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*Continuum Electromechanics*

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## Problems for Chapter 9

### For Section 9.3:

**Prob. 9.3.1** Plane Couette flow exists in a planar channel if there are no electromagnetic stresses and no longitudinal pressure gradients. (This would be the case if the channel were a model for a reentrant flow.)

- (a) What is the velocity profile?
- (b) With both boundaries fixed and no electromagnetic stresses, the flow is driven by the pressure gradient and called plane Poiseuille flow. Describe the velocity profile and use it to relate the volume rate of flow,  $Q_v$ , through a channel of length  $\ell$  and width  $w$  to the pressure drop.

**Prob. 9.3.2** Carry out the derivation of Eq. 9.3.5 described in the paragraph following that equation.

**Prob. 9.3.3** The circulating flow shown in Table 9.3.1 is reentrant, and hence has no azimuthal hybrid pressure gradient. Show that the radial dependence of the azimuthal velocity is given by Eq. (b) of that table.

**Prob. 9.3.4** In the absence of electromagnetic forces, a rotor having radius  $b$  rotates with the angular velocity  $\Omega_b$ . It is surrounded by a viscous fluid in an annulus with an outer wall at the radius  $a$  having angular velocity  $\Omega_a$ . Hence, with  $\Omega_b = 0$ , the configuration is that of the Couette viscometer shown in Fig. 7.13.1.

- (a) Find the viscous torque acting on the inner rotor.
- (b) Show that in the limit where  $b \gg (a-b)$ , the flow reduces to plane Couette flow (Prob. 9.3.1).

**Prob. 9.3.5** Axial flow through an annular region with circular cylindrical boundaries is depicted in Table 9.3.1. Show that the velocity profile is as summarized by Eq. (c) of the table.

**Prob. 9.3.6** A pipe has radius  $R$ .

- (a) Use Eq. (c) of Table 9.3.1 to deduce the velocity profile as a function of the pressure gradient. This is Couette flow in cylindrical geometry.
- (b) Find the relationship between pressure drop and volume rate of flow  $Q_v$  for a pipe having length  $\ell$ .

### For Section 9.4:

**Prob. 9.4.1** A tank, shown in Fig. 9.4.2a, is made of insulating material and holds a semi-insulating liquid so that it forms a layer of depth  $b$  with a free surface at  $x = 0$ . At a distance  $a$  above the interface, an electrode structure runs parallel to the interface and imposes the traveling wave of potential  $\text{Re } \hat{V}_0 \exp j(\omega t - ky)$ . Thus, the experiment shown in Fig. 5.14.4a is modeled. The time average surface force density is derived in Section 5.14. Using the fully developed flow model, find an expression for the velocity profile as a function of the system parameters and the imposed voltage amplitude.

**Prob. 9.4.2** In the configuration of Fig. 9.4.2b, the electrodes are immersed in the liquid. The model is for the experiments shown in Fig. 5.14.4b. Thus there is a layer of liquid above the structure having a depth  $a$ ; a free upper surface; and a layer of the returning liquid below having a depth  $b$  and bounded from below by a rigid equipotential surface. Take the lower surface of the box to be an equipotential surface, and the region of the free interface as extending to infinity. Use fully developed flow models for the regions above and below the electrodes to approximate the volume rate of flow for the circulation around the electrode structure.

**Prob. 9.4.3** A layer of liquid metal has an interface carrying skin currents induced by means of a traveling wave of surface current backed by an infinitely permeable material, as shown in Fig. 9.4.2c. Use the sinusoidal steady-state skin-effect model of Section 6.8 and the fully developed flow model to find the surface velocity of the liquid in the tank.

**Prob. 9.4.4** The configuration shown in Fig. 9.3.2d is a model for circulation in liquid metals by non-uniformities in a high frequency imposed magnetic field. The magnetic skin depth is much less than  $b$ . Fluid motions are slow enough that they have little effect on the fields. The upper bus-electrode is designed in Prob. 6.9.2 to give a uniformly distributed surface force density. Using the stress derived in that problem, find the interfacial velocity induced by the nonuniformity in field.

