\oint_S \mathbf{J}_f \cdot \mathbf{n}\, da = 0$	(1.1.22)
	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(1.1.5)	$\oint_C \mathbf{E}' \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot \mathbf{n}\, da$ where $\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$	(1.1.23)
Electric field system	$\nabla \times \mathbf{E} = 0$	(1.1.11)	$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	(1.1.24)
	$\nabla \cdot \mathbf{D} = \rho_f$	(1.1.12)	$\oint_S \mathbf{D} \cdot \mathbf{n}\, da = \int_V \rho_f\, dV$	(1.1.25)
	$\nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}$	(1.1.14)	$\oint_S \mathbf{J}_f' \cdot \mathbf{n}\, da = -\frac{d}{dt}\int_V \rho_f\, dV$	(1.1.26)
	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	(1.1.15)	$\oint_C \mathbf{H}' \cdot d\mathbf{l} = \int_S \mathbf{J}_f' \cdot \mathbf{n}\, da + \frac{d}{dt}\int_S \mathbf{D} \cdot \mathbf{n}\, da$ where $\mathbf{J}_f' = \mathbf{J}_f - \rho_f \mathbf{v}$ $\mathbf{H}' = \mathbf{H} - \mathbf{v} \times \mathbf{D}$	(1.1.27)

relation to more general electromagnetic theory should also be known. Both topics are discussed in Appendix B. A study of that material may be more appropriate as questions are raised in the course of the developments to follow.

With the equations in Table 1.2 accepted on a postulational basis, we can—and should—proceed forthwith to study electromechanical dynamics.