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Continuum Electromechanics

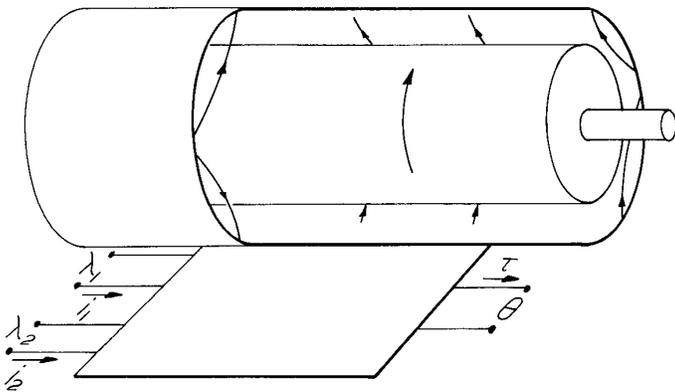
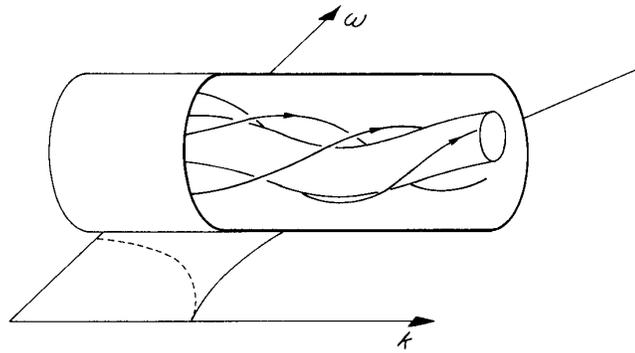
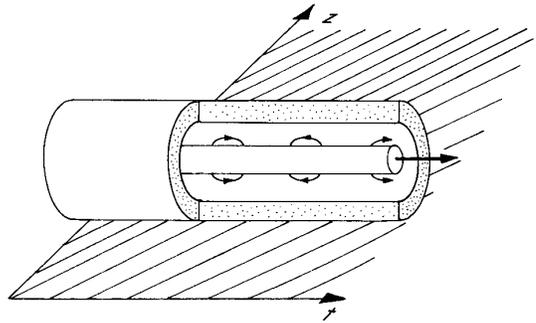
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Introduction to Continuum Electromechanics



1.1 Background

There are two branches to the area of electromagnetics. One is primarily concerned with electromagnetic waves. Typically of interest are guided and propagating waves ranging from radio to optical frequencies. These may propagate through free space, in plasmas or through optical fibers. Although the interaction of electromagnetic waves with media of great variety is of essential interest, and indeed the media modify these waves, it is the electromagnetic wave that is at center stage in this branch. Dynamical phenomena of interest to this branch are typified by times, τ , shorter than the transit time of an electromagnetic wave propagating over a characteristic length of the "system" being considered. For a characteristic length ℓ and wave velocity c (in free space, the velocity of light), this transit time is ℓ/c .

In the chapters that follow, it is the second branch of electromagnetics that plays the major role. In the sense that electromagnetic wave transit times are short compared to times of interest, the electric and magnetic fields are quasistatic: $\tau \gg \ell/c$. The important dynamical processes relate to conduction phenomena, to the mechanics of ponderable media, and to the two-way interaction created by electromagnetic forces as they elicit a mechanical response that in turn alters the fields.

Because the mechanics can easily upstage the electromagnetics in this second branch, it is likely to be perceived in terms of a few of its many parts. For example, from the electromagnetic point of view there is much in common between issues that arise in the design of a synchronous alternator and of a fusion experiment. But, on the mechanical side, the rotating machine, with its problems of vibration and fatigue, seems to have little in common with the fluid-like plasma continuum. So, the two areas are not generally regarded as being related.

In this text, the same fundamentals bear on a spectrum of applications. Some of these are reviewed in Sec. 1.2. The unity of these widely ranging topics hinges on concepts, principles and techniques that can be traced through the chapters that follow. By way of a preview, Secs. 1.3-1.7 are outlines of these chapters, based on themes designated by the section headings.

Chapters 2 and 3 are concerned with fundamentals. First the laws and approximations are introduced that account for the effect of moving media on electromagnetic fields. Then, the force densities and associated stress tensors needed to account for the return influence of the fields on the motion are formulated.

Chapter 4 takes up the class of devices and phenomena that can be described by models in which the distributions (or the relative distributions) of both the material motion and of the field sources are constrained. This subject of electromechanical kinematics embraces lumped parameter electromechanics. The emphasis here is on using the field point of view to determine the relationship between the lumped parameters and the physical attributes of devices, and to determine the distribution of stress and force density.