For Section 9.5:

Prob. 9.5.1 The planar fluid layer shown in Fig. (a) of Table 9.3.1 is a liquid metal driven by a traveling magnetic-field wave that imposes a tangential field  $H_y^\alpha = \text{Re } \hat{H}_y^\alpha \exp j(\omega t - ky)$  at the upper surface. The structure used to produce this field might be like that of Fig. 9.5.1, or the layer might be embedded in a heterogeneous system. The skin depth  $\delta = \sqrt{2/\omega\mu\sigma}$  is much less than both the layer width  $\Delta$  and the wavelength  $2\pi/k$ .

(a) With the velocities at the upper and lower surfaces and the pressure gradient left arbitrary, show that the velocity profile is approximately

$$v = v^\beta \left(1 - \frac{x}{\Delta}\right) + \frac{x}{\Delta} v^\alpha + \frac{\Delta^2}{2\eta} \frac{\partial p'}{\partial y} \left[\left(\frac{x}{\Delta}\right)^2 - \frac{x}{\Delta}\right] - \frac{\mu k |\hat{H}_y^\alpha|^2 \delta^2}{8\eta} \left(e^{-\frac{2(x-\Delta)}{\delta}} - \frac{x}{\Delta}\right)$$

(b) Sketch the magnetic contribution to this profile and compare it to the high frequency profile shown in Fig. 9.5.2.

Prob. 9.5.2 The cross section of a liquid metal induction pump is shown in Fig. P9.5.2. As the circular analogue of the planar configuration considered in this section, it consists of liquid metal in the annulus between highly permeable coaxial cylinders. The inner cylinder has outer radius  $b$  while the outer one has inner radius  $a$ . A winding, disposed essentially on the surface at  $r = a$ , imposes a surface current  $\vec{K} = \hat{i}_z \text{Re } \hat{K}_0 \exp j(\omega t - m\theta)$  so that the fluid is pumped azimuthally. Use the velocity profile of Table 9.3.1, Eq. b, and the magnetic diffusion relations summarized by Eq. 6.5.10 and Table 6.5.1 to determine the velocity of the fluid in the annulus. Set up the integrations so that they can be evaluated numerically, as in this section. Include an evaluation of the volume rate of flow.

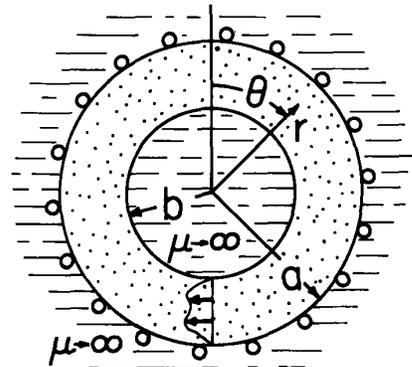


Fig. P9.5.2

Prob. 9.5.3 Table 9.3.1c shows the geometry of a circular induction pump. The liquid metal is in the annulus between coaxial walls at  $r = a$  and  $r = b$ . The region inside the inner wall can be taken as infinitely permeable while that outside the outer wall is a traveling wave structure backed by an infinitely permeable material. The winding is excited so that at  $r = a$  there is a surface current  $\vec{K} = \text{Re } \hat{K}_0 e^{j(\omega t - kz)} \hat{i}_\theta$ . The fluid is pumped in the axial direction. Use the velocity profile of Table 9.3.1, Eq. c, and the magnetic diffusion relations summarized by Eqs. 6.5.15 and Table 6.5.1 to determine the fully developed velocity profile. Set up the integrations so that they may be conveniently evaluated numerically, including the relation between pressure gradient and volume rate of flow.

For Section 9.6:

Prob. 9.6.1 A reentrant flow is modeled as in this section by a plane flow. When  $t = 0$ , the fluid is static and a uniform force density  $\vec{F} = F_0 \hat{i}_y$  is suddenly applied. Walls at  $x = 0$  and  $x = \Delta$  are fixed. Find the fluid response.

Prob. 9.6.2 Find a force density profile  $F_y(x)$  such that the fluid velocity profile has the same relative distribution as the fluid comes up to speed. Assume that this force density is suddenly applied when  $t = 0$  and remains constant in time thereafter.

