For Section 2.3:

Prob. 2.3.1 Perfectly conducting plane parallel plates are shorted at $z = 0$ and driven by a distributed current source at $z = -\ell$, as shown in Fig. P2.3.1.

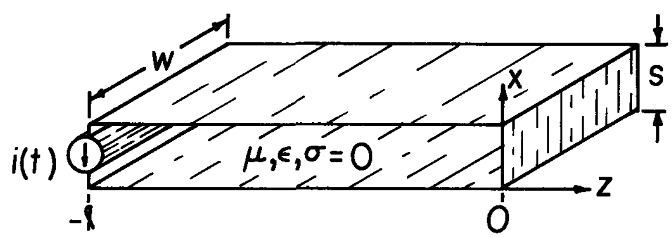


Fig. P2.3.1

(a) Apply the normalization of Eq. 4b to Maxwell's equations used to represent the fields between the plates. There is no material between the plates, so magnetization, polarization and conduction between the plates are ignorable.

(b) Simplify these equations by assuming that $\vec{\underline{E}} = \underline{E}_x(\underline{z},\underline{t})\vec{i}_x$ and $\vec{\underline{H}} = \underline{H}_y(\underline{z},\underline{t})\vec{i}_y$.

(c) The driving current is $i(t) = \text{Re}\, I_o \exp j\omega t$. Find $\underline{E}_x$, $\underline{H}_y$, the surface current and surface charge on the lower plate to second order.

(d) Convert the results of (c) to dimensional expressions.

(e) Solve for the exact fields and expand in β to check the results of (d).

Prob. 2.3.2 The parallel plates of Prob. 2.3.1 are now driven along their left edges by a voltage source v(t). They are open along their right edges. Carry out the steps analogous to those of Prob. 2.3.1. A normalization that makes the EQS limit the zero order approximation is appropriate.

Prob. 2.3.3 Perfectly conducting plane parallel electrodes in the planes $x = a$ and $x = 0$ "sandwich" and make electrical contact with a layer of material having conductivity σ and thickness a. These plates are driven along their edges so that the surface current is $\text{Re}\, \hat{K} \exp(j\omega t)\vec{i}_z$ in the lower plate at $z = -\ell$ and the negative of this in the upper plate. The edges of the plates at $z = 0$ are "open-circuit." In the conductor, fields take the form $E_x(z,t)$, $H_y(z,t)$.

(a) Show that all of Maxwell's equations are satisfied if

$$\frac{d^2\hat{H}_y}{d^2z} + k^2\hat{H}_y = 0; \quad k \equiv \sqrt{\omega^2\mu_o\varepsilon_o - j\omega\mu_o\sigma}; \quad \hat{E}_x = \frac{-1}{(\sigma + j\omega\varepsilon_o)}\frac{d\hat{H}_y}{dz}$$

(b) Show that

$$H_y = \text{Re}\, \hat{K}\frac{e^{-jkz} - e^{jkz}}{e^{jk\ell} - e^{-jk\ell}}e^{j\omega t}; \quad E_x = \frac{\text{Re}\, \hat{K}jk(e^{-jkz} + e^{jkz})e^{j\omega t}}{(\sigma + j\omega\varepsilon_o)(e^{jk\ell} - e^{-jk\ell})}$$

(c) In Fig. 2.3.1, $\tau \rightarrow 1/\omega$ and provided $\tau_e \neq \tau_m$, there are two possibilities:

(i) $\omega\tau_{em} << 1$ and $\omega\tau_m << 1$. Show that in this case $k\ell << 1$ and

$$E_x \rightarrow \text{Re}\frac{\hat{K}\, e^{j\omega t}}{(\sigma + j\omega\varepsilon_o)\ell}$$

so that the system is equivalent to a capacitor shorted by a resistor (what values?).

(ii) $\omega\tau_{em} << 1$, $\omega\tau_e << 1$. Show that in this case $k \rightarrow (-1 + j)/\delta_m$, where the skin depth

$\delta_m \equiv \sqrt{2/\omega\mu\sigma}$, and that H_y is the superposition of "skin-effect" waves decaying in the direction of phase propagation.

(d) Now, consider the EQS model from the outset. Under what conditions are the laws (Eqs. 23a - 27a) valid? Show that the solution for E_x is consistent with part (c).