Chapters 5 and 6 retain the mechanical kinematics, but delve into the self-consistent evolution of fields and sources. Motions of charged microscopic and macroscopic particles entrained in moving media are of interest in their own right, but also underlie the limitations of commonly used conduction constitutive laws. These chapters both introduce basic concepts, such as the Method of Characteristics and temporal and spatial modes, and model practical devices ranging from the electrostatic precipitator to the linear induction machine.

Chapters 7-11 treat interactions of fields and media where not only the field sources are free to evolve in a way that is consistent with the effect of deforming media, but the mechanical systems respond on a continuum basis to the electric and magnetic forces.

Chapter 7 introduces the basic laws and approximations of fluid mechanics. The formulation of laws, deduction of boundary conditions and use of transfer relations is a natural extension of the viewpoint introduced in the context of electromagnetics in Chap. 2.

Chapter 8 is concerned with electromechanical static equilibria and the dynamics resulting from perturbing these equilibria. Illustrated are a range of electromechanical models motivated by Chaps. 5 and 6. It is here that temporal instability first comes to the fore.

Chapter 9 is largely devoted to electromechanical flows. Included is a discussion of flow development, understood in terms of the same physical processes represented by characteristic times

in the previous four chapters. Flows that display super- and sub-critical behavior presage causal effects of wave propagation taken up in Chap. 11. The last half of this chapter is an introduction to "direct" thermal-to-electric energy conversion.

Chapter 10 is divided into parts that are each concerned with diffusion processes. Thermal diffusion, together with convective heat transfer, is considered first. Electrical dissipation accompanies almost all electromechanical processes, so that heat transfer often poses an essential limitation on invention and design. Because fields are often used for dielectric or induction heating, this is a subject in its own right. This part begins with examples where the coupling is "one-way" and ends by considering some of the mechanisms for two-way coupling between the thermal and electromechanical subsystems. The second part of this chapter serves as an introduction to electromechanical processes that occur on a spatial scale small enough that molecular diffusion processes come into play. Here introduced is the interplay between electric and mechanical stresses that makes it possible for particles to undergo electrophoresis rather than migrate in an electric field. The concepts introduced in this second part are applicable to physicochemical systems and point to the electromechanics of biological systems.

Chapter 11 brings together models and concepts from Chaps. 5-10, emphasizing streaming interactions, in which ordered kinetic energy is available for participation in the energy conversion process. Included are fluid-like continua such as electron beams and plasmas.

1.2 Applications

Transducers and rotating machines that are described by the lumped parameter models of Chap. 4 are so pervasive a part of modern day technology that their development might be regarded as complete. But, with new technologies outside the domain of electromechanics, there come new needs for electromechanical devices. The transducers used to drive high-speed computer print-outs are an example. New devices in other areas also result in electromechanical innovations. For example, high power solid-state electronics is revolutionizing the design and utilization of rotating machines.

As energy needs press the capabilities of electric power systems, rotating machines continue to be the mainstay of energy conversion to electrical form. Synchronous generators are subject to increasingly stringent demands. To improve capabilities, superconducting windings are being incorporated into a new class of generators. In these synchronous alternators, magnetic materials no longer play the essential role that they do in conventional machines, and new design solutions are required.

The Van de Graaff machine also considered in Chap. 4 should not be regarded as a serious approach to bulk power generation, but nevertheless represents an important approach to the generation of extremely high potentials. It is also the grandfather of proposed energy conversion approaches. An example is the electrodynamic "thermal-to-electric" energy converter of Chap. 9, Sec. 9.

Chapters 5 and 6 begin to hint at the diversity of applications outside the domain of lumped parameter electromechanics. The behavior of charged particles in moving fluids is important for understanding liquid insulation in transformers and cables. Again, in the area of power generation and distribution, ions and charged macroscopic particles contribute to the contamination of high-voltage insulators. Also related to the overhead line transmission of electric power is the generation of audible noise. In this case, the charged particles considered in Chap. 5 contribute to the transduction of electrical energy into acoustic form, the result being a sufficient nuisance that it figures in the determination of rights of way.

Some examples in Chap. 5 are intended to give basic background relevant to the control of particulate air pollution. The electrostatic precipitator is widely used for air pollution control. Gases cleaned range from the recirculating air within a single room to the exhaust of a utility. With industries of all sorts committed to the use of increasingly dirtier fuels, new devices that also exploit electrical forces are under development. These include not only air pollution control equipment, but devices for painting, agricultural spraying, powder deposition and the like.