For Section 9.7:

Prob. 9.7.1 There are electromechanical situations where a fluid essentially "slips" relative to a fixed boundary. An example results when a double layer exists between an insulating boundary plate and an electrolyte and a tangential electric field is applied. The resulting flow, which is taken up in Chap. 10, is dominated by viscous stresses within the double layer. Insofar as the bulk flow is concerned, the fluid in the vicinity of the plate is moving with a uniform velocity  $v_y = U$  (i.e., the velocity is independent of  $y$ ). Suppose that  $v_y(0, y) = U$  for  $y > 0$ , that the rigid plate requires that  $v_x(0, y) = 0$  and that the fluid is stagnant as  $x \rightarrow \infty$ . Formulate the similarity problem. What is the viscous stress acting on the plate and what is the total force acting on a length  $L$  of the plate?

Prob. 9.7.2 For the stress-constrained boundary layer, what is the transit time between  $y = 0$  where the stress begins and  $y = y$  for a particle on the surface? Show that the similarity parameter, Eq. 25, is the square root of the ratio of the viscous diffusion time to this transit time.

**Prob. 9.7.3** The fluid interface shown in Fig. 9.7.6 is subject to the imposed surface force density  $T(y) = T_0(y/a)^k$ , where  $T_0$ , and  $k$  are constants.

(a) Show that appropriate similarity parameter and function are

$$\xi = \left( \frac{T_0 \rho}{k a^2} \right)^{1/3} xy^{\frac{k-1}{3}} ; \quad A_v = - \left( \frac{T_0 \eta}{\rho a^2 k} \right)^{1/3} f(\xi) y^{\frac{k+2}{3}}$$

(b) Show that the boundary layer equations are

$$\frac{d}{d\xi} \begin{bmatrix} f \\ g \\ h \end{bmatrix} = \begin{bmatrix} g \\ h \\ \left(\frac{2k+1}{3}\right)g^2 - \left(\frac{k+2}{3}\right)fh \end{bmatrix}$$

(c) Argue that appropriate boundary conditions are given by Eq. 27.

**Prob. 9.7.4** The configuration shown in Fig. P9.7.4 has a planar layer of relatively inviscid ohmic liquid having depth  $b$  and charge relaxation time short compared to transport times of interest. The liquid has a "free" surface at  $x = 0$  which, because the mass density of the liquid is much greater than that of the air above, is held flat by gravity. Electrodes in the plane  $x = b$  constrain the potential of the liquid as shown in the figure, where  $V_b$  and  $b$  are constants.

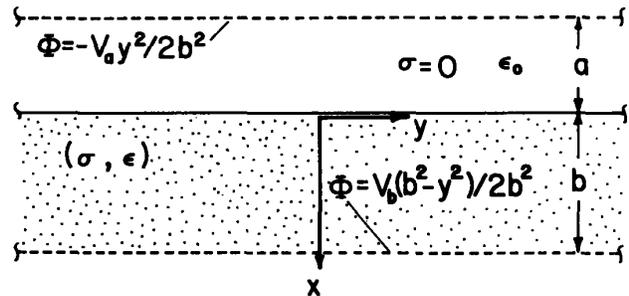


Fig. P9.7.4

(a) Show that in the liquid the electric potential is

$$\phi = \frac{V_b}{2b} (x^2 - y^2)$$

- (b) At a distance  $a$  above the interface, electrodes constrain the potential to be as shown in the figure. Assume that  $a$  is small enough so that  $E_x$  in the air can be approximated as the voltage divided by the spacing. What is the electric shearing surface force density acting on the interface?
- (c) Show that the boundary layer resulting from this surface force density can be represented as in Prob. 9.7.3. Assume that  $b$  is so much greater than the boundary layer thickness that the fluid outside this layer can be regarded as stagnant. What is the value of  $k$ ?

**For Section 9.8:**

**Prob. 9.8.1** Two semi-insulating liquid layers having ohmic conductivities  $(\sigma_a, \sigma_b)$ , permittivities  $(\epsilon_a, \epsilon_b)$  and viscosities  $(\eta_a, \eta_b)$ , respectively, are shown in Fig. 9.8.2. Assume that the flow has little effect on the distribution of fields, that gravity holds the interface flat and that the Reynolds number is small.

- (a) What is the shearing surface force density  $T_y(y)$  due to the field?
- (b) Sketch the expected cellular flow pattern.
- (c) What is the velocity of the interface  $v_y(y)$  as a function of the driving voltage  $V_0$ ?
- (d) What conditions must prevail to insure that effects of motion on the field are negligible and that  $R_y$  is small?