(e) Consider the magnetoquasistatic laws (Eqs. 23b - 27b) from the outset and show that the result is consistent with part (c). For what conditions are these laws valid?

Prob. 2.3.4 Given the EQS laws, Eqs. 23a - 25a, together with conduction and polarization constitutive laws and the material motions, $\vec{E}$, $\vec{P}$ and ρ_f can be determined. This is generally possible because the constitutive laws do not typically involve $\vec{H}$. Then, if $\vec{H}$ is required, Eqs. 26a and 26b, together with a magnetization constitutive law, can be used. It is clear that these relations uniquely define $\vec{H}$, because they stipulate both $\nabla \times \vec{H}$ and $\nabla \cdot \vec{H}$. Consider now the analogous question of uniquely determining $\vec{E}$ in an MQS system. In such a system the conduction and magnetization constitutive laws respectively take the form

$$\vec{J}_f = \sigma(\vec{r},t)(\vec{E} + \vec{v}\times\mu_o\vec{H}) \quad ; \quad \vec{M} = \vec{M}(\vec{H},\vec{v})$$

and Eqs. 23b - 25b together with a knowledge of the material motion can be used to find $\vec{H}$ and $\vec{M}$. Show that $\vec{E}$ is then uniquely specified and that recourse to Gauss' Law is made only to make an "after the fact" evaluation of the charge density.

For Section 2.4:

Prob. 2.4.1 A material suffers a rigid-body rotation about the z axis with constant angular velocity Ω. The particle at the position (r_o,θ_o) when $t = 0$ is found at

$$\vec{\xi}(r_o,\theta_o,t) = r_o\cos(\Omega t + \theta_o)\vec{i}_x + r_o\sin(\Omega t + \theta_o)\vec{i}_y$$

at a subsequent time t. This Lagrangian description is pictured in Fig. P2.4.1. Use Eqs. 2.4.1 and 2.4.2 to show that the velocity and acceleration are respectively

$$\vec{v} = r_o\,\Omega[-\sin(\Omega t + \theta_o)\vec{i}_x + \cos(\Omega t + \theta_o)\vec{i}_y]$$

$$\vec{a} = -\Omega^2\,\vec{\xi}$$

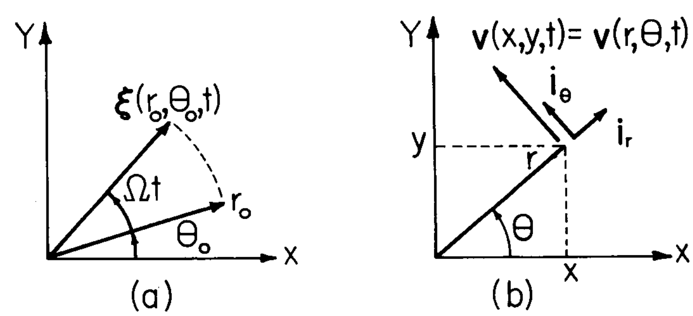


Fig. P2.4.1. Specific example in which rigid-body steady rotation is represented in (a) Lagrangian coordinates and (b) Eulerian coordinates.

Prob. 2.4.2 One incentive for using an Eulerian representation is that motions which are time dependent in Lagrangian coordinates can become independent of time. To illustrate, consider the alternative representation of the rigid body rotation of Prob. 2.4.1.

The material velocity at a given point (r,θ) or (x,y) is

$$\vec{v} = \vec{i}_\theta\Omega r = \Omega(-r\sin\theta\vec{i}_x + r\cos\theta\vec{i}_y) = \Omega(-y\vec{i}_x + x\vec{i}_y)$$

i.e., the velocity is independent of time. Clearly the acceleration is not obtained by taking the partial derivative with respect to time, as might be suggested by the misuse of Eq. 2.4.2. Use Eq. 2.4.4 to find $\vec{a}$ and compare to the result of Prob. 2.4.1.