Image processing is an application of charged particle dynamics, as are other matters taken up in later chapters. Charged droplet printing is under development as a means of marrying the computer to the printed page. Xerographic and aerosol printing of considerable variety exploit electrical forces on particles.

A visit to a printing plant, to a paper mill or to a textile factory makes the importance of charges and associated electrical forces on moving materials obvious. The charge relaxation processes considered in Chap. 5 are fundamental to understanding such phenomena.

The induction machines considered in Chap. 6 are the most common type of rotating motor. But related interactions between moving conductors and magnetic fields also figure in a host of other applications. The development of high-speed ground transportation has brought into play the linear induction machine as a means of propulsion, and induced magnetic forces as a means of producing mag-

netic lift. Even if these developments do not reach maturity, the induction type of interaction would remain important because of its application to material transport in manufacturing processes, and to melting, levitation and pumping in metallurgical operations. The application of induced magnetic forces to the sorting of refuse is an example of how such processes can figure in seemingly unrelated areas.

Chapter 7 plays a role relative to fluid mechanics that Chap. 2 does with respect to electromagnetics. Without a discourse on the applications of this material in its own right, consider the relevance of topics that are taken up in the subsequent chapters.

Fields can be used to position, levitate and shape fluids. In many cases, a static equilibrium is desired. Examples treated in Chap. 8 include the levitation of liquid metals for metallurgical purposes, shaping of interfaces in the processing of plastics and glass, and orientation of ferrofluid seals and of cryogenic liquids in zero gravity environments.

The electromechanics of systems having a static equilibrium is often dominated by instabilities. The insights gained in Chap. 8 are a starting point in understanding atomization processes induced by means of electric fields. Here, droplets formed by means of electric fields figure in electrostatic paint spraying and corona generation from conductors under foul weather conditions. Internal instabilities also taken up in Chap. 8 are basic to mixing of liquids by electrical means and for electrical control of liquid crystal displays. Both two-phase (boiling and condensation) and convective heat transfer can be augmented by electromechanical coupling, usually through the mechanism of instability. Perhaps not strictly in the engineering domain is thunderstorm electrification. The stability of charged drops and the electrohydrodynamics of air entrained collections of charged drops are topics touched upon in Chap. 8 that have this meteorological application.

The statics and dynamics of hydromagnetic equilibria is now a subject in its own right. Largely because of its relevance to fusion machines, the discussion of hydromagnetic waves and surface instabilities serves as an introduction to an area of active research that, like other applications, has important implications for the energy posture. Internal modes taken up in Chap. 8 also have counterparts in hydromagnetics.

Magnetic pumping of liquid metals, taken up in Chap. 9, has found application in nuclear reactors and in metallurgical operations. Electrically induced pumping of semi-insulating and insulating liquids, also discussed in Chap. 9, has seen application, but in a range of modes. A far wider range of fluids have properties consistent with electric approaches to pumping and hence there is the promise of innovation in manufacturing and processing.

Magnetohydrodynamic power generation is being actively developed as an approach to converting thermal energy (from burning coal) to electrical form. The discussion of this approach in Chap. 9 is not only intended as an introduction to MHD energy conversion, but to the general issues confronted in any approach to thermal-to-electrical energy conversion, including turbine-generator systems. The electrohydrodynamic converter also discussed there is an alternative to the MHD approach that sees periodic interest. For that reason, its applicability is a matter that needs to be understood.

Inductive and dielectric heating, even of materials at rest and with no electromechanical considerations, are the basis for important technologies. These topics, as well as the generation and transport of heat in electromechanical systems where thermal effects often pose primary design limitations, are part of the point of the first half of Chap. 10. But, thermal effects can also be central to the electromechanical coupling itself. Examples where thermally induced property inhomogeneities result in such coupling include electrothermally induced convection of liquid insulation.

Electromechanical coupling seated in double layers, also taken up in Chap. 10, relates to processes (such as electrophoretic particle motions) that see applications ranging from the painting of automobiles to the chemical analysis of large molecules. One of the reasons for including electrokinetic and electrocapillary interactions is the suggestion it gives of mechanisms that can come into play in biological systems, a subject that draws heavily on physicochemical considerations. The purely electromechanical models considered here serve to identify this developing area.

The electromechanics of streaming fluids and fluid-like systems, taken up in Chap. 11, has perhaps its best known applications in the domain of electron beam engineering. Klystrons, traveling-wave tubes, resistive-wall amplifiers and the like are examples of interactions between streams of charged particles (electrons) and various types of structures. The space-time issues of Chap. 11 have general application to problems ranging from the stimulation of liquid jets used to form drops, to electromechanical processes for making synthetic fibers, to understanding liquid flow through "wall-less" pipes (in which electric or magnetic fields play the role of a duct wall), to beam-plasma interactions that result in instabilities that are used as a mechanism for heating plasmas.

