Topic 12

Demonstrative Example Solutions in Static Analysis

Contents:

- **Analysis of various problems to demonstrate, study, and evaluate solution methods in statics**
- **Example analysis: Snap-through of an arch**
- **Example analysis: Collapse analysis of an elastic-plastic cylinder**
- **Example analysis: Large displacement response of a shell**
- **Example analysis: Large displacements of a cantilever subjected to deformation-independent and deformation-dependent loading**
- **Example analysis: Large displacement response of a diamond-shaped frame**
- **Computer-plotted animation: Diamond-shaped frame**
- **Example analysis: Failure and repair of a beam/cable structure**

Textbook: Sections 6.1, 6.5.2, 8.6, 8.6.1, 8.6.2, 8.6.3

IN THIS LECTURE, WE WANT TO STUDY SOME EXAMPLE SOLUTIONS

EX. 1 SNAP-THROUGH OF A TRUSS ARCH

EX. 2 COLLAPSE ANALYSIS OF AN ELASTO-PLASTIC CYLINDER

EX. 3 LARGE DISPLACEMENT SOLUTION OF A SHPERICAL SHELL

EX. 4 CANTILEVER UNDER PRESSURE LOADING

EX. 5 ANALYSIS OF DIAMOND-SHAPED FRAME

EX. 6 FAILURE AND REPAIR OF A BEAM/CABLE STRUCTURE

Markerboard
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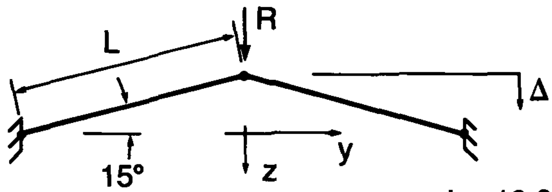
Example: Snap-through of a truss arch



$L = 10.0$

$k = \frac{EA}{L} = 2.1 \times 10^5$

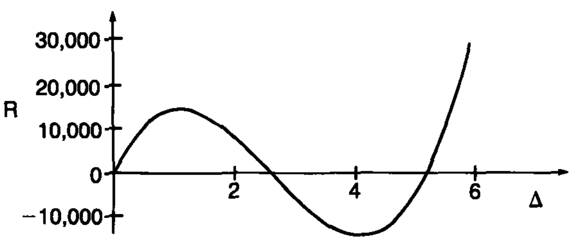
- Perform post-buckling analysis using automatic load step incrementation.
- Perform linearized buckling analysis.

Transparency 12-2

Postbuckling analysis:

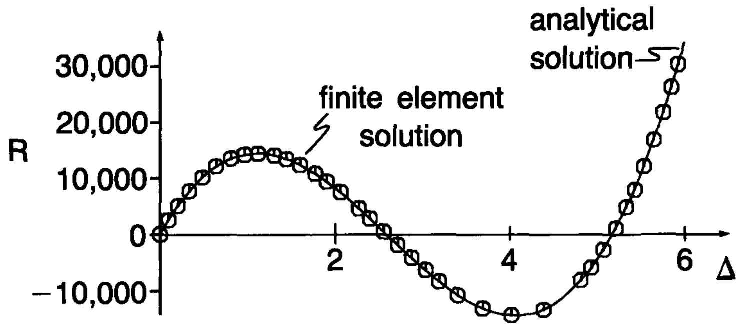
The analytical solution is

$$R = 2kL\left[\frac{1}{\sqrt{1 - 2\left(\frac{\Delta}{L}\right)\sin 15^\circ + \left(\frac{\Delta}{L}\right)^2}} - 1\right]\left(\sin 15^\circ - \frac{\Delta}{L}\right)$$



The automatic load step incrementation procedure previously described may be employed.