For Section 2.5:

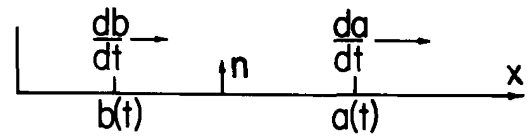
Prob. 2.5.1 A scalar function takes the traveling-wave form $\Phi = \text{Re}\hat{\Phi}(x,y)\exp^{j(\omega t-kz)}$ in the frame of reference $(\vec{r},t)$. The primed frame moves in the z direction relative to the unprimed frame with the velocity U. Use the convective derivative to find the rate of change of Φ for an observer moving with the velocity $U\vec{i}_z$. Compute this same time rate of change by expressing $\Phi = \Phi(x',y',z',t')$ and finding $\partial/\partial t'$. Use these results to deduce the transformation $\omega' = \omega - kU$. If $\omega' = 0$, $\omega = kU$. Explain in physical terms.

Prob. 2.5.2 A vector function $\vec{A}(x,y,z,t)$ can also be evaluated as $\vec{A}(x',y',z',t')$ where the prime coordinates are related to the unprimed ones by Eq. 2.5.1. Show that Eq. 2.5.2b holds.

For Section 2.6:

Prob. 2.6.1 The one-dimensional form of Leibnitz' rule pertains to taking an integral between end-points (b) and (a) which are themselves a function of time, as sketched in Fig. P2.6.1.

Fig. P2.6.1. One-dimensional form of Leibnitz' rule specifies how derivative can be taken of the integral between time-varying endpoints.



Define $\vec{A} = f(x,t)\vec{i}_z$ and use Eq. 2.6.4 with a suitable surface to show that, for the one-dimensional case, Leibnitz' rule becomes

$$\frac{d}{dt}\int_{b(t)}^{a(t)} f(x,t)\,dx = \int_b^a \frac{\partial f}{\partial t}\,dx + f(a,t)\frac{da}{dt} - f(b,t)\frac{db}{dt}$$

Prob. 2.6.2 The following steps lead to a derivation of the generalized Leibnitz rule, Eq. 2.6.4 where S is pictured as S_2, and S_1 at the times $t + \Delta t$ and t, respectively. The vector function $\vec{A}$ depends on both space and time. However, for convenience, the spatial dependence is not explicitly indicated in the following. By definition:

$$\frac{d}{dt}\int_S \vec{A}\cdot\vec{n}\,da = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\int_{S_2}\vec{A}(t+\Delta t)\cdot\vec{n}da - \int_{S_1}\vec{A}(t)\cdot\vec{n}da\right) \qquad (1)$$

so the first integral in brackets on the right must be evaluated to first order in Δt. To that end,

(a) Apply Gauss theorem to the volume V swept out by S during the time Δt. Note that $\vec{n}$ is the normal to the open surface S and show that to first order in Δt,

$$\int_V \nabla\cdot\vec{A}dV = \int_{S_2}\vec{A}(t)\cdot\vec{n}da - \int_{S_1}\vec{A}(t)\cdot\vec{n}da - \Delta t\oint_{C_1}\vec{A}\cdot\vec{v}\times d\vec{\ell} \qquad (2)$$

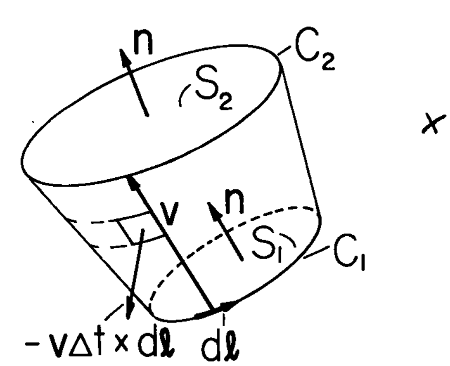


Fig. P2.6.2

(b) Argue that also to first order in Δt,

$$\int_{S_2}\vec{A}(t+\Delta t)\cdot\vec{n}da \simeq \int_{S_2}\vec{A}(t)\cdot\vec{n}da + \int_{S_1}\frac{\partial\vec{A}}{\partial t}(t)\Delta t\cdot\vec{n}da + \cdots \qquad (3)$$

(c) Finally, show that the volume element dV, called for in evaluating the left side of Eq. 2, is $dV = \Delta t\vec{v}\cdot\vec{n}da$.

(d) Combine these results to evaluate the right-hand side of Eq. 1 and deduce Eq. 2.6.4.