1.3 Energy Conversion Processes

A theme of the chapters to follow is conversion of energy between electrical and mechanical forms. The relation between electromechanical power flow and the product of electric or magnetic stress and material velocity is first emphasized in Chap. 4. Rotating machines deserve to be highlighted in this basic sense, because for bulk power generation they are a standard for comparison. But, even where kinematic systems are superseded by those involving self-consistent interactions, there is value in considering the kinematic examples. They make clear the basic objectives governing the engineering of materials and fields even when the objectives are achieved by more devious methods. For example, the synchronous interactions with constrained charged particles are not directly applicable to practical devices, but highlight the basically electroquasistatic electric shear stress interaction that underlies electron beam interactions in Chap. 11.

The classification of energy conversion processes made in Chap. 4 provides a frame of reference for many of the self-consistent interactions described in later chapters. Thus, d-c rotating machines from Chap. 4 have counterparts with fluid conductors in Chap. 9, and the Van de Graaff generator is a prototype for the gasdynamic models developed in Chaps. 5 and 9. Electric and magnetic induction machines, respectively taken up in Chaps. 5 and 6, are a prototype for induction interactions with fluids in Chap. 9. And, the synchronous interactions of Chap. 4 motivate the self-consistent electron beam interactions of Chap. 11.

1.4 Dynamical Processes and Characteristic Times

Rate processes familiar from electrical circuits are the discharge of a capacitor (C) or an inductor (L) through a resistor (R), or the oscillation of energy between a capacitor and an inductor. One way to characterize the dynamics is in terms of the times RC , L/R and \sqrt{LC} , respectively.

Characteristic times describing rate processes on a continuum basis are a recurring theme. The electromagnetic times summarized in Table 1.4.1 are the field analogues of those familiar from circuit theory. Rather than defining the variables, reference is made to the section where the characteristic times are introduced. Some of the mechanical and thermal ones also have lumped parameter counterparts. For example, the viscous diffusion time, which represents the mechanical damping of ponderable material, is the continuum version of the damping rate for a dash-pot connected to a mass.

The electromechanical characteristic times represent the competition between electric or magnetic forces and viscous or inertial forces. In specialized areas, they may appear in a different guise. For example, with the electric field intensity \vec{E} that due to the bunching of electrons in a plasma, the electro-inertial time is the reciprocal plasma frequency. In a highly conducting fluid stressed by a magnetic field intensity H , the magneto-inertial time is the transit time for an Alfvén wave.

Especially in fluid mechanics, these characteristic times are often brought into play as dimensionless ratios of times. Table 1.4.2 gives some of these ratios, again with references to the sections where they are introduced.

1.5 Models and Approximations

There are three classes of approximation, used repeatedly in the following chapters, that should be recognized as a recurring theme. Formally, these are based on time-rate, space-rate and amplitude-parameter expansions of the relevant laws.

The time-rate approximation gives rise to a quasistatic model, and exploits the fact that temporal rates of change of interest are slow compared to one or more times characterizing certain dynamical processes. Some possible times are given in Table 1.4.1. Both for electroquasistatics and magnetoquasistatics, the critical time is the electromagnetic wave transit time, τ_{em} (Sec. 2.3).

Space-rate approximations lead to quasi-one-dimensional (or two-dimensional) models. These are also known as long-wave models. Here, fields or deformations in a "transverse" direction can be approximated as being slowly varying with respect to a "longitudinal" direction. The magnetic field in a narrow but spatially varying air gap and the flow of a gas through a duct of slowly varying cross section are examples.

Amplitude parameter expansions carried to first order result in linearized models. Often they are used to describe dynamics departing from a static or steady equilibrium. Long-wave and linearized models are discussed and exemplified in Sec. 4.12, and are otherwise used repeatedly without formality.

Table 1.4.1. Characteristic times for systems having a typical length ℓ .