**For Section 9.9:**

**Prob. 9.9.1** Fully developed Hartmann flow exists in the half-space  $x > 0$ . In the plane  $x = 0$  there is an insulating rigid flat plate. Throughout the fluid, there is a uniform electric field  $\vec{E} = E_z \hat{z}$  and the pressure gradient in the  $y$  direction is constant. Determine the velocity profile  $\vec{v} = v_y(x) \hat{y}$ . What is the thickness of the Hartmann boundary layer?

For Section 9.14:

Prob. 9.14.1 Flow over an uneven bottom is shown in Fig. P9.14.1.

- Write the quasi-one-dimensional equations of motion in terms of  $\xi(y,t)$  and  $v_y(y,t)$ .
- Draw a "head" diagram analogous to Fig. 9.14.2 and discuss the steady transition of the fluid depth as it passes over an elevation in the bottom. How does the profile depend on the entrance velocity relative to the velocity of a gravity wave?

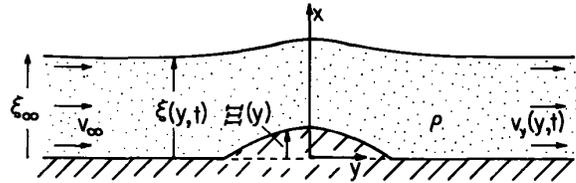


Fig. P9.14.1

Prob. 9.14.2 An alternative to the deduction of the quasi-one-dimensional derivation given here is to use the space-rate expansion illustrated in Sec. 4.12. For a gravity flow, where  $\xi = 0$  and  $\rho \vec{g} \cdot \vec{r} = -\rho g x$ , normalize variables such that

$$x = d \underline{x}, \quad y = \ell \underline{y}, \quad t = \ell t / \sqrt{dg}, \quad p = dg \rho \underline{p}$$

$$\underline{v}_y = \sqrt{dg} \underline{v}_y, \quad \underline{v}_x = \frac{d}{\ell} \sqrt{dg} \underline{v}_x$$

and deduce the quasi-one-dimensional model by expanding the dependent variables in powers of the space-rate parameter  $(d/\ell)^2$ . See Fig. P9.14.1 for the configuration.

Prob. 9.14.3 The cross section of an electromechanical flow structure is shown in Fig. P9.14.3. The applied voltage is high frequency a-c, so that free charge cannot accumulate in the highly insulating liquid. Under the assumption that the mechanical response is only to the time average of the field,  $V_a$  is taken as the rms of the applied voltage and henceforth regarded as being d-c. The flow dynamics in the  $z$  direction is to be described under the assumption that as the fluid cross section varies the interfaces remain in the regions to right and left, respectively, well removed from the position of minimum spacing between electrodes.

- For static equilibrium in directions transverse to  $z$ , what are  $p(\xi_a)$  and the cross-sectional area  $A(\xi_a)$ ? (Assume  $p = 0$  exterior to the liquid.)
- Show that the quasi-one-dimensional equations of motion are

$$\frac{\partial \xi_a^2}{\partial t} + \xi_a^2 \frac{\partial v}{\partial z} + v \frac{\partial \xi_a^2}{\partial z} = 0$$

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) + \frac{1}{2} \frac{(\epsilon - \epsilon_0)}{\alpha^2 (\xi_a^2)^2} V_a^2 \frac{\partial (\xi_a^2)}{\partial z} = 0$$

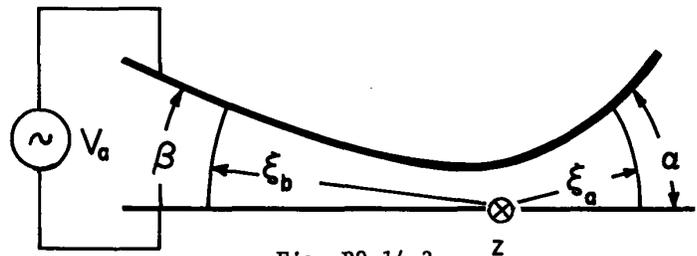


Fig. P9.14.3

For Section 9.16:

Prob. 9.16.1 Derive the area-velocity relation of Eq. 9.16.9.

Prob. 9.16.2 Along the trajectories a and o of Fig. 9.16.1, in both the subsonic nozzle and supersonic diffuser, the area ratio decreases in the direction of flow. Show that as the channel reaches the critical area ratio, defined as the minimum ratio consistent with isentropic steady flow, the Mach number is unity.