Prob. 2.6.3 It is sometimes necessary to evaluate the time rate of change of a line integral of a vector variable having time-varying end points. The problem is to evaluate the derivative

$$\frac{d}{dt}\int_{\vec{a}(t)}^{\vec{b}(t)} \vec{A}\cdot d\vec{\ell} = \lim_{\Delta t\to 0}\frac{\left[\displaystyle\int_{\vec{a}(t+\Delta t)}^{\vec{b}(t+\Delta t)} \vec{A}(t+\Delta t)\cdot d\vec{\ell} - \int_{\vec{a}(t)}^{\vec{b}(t)} \vec{A}(t)\cdot d\vec{\ell}\right]}{\Delta t}$$

Here a and b denote time-dependent vector positions in space. What is meant by the line integration is indicated by Fig. P2.6.3.

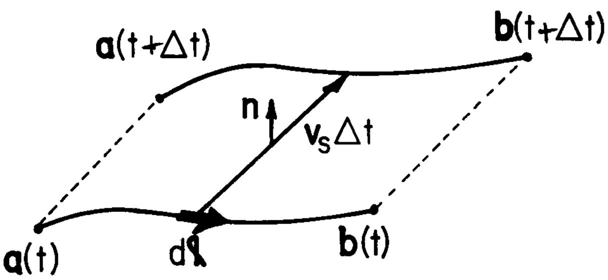


Fig. P2.6.3. Time-varying contour of line integration.

The contour of integration at the time t is instantaneously sketched. At that instant each point on the contour has a velocity $\vec{v}_s$ so that in a time Δt the contour has moved by an amount $\vec{v}_s\Delta t$. By definition, the velocity of the end point is $\vec{v}_s$ evaluated at the end point.

The theorem to be derived shows how the integration can be carried out after the time derivative has been taken. Thus it is analogous to the generalized Leibnitz rule for differentiation of a surface integral having time-varying geometry. The desired theorem states that

$$\frac{d}{dt}\int_{\vec{a}(t)}^{\vec{b}(t)} \vec{A}\cdot d\vec{\ell} = \int_{\vec{a}(t)}^{\vec{b}(t)} \frac{\partial\vec{A}}{\partial t}\cdot d\vec{\ell} + \vec{A}(\vec{b},t)\cdot\vec{v}_s(\vec{b},t) - \vec{A}(\vec{a},t)\cdot\vec{v}_s(\vec{a},t) + \int_{\vec{a}}^{\vec{b}} (\nabla\times\vec{A})\times\vec{v}_s\cdot d\vec{\ell}$$

Show that this rule can be derived following steps motivated by those used in the derivation of the generalized Leibnitz rule for a time-varying surface integration.

For Section 2.8:

Prob. 2.8.1 To illustrate how the steady-state motion of dipoles results in a $\vec{J}_p$ and hence an induced magnetic field, consider a slab of material extending to infinity in the y and z directions between infinitely permeable surfaces at $x = \pm a$. The slab has a thickness 2a, moves in the y direction with uniform velocity U and supports the polarization $\vec{P} = -(\rho_o a/\pi)\sin(\pi x/a)\vec{i}_x$, where ρ_o is a given constant. Fields are in the steady state and there is no free current density.

(a) Observe that Ampere's law, Eq. 2.2.2, and the boundary conditions are satisfied by making $\vec{H} = \vec{P}\times\vec{v}$. What is $\vec{H}$?

(b) Compute $\vec{J}_p$ and then use Ampere's law to find $\vec{H}$ in much the same way as if $\vec{J}_p$ were a free current density.

(c) Find ρ_p and show that in this case $\vec{J}_p$ is simply the result of polarization charge in motion

For Section 2.9:

Prob. 2.9.1 To someone not appreciating the importance of keeping field transformations consistent with the fundamental laws, it might appear that Faraday's law written in the Chu formulation (Eq. 2.2.1) would imply that a magnetized and conducting material set into motion would automatically support an electric field that would drive a free current density. In fact, there is an $\vec{E}$, but no $\vec{J}_f$. Consider as a specific case a magnetized slab, having $\vec{M} = -(\rho_o a/\pi\mu_o)\sin(\pi x/a)\vec{i}_x$, extending to infinity in the y and z directions, having boundaries at $x = \pm a$ in the x direction and suffering a uniform y-

Prob. 2.9.1 (continued)

directed translation with velocity U. Perfectly conducting walls bound the slab at $x = \pm a$. Steady state conditions prevail.