Using ${}^{1}\Delta = {}^{1}U = -0.1$, we obtain



Transparency 12-3

Solution details for load step 7:

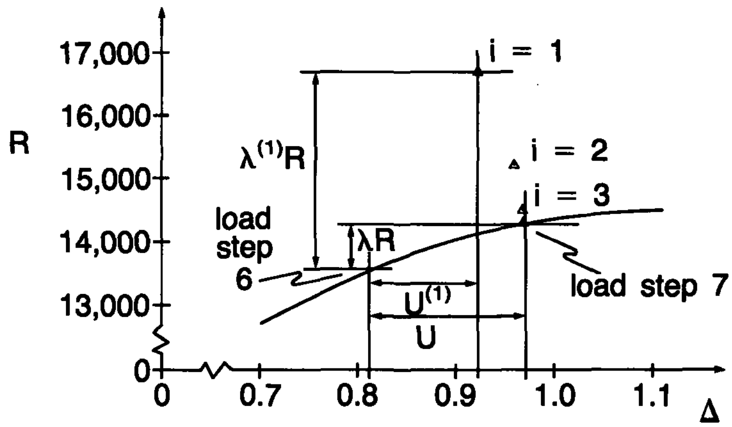
- The spherical constant arc-length algorithm is employed.
- The initial stiffness matrix is employed for all iterations, ${}^{t}U = .8111$, ${}^{t}R = 13{,}580$.

i	${}^{t+\Delta t}U^{(i)}$	${}^{t+\Delta t}\lambda^{(i)} R$	$U^{(i)}$	$\lambda^{(i)} R$
1	.9220	16,690	.1109	3,120
2	.9602	15,220	.1491	1,640
3	.9686	14,510	.1575	936
4	.9699	14,340	.1588	763
5	.9701	14,310	.1590	734
6	.9701	14,310	.1590	731

Transparency 12-4

Transparency 12-5

Pictorially, for load step 7,



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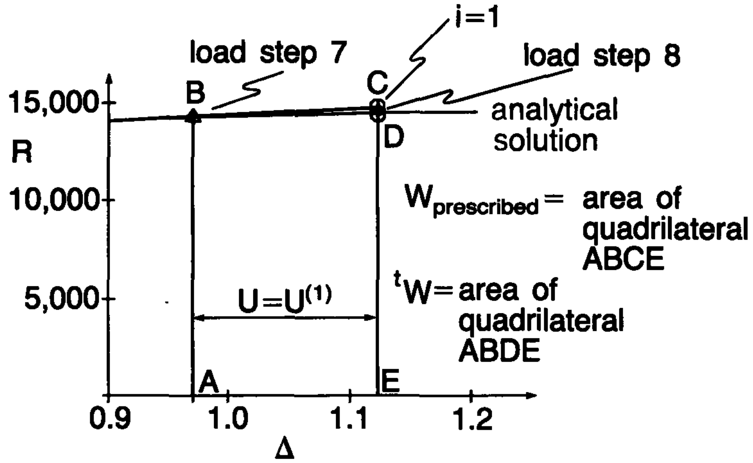
Solution details for load step 8:

- The constant increment of external work algorithm is employed.
- Modified Newton iterations are used, ${}^{t}U = .9701$, ${}^{t}R = 14{,}310$.

i	${}^{t+\Delta t}U^{(i)}$	${}^{t+\Delta t}\lambda^{(i)} R$	$U^{(i)}$	$\lambda^{(i)} R$
1	1.1227	14,740	.1526	440
2	1.1227	14,500	.1526	200

Pictorially, for load step 8,



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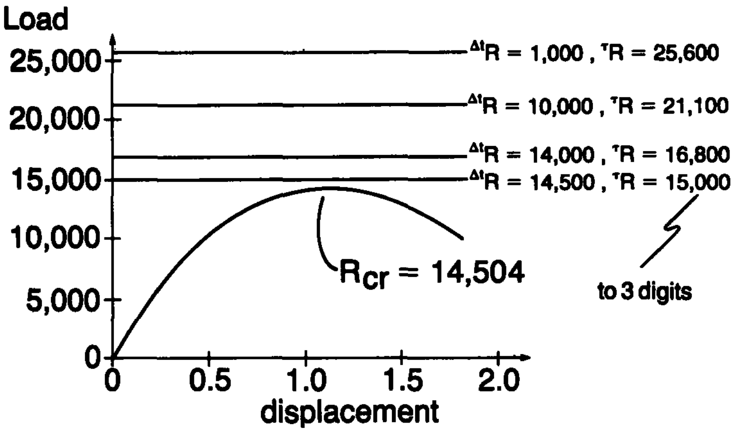
We now employ a linearized buckling analysis to estimate the collapse load for the truss arch.