Prob. 9.16.3 Use Eqs. (b) - (e) of Table 9.15.1, with no external coupling ( $F = 0, J = 0$ ) to show that

$$\frac{d}{dz}(\rho \rho^{-\gamma}) = 0$$

for the quasi-one-dimensional flow described in this section.

For Section 9.17:

Prob. 9.17.1 Because constant velocity implies an increasing Mach number, the flow discussed in this section approaches sonic velocity even if initially subsonic. To avoid the associated losses in the

Prob. 9.17.1 (continued)

subsequent diffuser, used to bring the gas to rest after passage through the MHD duct, it can be advantageous to make  $A(z)$  such that the Mach number remains constant:  $M^2 \equiv v^2/\gamma RT = M_0^2$ . Assume that  $B$  is also constant with respect to  $z$  and observe that  $vdv = \gamma RM_0^2 dT/2$ . Use conservation of energy and momentum to show that

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^\alpha; \quad \alpha \equiv \frac{\gamma}{\kappa(\gamma-1)} \left[1 - \frac{1}{2}(\gamma-1)M_0^2(\kappa-1)\right]$$

In turn, find  $p$ ,  $\rho$  and  $v$  in terms of  $T$ . Then find and integrate a differential equation for  $T$ . Finally, what is  $A(z)$  and the specific entropy  $S_T(z)$ ?

Prob. 9.17.2 In general, the  $z$  dependence of flow variables cannot be found in analytical form. However, numerical integration of the equations from given inlet conditions is relatively straightforward once the differential equations have been written as a system of first order equations.

The loading is allowed to be arbitrary so that  $E$  is now independent of  $vB$ . Write the quasi-one-dimensional laws in the systematic form ( $\rho' \equiv dp/dz$ )

$$\begin{bmatrix} \rho v & 0 & \rho v & 0 & 0 \\ 0 & p & \rho v^2 & 0 & 0 \\ 0 & 0 & \rho v^3 & \frac{\rho v^3}{M^2(\gamma-1)} & 0 \\ p & -p & 0 & p & 0 \\ 0 & 0 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{\rho'}{\rho} \\ \frac{p'}{p} \\ \frac{v'}{v} \\ \frac{T'}{T} \\ \frac{(M^2)'}{M^2} \end{bmatrix} = \begin{bmatrix} -\rho v \frac{A'}{A} \\ -\sigma B(E+vB) \\ \sigma E(E+vB) \\ 0 \\ 0 \end{bmatrix}$$

where, from top to bottom, these equations represent mass, momentum and energy conservation, the differential forms of the mechanical equation of state and definition of  $M^2$ . Under the assumption that  $A(z)$  is given, invert these equations and show that written in terms of "influence coefficients" they are

$$\begin{bmatrix} \frac{\rho'}{\rho} \\ \frac{p'}{p} \\ \frac{v'}{v} \\ \frac{T'}{T} \\ \frac{(M^2)'}{M^2} \end{bmatrix} = \frac{1}{1-M^2} \begin{bmatrix} M^2 & -\frac{1}{p} & -\frac{(\gamma-1)}{\gamma v p} \\ \gamma M^2 & -\frac{1}{p} [1+M^2(\gamma-1)] & -\frac{M^2(\gamma-1)}{p v} \\ -1 & \frac{1}{p} & \frac{(\gamma-1)}{\gamma v p} \\ M^2(\gamma-1) & -\frac{M^2(\gamma-1)}{p} & -\frac{(\gamma-1)(\gamma M^2-1)}{\gamma p v} \\ -M^2(\gamma-1)-2 & \frac{M^2(\gamma-1)+2}{p} & \frac{(\gamma M^2+1)(\gamma-1)}{\gamma p v} \end{bmatrix} \begin{bmatrix} \frac{A'}{A} \\ \sigma B(E+vB) \\ \sigma E(E+vB) \end{bmatrix}$$

Discuss how these equations would be integrated numerically. Describe a systematic approach to specifying  $A(z)$  such that one of the flow variables has a prescribed evolution with  $z$ .