(a) Find the $\vec{H}$ induced by the given magnetization.

(b) Use Faraday's law to deduce $\vec{E}$.

(c) Now, if the material also has a conductivity σ, so that an observer at rest in the conductor can apply Ohm's law in the form $\vec{J}_f' = \sigma \vec{E}'$, because $\vec{J}_f = \vec{J}_f'$ but $\vec{E}' = \vec{E} + \vec{v}\times\mu_o\vec{H}$ (Eqs. 2.5.11 and 2.5.12), $\vec{J}_f = \sigma(\vec{E} + \vec{v}\times\mu_o\vec{H})$. Show that in fact $\vec{J}_f = 0$.

For Section 2.11:

Prob. 2.11.1 A plane parallel capacitor with electrodes at potentials v_1 and v_2 is used to impose a field on a third electrode that is grounded and free to move either longitudinally or transversely with displacements (ξ_1, ξ_2). The electrodes, shown in Fig. P2.11.1, have depth d into paper. Ignore fringing fields and find the capacitance matrix relating the charges (q_1,q_2) to the voltages (v_1,v_2).

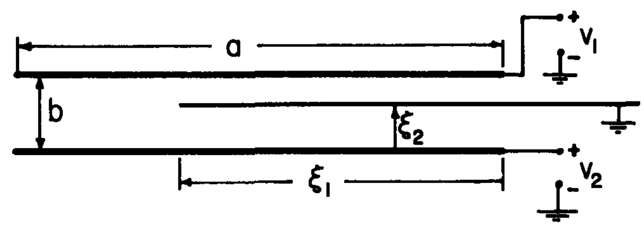


Fig. P2.11.1

For Section 2.12:

Prob. 2.12.1 A pair of perfectly conducting coaxial one-turn coils have the shape of circular cylinders of radius a and ξ, each with a length $d \gg a$. Currents i_1 and i_2 are fed to the coils through parallel electrodes having a spacing that is negligible compared to other dimensions of interest. Determine the inductance matrix, Eq. 2.12.5, relating (λ_1, λ_2) to (i_1,i_2).

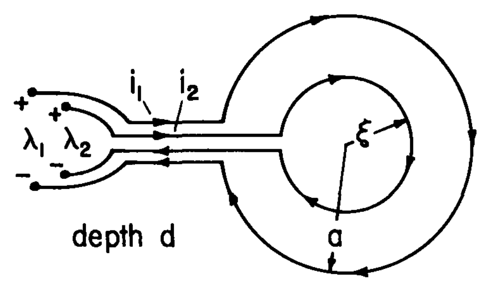


Fig. P2.12.1

For Section 2.13:

Prob. 2.13.1 For the system of Prob. 2.11.1, find the total coenergy storage $w'(v_1,v_2,\xi_1,\xi_2)$ by integrating Eq. 2.13.10.

Prob. 2.13.2 The dielectric slab shown in Fig. P2.13.2 is composed of material having the constitutive law $\vec{D} = \varepsilon_o\vec{E} + \vec{E}/\alpha_1\sqrt{\alpha_2^2 + E^2}$. The slab has depth d into the paper. Under the assumption that $\rho_f=0$ in the dielectric and that its edges remain well removed from the fringing fields, find the dependence of the coenergy on (v,ξ).

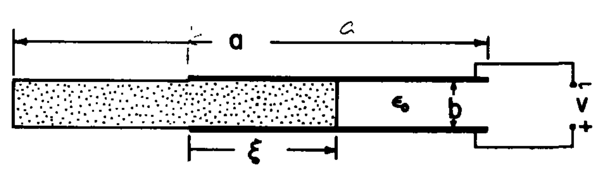


Fig. P2.13.2

For Section 2.14:

Prob. 2.14.1 For the system described in Prob. 2.12.1,

(a) Find the energy, $w = w(\lambda_1,\lambda_2,\xi)$, (b) the coenergy $w' = w'(i_1,i_2,\xi)$.

For Section 2.15:

Prob. 2.15.1 Show that the Fourier coefficients given by Eq. 2.15.8 follow from the procedure outlined in the paragraph following Eq. 2.15.7.