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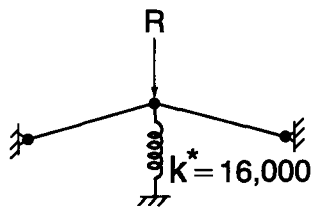
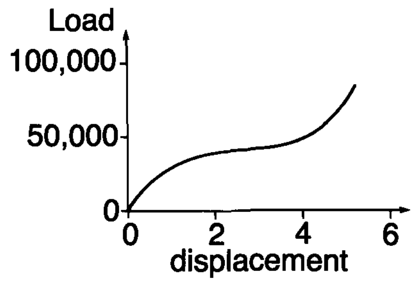
There are cases for which linearized buckling analysis gives buckling loads for stable structures. Consider the truss arch reinforced with a spring as shown:





This structure is always stable.

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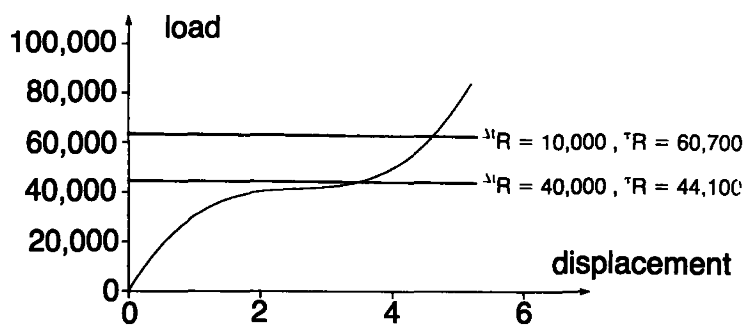
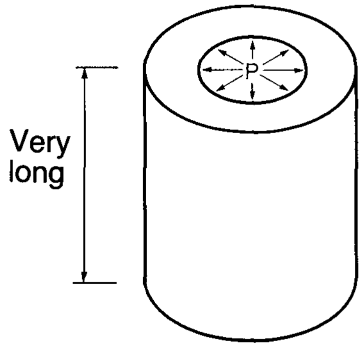
We perform a linearized buckling analysis. When the load level is close to the inflection point, the computed collapse load is also close to the inflection point.



Example: Elastic-plastic cylinder under internal pressure

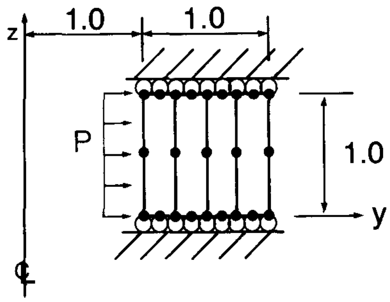


$E = 8667$
$\nu = 0.3$
$E_T = 0$
$\sigma_y = 17.32$
Inner radius = 1
Outer radius = 2

— Goal: Determine the limit load.

Transparency 12-11

Finite element mesh: Four 8-node axisymmetric elements



We note that, due to the boundary conditions and loading used, all stresses are constant in the z direction. Hence, 6-node elements could also have been used.

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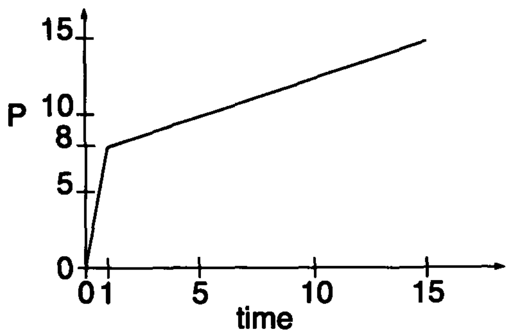
Transparency 12-13

Since the displacements are small, we use the M.N.O. formulation.

- We employ the following load function:



(the elastic limit load is P=7.42)

Transparency 12-14

Now we compare the effectiveness of various solution procedures:

- — Full Newton method with line searches
- — Full Newton method without line searches
- — BFGS method
- — Modified Newton method with line searches
- — Modified Newton method without line searches
- — Initial stress method