Prob. 9.17.3 The three modes of operation for a d-c machine with a rigid conductor are summarized in Fig. 4.10. To say whether the MHD duct as a whole gives generation, braking or pumping, the distribution of flow variables and load must be determined. In general, for a compressible conductor, solutions

Prob. 9.17.3 (continued)

to the equations found in Prob. 9.17.2 are required. Consider here the local interaction, the effect of specifying E and B at a given location by means of segmented electrodes. Use the results of Prob. 9.17.2 specialized to a channel of uniform cross section to find the signs of the rates of change of flow parameters for the following cases:

- (a) Generator operation with local electrical power out,  $EJ < 0$ , and a retarding magnetic force,  $JB > 0$ , for  $M^2 \gtrsim 1$ .
- (b) Pump or accelerator operation with  $EJ > 0$  and  $JB < 0$  for  $M^2 \lesssim 1$ .
- (c) In both of the above, identify those cases where acceleration is reversed from what would be expected for the assigned JB and explain.

For Section 9.18:

Prob. 9.18.1 Use the procedure outlined before Eq. 9.18.23 to show that the zero mobility flow is isentropic and hence satisfies Eq. 9.18.23.

Prob. 9.18.2 The zero mobility generator is to be designed for constant temperature throughout. Show that the pressure and mass density are then also constant. Given the outlet conditions denoted by subscripts d, find v and  $\phi$  in terms of the channel area  $A = \pi\xi^2$ . In turn, show that the area is governed by the equation

$$\frac{d^2}{dz^2} A^{-1} - \frac{\rho_{fd} I_0^2}{\epsilon_o (A_d \rho_d v_d)^3} A = 0$$

Show that this expression can be integrated, with boundary conditions  $E(\ell) = 0$  and  $A(\ell) = A_d$ , to obtain the implicit dependence of A on z:

$$F(x)e^{x^2} = (\ell - z) \left[ \frac{\rho_{fd}^2}{\epsilon_o v_d^2 \rho_d} \right]^{1/2}$$

$$F(x) \equiv e^{-x^2} \int_0^x e^{x^2} dx ; \quad x \equiv \left[ \ln\left(\frac{A_d}{A}\right) \right]^{1/2}$$

where F(x) is tabulated as the Dawson integral.<sup>1</sup> Use subscripts to denote inlet variables and show that

$$\frac{\epsilon_o E_o^2 / 2}{\rho_o v_o^2} = \ln \left[ \left( \frac{A_o}{A_d} \right) \right]$$

Show that the electrical output power VI can be written in terms of the inlet electric pressure as

$$VI = \frac{v_o A_o}{r} \left( \frac{1}{2} \epsilon_o E_o^2 \right) [1 - \exp(-r)]$$

where

$$r \equiv \left( \frac{1}{2} \epsilon_o E_o^2 \right) / \left( \frac{1}{2} \rho_o v_o^2 \right).$$

Prob. 9.18.3 A systematic approach to writing the quasi-one-dimensional equations in terms of influence coefficients is outlined in Prob. 9.17.2. Consider here the analogous electrohydrodynamic flow with

1.M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, NBS Applied Math. Series 55, U.S. Printing Office, Washington, DC, 1964, p. 319.

Prob. 9.18.3 (continued)

finite mobility and wall conductivity. Write the appropriate flow equations in a form analogous to the first equations in Prob. 9.17.2. The geometry can be taken as given so that  $\xi'/\xi$  is known, and the unknowns are  $\rho'_f/\rho_f$ ,  $E'/E$ ,  $v'/v$ ,  $\rho'/\rho$ ,  $p'/p$ ,  $T'/T$  and  $(M^2)'/M^2$ . Invert this system of seven equations to show that the influence-coefficient representation of the equations is

$$\begin{bmatrix} v'/v \\ p'/p \\ T'/T \\ (\frac{bE}{v} + 1) \rho'_f/\rho_f \\ E'/E \\ \rho'/\rho \\ M^2'/M^2 \end{bmatrix} = \frac{1}{M^2 - 1} \begin{bmatrix} A_{ij} \end{bmatrix} \begin{bmatrix} -2\xi'/\xi \\ \rho_f E/\rho \\ \frac{\rho_f E}{\rho v} (\frac{\gamma-1}{\gamma})(bE + v) \\ -2(bE + v + \frac{\sigma_s E}{\xi \rho_f}) (\frac{\xi'}{\xi}) \\ Q \end{bmatrix}$$

where

$$A_{14} = A_{15} = A_{24} = A_{25} = A_{34} = A_{35} = A_{51} = A_{52} = A_{53} = A_{54} = A_{64} = A_{65} = A_{74} = A_{75} = 0$$