Prob. 2.15.2 A function $\Phi(z,t)$ is a square-wave function of z with magnitude $V_o(t)$. That is, $\Phi = V_o(t)$, $-\ell/4 < z < \ell/4$ and $\Phi = -V_o(t)$, $\ell/4 < z < 3\ell/4$. Show that the Fourier coefficients are

$$\tilde{\Phi}_m = 0, \text{ m even and } \tilde{\Phi}_m = 4V_o(t)\sin\left(\frac{k_m\ell}{4}\right)/(k_m\ell), \text{ m odd}$$

Prob. 2.15.3 A function $\Phi(z,t)$ is zero except in the interval $-\ell/2 < z < \ell/2$, where it is $V_o(t)$. Show that its Fourier transform is $\tilde{\Phi}(k,t) = \ell V_o(t)\sin\left(\frac{k\ell}{2}\right)/(k\ell/2)$.

Prob. 2.15.4 Carry out the spatial average of the product of two Fourier series, as called for in completing Eq. 2.15.17.

For Section 2.16:

Prob. 2.16.1 Start with Eq. 2.16.14 and the relation between potential and flux, Eq. 2.16.5 and deduce the transfer relations of Table 2.16.1 for a planar layer.

Prob. 2.16.2 Start with Eqs. 2.16.20, 2.16.21 and 2.16.25 and deduce the transfer relations of Table 2.16.2. Use the properties of the Bessel functions as $r \to 0$ and $r \to \infty$ to deduce the limiting cases of Eqs. c and d.

Prob. 2.16.3 Start with Eq. 2.16.36 and deduce the transfer relations of Table 2.16.3. Evaluate the appropriate limits to arrive at Eqs. c and d.

Prob. 2.16.4 A region of free space is bounded by fictitious parallel planes at $x = \Delta$ and $x = 0$, as shown in Fig. P2.16.4.

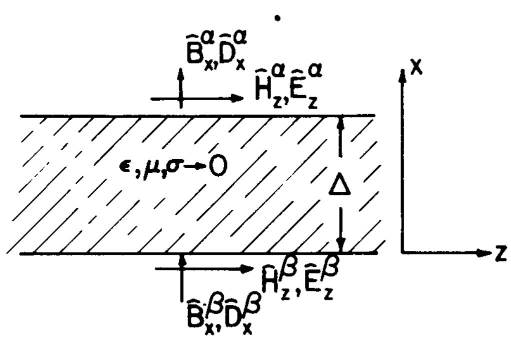


Fig. P2.16.4

Fields take the form
$\vec{E} = \text{Re}\, \hat{\vec{E}}(x)\, e^{j(\omega t - kz)}$;
$\vec{H} = \text{Re}\, \hat{\vec{H}}(x)\, e^{j(\omega t - kz)}$

so that there is no dependence on y and the time dependence is explicitly taken as exp $(j\omega t)$. The objective is to obtain transfer relations between tangential and perpendicular field components at the α and β surfaces without the quasistatic approximation.

(a) With fields taking the given form, show that all components of $\vec{E}$ and $\vec{H}$ can be written in terms of the axial components of $\hat{E}_z$ and $\hat{H}_z$. (This follows from Ampere's and Faraday's laws). Also show that E_z and H_z satisfy the wave equation.

(b) Write $\hat{E}_z$ and $\hat{H}_z$ in terms of the amplitudes $\hat{E}_z^\alpha$, $\hat{E}_z^\beta$ and $\hat{H}_z^\alpha$, $\hat{H}_z^\beta$ defined as these quantities evaluated on the respective surfaces.

(c) Show that the transfer relation for the layer is

$$\begin{bmatrix} \epsilon\hat{E}_x^\alpha \\ \epsilon\hat{E}_x^\beta \\ \mu\hat{H}_x^\alpha \\ \mu\hat{H}_x^\beta \end{bmatrix} = \begin{bmatrix} j\frac{\epsilon k}{\gamma}\coth(\gamma\Delta) & -j\frac{\epsilon k}{\gamma}\frac{1}{\sinh(\gamma\Delta)} & 0 & 0 \\ j\frac{\epsilon k}{\gamma}\frac{1}{\sinh(\gamma\Delta)} & -j\frac{\epsilon k}{\gamma}\coth(\gamma\Delta) & 0 & 0 \\ 0 & 0 & j\frac{\mu k}{\gamma}\coth(\gamma\Delta) & -j\frac{\mu k}{\gamma}\frac{1}{\sinh(\gamma\Delta)} \\ 0 & 0 & j\frac{\mu k}{\gamma}\frac{1}{\sinh(\gamma\Delta)} & -j\frac{\mu k}{\gamma}\coth(\gamma\Delta) \end{bmatrix} \begin{bmatrix} \hat{E}_z^\alpha \\ \hat{E}_z^\beta \\ \hat{H}_z^\alpha \\ \hat{H}_z^\beta \end{bmatrix}$$