Transparency 12-15

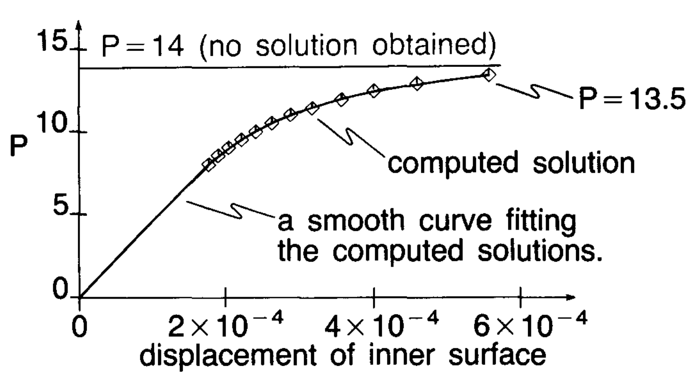
The following convergence tolerances are employed:

$$\frac{\Delta\underline{U}^{(i)T}\,[{}^{t+\Delta t}\underline{R} - {}^{t+\Delta t}\underline{F}^{(i-1)}]}{\Delta\underline{U}^{(1)T}\,[{}^{t+\Delta t}\underline{R} - {}^{t}\underline{F}]} \leq \underbrace{0.001}_{\text{ETOL}}$$

$$\frac{\|{}^{t+\Delta t}\underline{R} - {}^{t+\Delta t}\underline{F}^{(i-1)}\|_2}{\underbrace{1.0}_{\text{RNORM}}} \leq \underbrace{0.01}_{\text{RTOL}}$$

Transparency 12-16

When any of these procedures are used, the following force-deflection curve is obtained. For P = 14, no converged solution is found.



Transparency 12-17

We now compare the solution times for these procedures. For the comparison, we end the analysis when the solution for P = 13.5 is obtained.

Method	**Normalized time**
Full Newton method with line searches	1.2
Full Newton method	1.0
BFGS method	0.9
Modified Newton method with line searches	1.1
Modified Newton method	1.1
Initial stress method	2.2

Transparency 12-18

Now we employ automatic load step incrementation.

- No longer need to specify a load function
- Softening in force-deflection curve is automatically taken into account.

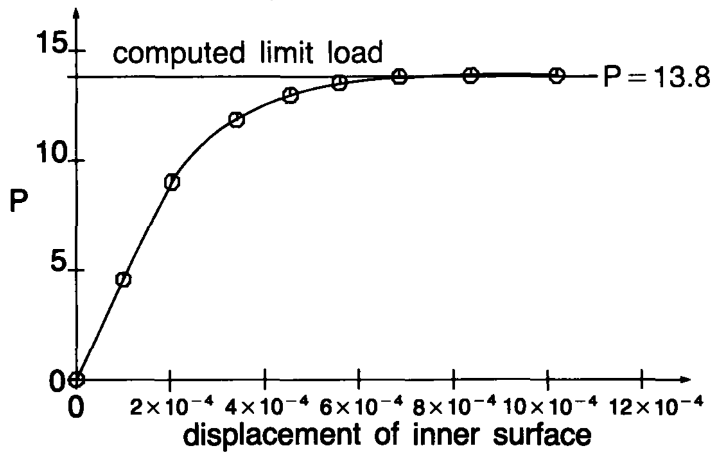
Here we use

$ETOL = 10^{-5}$

$RTOL = 0.01$

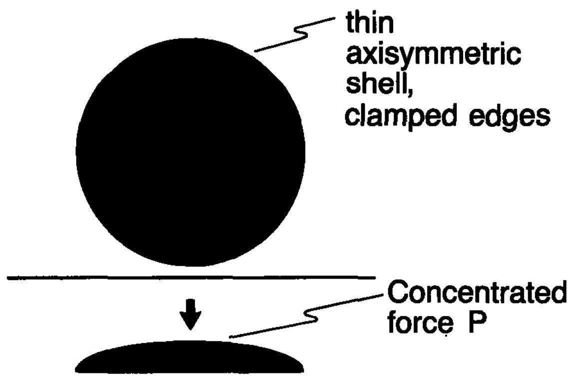
$RNORM = 1.0$

Result: Here we selected the displacement of the inner surface for the first load step to be 10^{-4}.