Time	Nomenclature	Section reference
Electromagnetic		
$\tau_{em} = \ell/c$	Electromagnetic wave transit time	2.3
$\tau_e = \epsilon/\sigma$	Charge relaxation time	2.3, 5.10
$\tau_m = \mu\sigma\ell^2$	Magnetic diffusion time	2.3, 6.2
$\tau_{mig} = \ell/bE$	Particle migration time	5.9
Mechanical and thermal		
$\tau_a = \ell/a$	Acoustic wave transit time	7.11
$\tau_v = \rho\ell^2/\eta$	Viscous diffusion time	7.18, 7.24
$\tau_c = \eta/\rho a^2$	Viscous relaxation time	7.24
$\tau_D = \ell^2/\kappa$	Molecular diffusion time	10.2
$\tau_T = \ell^2\rho c_v/k_T$	Thermal diffusion time	10.2
Electromechanical		
$\tau_{EV} = \eta/\epsilon E^2$	Electro-viscous time	8.7
$\tau_{MV} = \eta/\mu H^2$	Magneto-viscous time	8.6
$\tau_{EI} = \ell\sqrt{\rho/\epsilon E^2}$	Electro-inertial time	8.7
$\tau_{MI} = \ell\sqrt{\rho/\mu H^2}$	Magneto-inertial time	8.6

Table 1.4.2. Dimensionless numbers as ratios of characteristic times. The material transit or residence time is $\tau = \ell/U$, where U is a typical material velocity.

Number	Symbol	Nomenclature	Sec. ref.
Electromagnetic			
$\tau_e/\tau = \epsilon U/\sigma$	R_e	Electric Reynolds number	5.11
$\tau_m/\tau = \mu\sigma\ell U$	R_m	Magnetic Reynolds number	6.2
Mechanical and thermal			
$\tau_a/\tau = U/a$	M	Mach number	9.19
$\tau_v/\tau = \rho\ell U/\eta$	R_y	Reynolds number	7.18
$\tau_D/\tau = \ell U/\kappa$	R_D	Molecular Peclet number	10.2
$\tau_T/\tau = \rho c_p \ell U/k_T$	R_T	Thermal Peclet number	10.2
$\tau_D/\tau_v = \eta/\rho\kappa$	P_D	Molecular-viscous Prandtl number	10.2
$\tau_T/\tau_v = c_p\eta/k_T$	P_T	Thermal-viscous Prandtl number	10.2
Electromechanical			
$\sqrt{\tau_m/\tau_{MV}} = \mu H\ell\sqrt{\sigma/\eta}$	H_m	Magnetic Hartmann number	8.6
$\sqrt{\tau_{mig}/\tau_{EV}}; \tau_e/\tau_{EV}$	H_e	Electric Hartmann number	9.12
$\tau_m/\tau_v = \eta\mu\sigma/\rho$	P_m	Magnetic-viscous Prandtl number	8.6

1.6 Transfer Relations and Continuum Dynamics of Linear Systems

Fields, flows and deformations in systems that are uniform in one or more "longitudinal" directions can have the dependence on the associated coordinate represented by complex amplitudes, Fourier series, Fourier transforms, or the appropriate extension of these in various coordinate systems. Typically, configurations are nonuniform in the remaining "transverse" coordinate. The dependence of variables on this direction is represented by "transfer relations." They are first introduced in Chap. 2 as flux-potential relations that encapsulate Laplacian fields in coordinate systems for which Laplace's equation is variable separable.

At the risk of having a forbidding appearance, most chapters include summaries of transfer relations in the three common coordinate systems. This is done so that they can be a resource, helping to obviate tedious manipulations that tend to obscure what is essential in the derivation of a model. The transfer relations help in organizing a development. Once the way in which they represent the space-time dynamics of a given medium is appreciated, they are also a way of quickly communicating the physical nature of a continuum.

Applications in Chap. 4 begin to exemplify how the transfer relations can help to organize the representation of configurations involving piece-wise uniform media. The systems considered there are spatially periodic in the "longitudinal" direction.

With each of the subsequent chapters, the application of the transfer relations is broadened. In Chap. 5, the temporal transient response is described in terms of the temporal modes. Then, spatial transients for systems in the temporal sinusoidal steady state are considered. In Chap. 6, magnetic diffusion processes are represented in terms of transfer relations, which take a form equally applicable to thermal and particle diffusion.

Much of the summary of fluid mechanics given in Chap. 7 is couched in terms of transfer relations. There, the variables are velocities and stresses. In a wealth of electromechanical examples, coupling between fields and media can be represented as occurring at boundaries and interfaces, where there are discontinuities in properties. Thus, in Chap. 8, the purely mechanical relations of Chap. 7 are combined with the electrical relations from Chap. 2 to represent electromechanical systems. More specialized are electromechanical transfer relations representing charged fluids, electron beams, hydro-magnetic systems and the like, derived in Chaps. 8-11.

A feature of many of the examples in Chap. 8 is instability, so that again the temporal modes come to the fore. But with effects of streaming brought into play in Chap. 11, there is a question of whether the instability is absolute in the sense that the response becomes unbounded with time at a given point in space, or convective (amplifying) in that a sinusoidal steady state can be established but with a response that becomes unbounded in space. These issues are taken up in Chap. 11.