where the other components of $\vec{E}$ and $\vec{H}$ are found from

$$\hat{H}_y = \frac{\omega\epsilon_o}{k}\hat{E}_x \quad , \quad \hat{E}_y = \frac{-\omega\mu_o}{k}\hat{H}_x \quad , \quad \text{and } \gamma \equiv \sqrt{k^2 - \omega^2\mu\epsilon}$$

Prob. 2.16.4 (continued)

(d) Show that in the quasistatic limit the relation reduces to the electroquasistatic and magnetoquasistatic transfer relations of Table 2.16.1 with appropriate identification of variables for the electric and magnetic relations.

(e) To make a connection with TE and TM modes in a plane parallel plate waveguide, let the α and β surfaces be perfectly conducting electrodes. Thus, the boundary conditions are

$$\hat{E}_z^\alpha = \hat{E}_z^\beta = 0 \qquad \text{TM modes}$$

$$\hat{B}_x^\alpha = \hat{B}_x^\beta = 0 \qquad \text{TE modes}$$

where the transverse magnetic and transverse electric modes can be separated because of the form taken by the transfer relations. Use these relations to argue that fields within that satisfy these homogeneous boundary conditions must also satisfy the dispersion equations

$$\omega^2\mu\varepsilon = k^2 + \left(\frac{n\pi}{\Delta}\right)^2 \quad ; \quad n = 1, 2, 3\ldots$$

Prob. 2.16.5 A planar region, shown in Table 2.16.1, is filled by an inhomogeneous dielectric, with a permittivity that depends on x:

$$\varepsilon(x) = \varepsilon_\beta \exp 2\eta x, \quad \eta \equiv \ln(\varepsilon_\alpha/\varepsilon_\beta)/2\Delta$$

The free charge density is zero.

(a) Show that the potential distribution is

$$\tilde{\Phi} = \tilde{\Phi}^\alpha e^{-\eta(x-\Delta)} \frac{\sinh\lambda x}{\sinh\lambda\Delta} - \tilde{\Phi}^\beta e^{-\eta x} \frac{\sinh\lambda(x-\Delta)}{\sinh\lambda\Delta}$$

where

$$\lambda \equiv \sqrt{k^2 + \eta^2}$$

(b) Show that the transfer relations are

$$\begin{bmatrix} \tilde{D}_x^\alpha \\ \tilde{D}_x^\beta \end{bmatrix} = \varepsilon_\beta\lambda \begin{bmatrix} \left(\frac{\eta}{\lambda} - \coth\lambda\Delta\right)e^{\eta 2\Delta} & \frac{e^{\eta\Delta}}{\sinh\lambda\Delta} \\ \frac{-e^{\eta\Delta}}{\sinh\lambda\Delta} & \frac{\eta}{\lambda} + \coth\lambda\Delta \end{bmatrix} \begin{bmatrix} \tilde{\Phi}^\alpha \\ \tilde{\Phi}^\beta \end{bmatrix}$$

Prob. 2.16.6 A planar region, shown in Table 2.16.1, is filled by an anisotropic material having the constitutive law $D_i = \varepsilon_{ij}E_j$. The permittivity coefficients are uniform throughout. Determine the transfer relations in the form of Eqs. (a) of Table 2.16.1.

For Section 2.17:

Prob. 2.17.1 In developing conditions on coefficients in the transfer relations with the potentials expressed as functions of the "flux" variables, it is natural to use the energy function as exemplified in this section. The coenergy function is more convenient in dealing with the potentials as the independent variables. For the transfer relations of Sec. 2.16 written in the form

$$\begin{bmatrix} \tilde{D}_n^\alpha \\ \tilde{D}_n^\beta \end{bmatrix} = \begin{bmatrix} -B_{11} & B_{12} \\ -B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \tilde{\Phi}^\alpha \\ \tilde{\Phi}^\beta \end{bmatrix}$$

derive conditions analogous to those of Eqs. 2.17.10 and 2.17.12.