Transparency 12-19

Example: Spherical Shell



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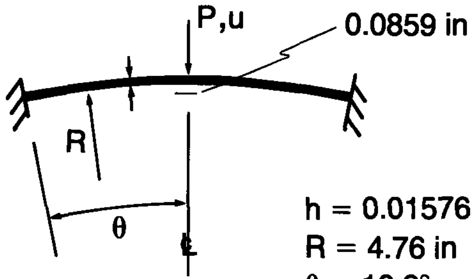
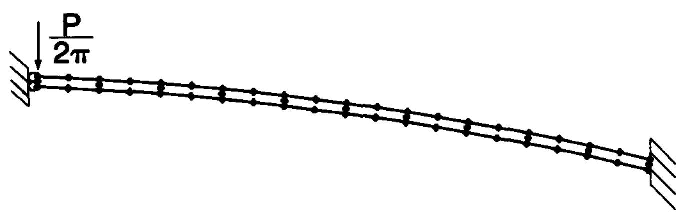
Transparency 12-21



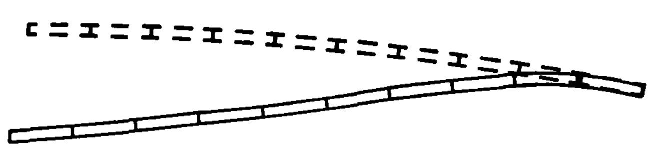
h = 0.01576 in
R = 4.76 in
θ = 10.9°
E = 10^7 psi
ν = 0.3

Transparency 12-22

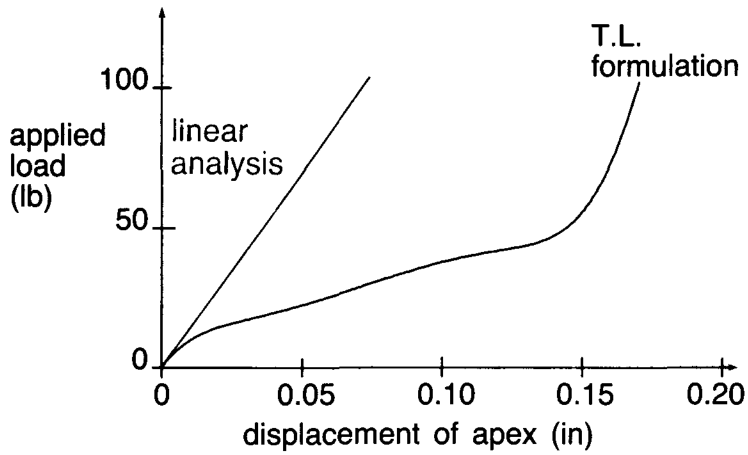
Finite element mesh: Ten 2-D axisymmetric elements



Deformed configuration for P = 100 lb:



Force-deflection curve obtained using 10 element mesh:



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Comparison of solution procedures:

1) Apply full load (100 lb) in 10 equal steps:

Solution procedure	Normalized solution time
Full Newton with line searches	1.4
Full Newton without line searches	1.0
BFGS method	did not converge
Modified Newton with line searches	did not converge
Modified Newton without line searches	did not converge

Transparency 12-24

Transparency 12-25

2) Apply full load in 50 equal steps:

Solution procedure	Normalized solution time
Full Newton with line search	1.3
Full Newton without line search	1.0
BFGS method	1.6
Modified Newton with line search	1.9
Modified Newton without line search	did not converge

Transparency 12-26

Convergence criterion employed:

$$\frac{\Delta \underline{U}^{(i)T}\left[{}^{t+\Delta t}\underline{R} - {}^{t+\Delta t}\underline{F}^{(i-1)}\right]}{\Delta \underline{U}^{(1)T}\left[{}^{t+\Delta t}\underline{R} - {}^{t}\underline{F}\right]} \le \underbrace{0.001}_{\text{ETOL}}$$

Maximum number of iterations permitted = 99

We may also employ automatic load step incrementation:

Here we use

$$\text{ETOL} = 10^{-5}$$

and

$$\underbrace{\frac{\| {}^{t+\Delta t}\underline{R} - {}^{t+\Delta t}\underline{F}^{(i-1)} \|_2}{1.0}}_{\text{RNORM}} \leq \underbrace{0.01}_{\text{RTOL}}$$

as convergence tolerances.