$$A_{12} = -A_{13} = A_{41} = -A_{42} = A_{43} = -A_{62} = A_{63} = 1; \quad -A_{71} = A_{72} = M^2(\gamma-1) + 2$$

$$A_{11} = A_{61} = M^2; \quad A_{21} = A_{23} = M^2$$

$$A_{22} = -M^2(\gamma-1) - 1; \quad A_{31} = -A_{32} = M^2(\gamma-1)$$

$$A_{33} = -A_{73} = M^2\gamma - 1; \quad A_{44} = 1/\sigma_s$$

$$A_{45} = -(M^2-1) \frac{E}{v} (b + \frac{2\sigma_s}{\xi \rho_f}); \quad A_{55} = M^2 - 1$$

and

$$Q \equiv -2(1 + \frac{\sigma_s}{\xi \rho_f b}) (\frac{\xi'}{\xi}) + \frac{\rho_f/\epsilon_0 E}{1+(2\sigma_s/\xi \rho_f b)}$$

**Prob. 9.18.4** In the configuration of Fig. 9.18.1, ions are injected at the left and collected at the right with no gas flow ( $v = 0$ ). The total current is  $I$  and the inlet radius is  $\xi_0$ . Determine the radius  $\xi(z)$  required to keep the electric field  $E = E_0$  independent of  $z$ . What is the associated space-charge distribution?

**For Section 9.19:**

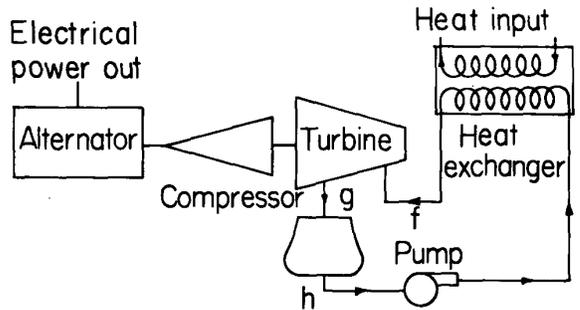
**Prob. 9.19.1** In the diffuser, from  $d$  to  $e$ , it is assumed that the pressure rises. Show that if the flow at the generator outlet is subsonic,  $p_e > p_d$ .

**Prob. 9.19.2** In a "conventional" thermal power plant, shaft power from a turbine is used to drive a synchronous alternator which generates electrical power. Thus the generator of Sec. 4.7 integrates into a system fundamentally like that of Fig. P9.19.2a. The turbine plays a role in this Rankine steam cycle analogous to that of the MHD generator, directly producing shaft rather than electrical power.<sup>2</sup> The steam cycle is summarized by the T-S<sub>T</sub> plot, which shows the demarcation between liquid, wet vapor and

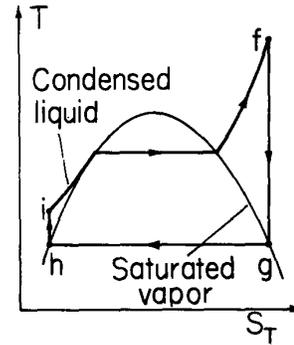
2. The analogy to a turbine extends to the manner in which frictional heat generated at one stage can be partially recovered downstream with the inefficiency showing up through the entropy production (ignored in this problem). See E. F. Church, Steam Turbines, McGraw-Hill Book Co., 1950, Chap. 14.

Prob. 9.19.2 (continued)

superheated gas phase. Clearly the perfect gas model is not appropriate. Use the enthalpy function defined at the marked stations, and assume that the turbine acts isentropically. Find the overall efficiency, defined as the electrical power output divided by thermal power input. Assume that the generator has an efficiency  $\eta_g$  for mechanical to electrical conversion and that the compressor is not used. Now the MHD generator has the disadvantage that relatively high outlet temperatures must be maintained in order that the thermal ionization responsible for the gas conductivity remains effective. Thus the cycle of Fig. 9.19.1 is operated as a topping unit with the rejected heat used to drive the steam cycle of Fig. P9.19.2a. Find the overall efficiency of the combined system in terms of the enthalpy function. Show that it can be written in the form of Eq. 9.19.13 where the heat rejected is that rejected by the steam cycle.



P9.19.2a



P9.19.2b