Prob. 2.17.2 Use the reciprocity condition, Eq. 2.17.10 to show

$$kx[H_m(jkx)\ J_m'(jkx) - J_m(jkx)\ H_m'(jkx)] = \text{constant}$$

Use Eqs. 2.16.22 and 2.16.23 to establish that the constant is $2/\pi$. Thus, the numerators of the functions g_m and G_m in the cases $k \neq 0$ of Table 2.16.2 are considerably simplified from what is obtained by direct evaluation.

Prob. 2.17.3 With Eq. 2.17.7, it is assumed that the excitations on the α and β surfaces are in spatial phase, and that the A_{ij} are real. By allowing the excitations to have arbitrary phase, it is possible to learn more about these coefficients. In general, the expression replacing Eq. 2.17.7 in Cartesian or cylindrical geometry is

$$\delta w = \frac{1}{2}\ C\ \text{Re}[-a^\alpha\tilde{\Phi}^\alpha\delta(\tilde{D}_n^\alpha)^* + \quad a^\beta\tilde{\Phi}^\beta\delta(\tilde{D}_n^\beta)^*\ ]$$

Because $\text{Re}\ \tilde{u}\ \delta\tilde{V} = \tilde{u}_r\delta\tilde{V}_r + \tilde{u}_i\delta\tilde{V}_i$, this expression becomes

$$\delta w = \frac{1}{2}\ C[-a^\alpha\tilde{\Phi}_r^\alpha\delta\tilde{D}_{nr}^\alpha - a^\alpha\tilde{\Phi}_i^\alpha\delta\tilde{D}_{ni}^\alpha + a^\beta\tilde{\Phi}_r^\beta\delta\tilde{D}_{nr}^\beta + a^\beta\tilde{\Phi}_i^\beta\delta\tilde{D}_{ni}^\beta]$$

That is, the real and imaginary parts of the excitations on each surface are independent variables. Use the fact that the energy is a state variable: $w = w(\tilde{D}_{nr}^\alpha, \tilde{D}_{ni}^\alpha, \tilde{D}_{nr}^\beta, \tilde{D}_{ni}^\beta)$ and show that

$$-a^\alpha\tilde{\Phi}_r^\alpha = \frac{\partial w}{\partial\tilde{D}_r^\alpha}\ ;\quad -a^\alpha\tilde{\Phi}_i^\alpha = \frac{\partial w}{\partial\tilde{D}_i^\beta}\ ;\quad a^\beta\tilde{\Phi}_r^\beta = \frac{\partial w}{\partial\tilde{D}_r^\beta}\ ;\quad a^\beta\tilde{\Phi}_i^\beta = \frac{\partial w}{\partial\tilde{D}_i^\beta}$$

From these relations, derive reciprocity relations between the derivatives of $(\tilde{\Phi}_r^\alpha, \tilde{\Phi}_i^\alpha, \tilde{\Phi}_r^\beta, \tilde{\Phi}_i^\beta,)$ with respect to $(\tilde{D}_{nr}^\alpha, \tilde{D}_{ni}^\alpha, \tilde{D}_{nr}^\beta, \tilde{D}_{ni}^\beta)$. Assume that the A_{ij} can have real and imaginary parts, and show from these reciprocity relations that A_{11} and A_{22} must be real and that $a^\alpha A_{12} = a^\beta A^*_{21}$.

Prob. 2.17.4 Use the results of Prob. 2.17.1 to show that the transfer relations of Prob. 2.16.5 satisfy the reciprocity relations.

For Section 2.18:

Prob. 2.18.1 For the axisymmetric cylindrical case of Table 2.18.1, show that Eq. (h) follows from Eq. (g) and that Eq. 2.18.2 can be used to deduce the expression for the total flux, Eq. (i).

Prob. 2.18.2 Show that Eq. (k) of Table 2.18.1 follows from Eq. (j).

For Section 2.19:

Prob. 2.19.1 Derive Eqs. (e) and (f) of Table 2.19.1.