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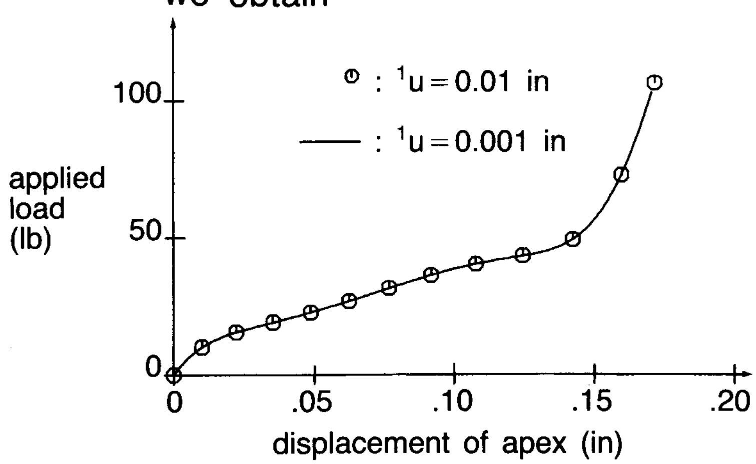
Results: Using different choices of initial prescribed displacements, we obtain



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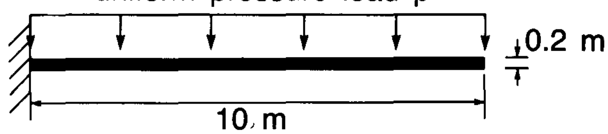
Transparency 12-29

Example: Cantilever under pressure loading



E = 207000 MPa
$\nu = 0.3$
Plane strain, width = 1.0 m

- Determine the deformed shape of the cantilever for p = 1 MPa.

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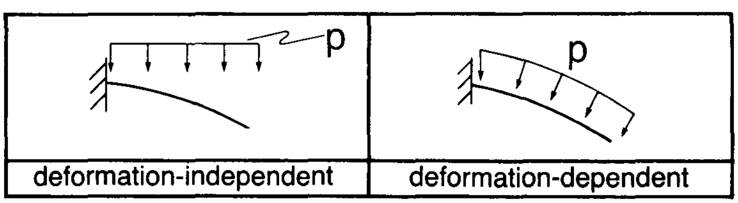
— Since the cantilever undergoes large displacements, the pressure loading (primarily the direction of loading) depends on the configuration of the cantilever:



The purpose of this example is to contrast the assumption of deformation-independent loading with the assumption of deformation-dependent loading.

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Finite element model: Twenty-five two-dimensional 8-node elements (1 layer, evenly spaced)

Solution details:

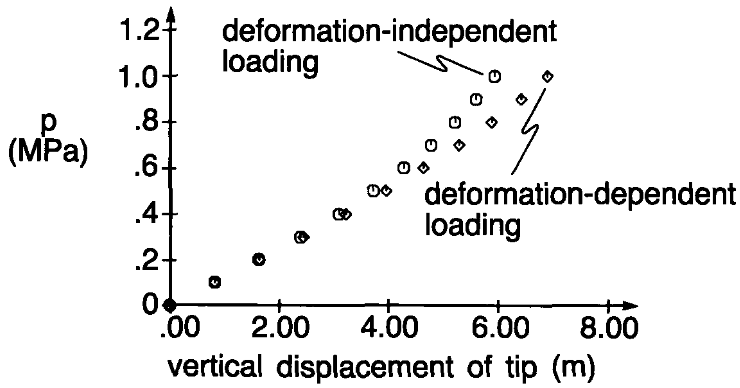
- Full Newton method without line searches is used.
- Convergence tolerances are
  - $ETOL = 10^{-3}$
  - $RTOL = 10^{-2}$, $RNORM = 1.0$ MN

Transparency 12-32

Transparency 12-33

Results: Force-deflection curve

- For small deflections, there are negligible differences between the two assumptions.



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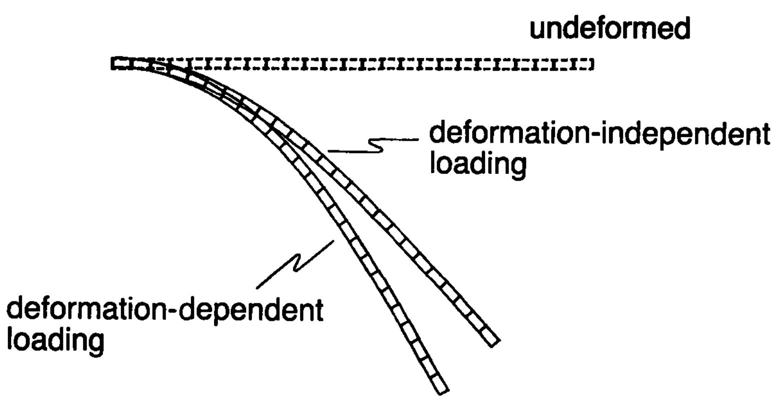
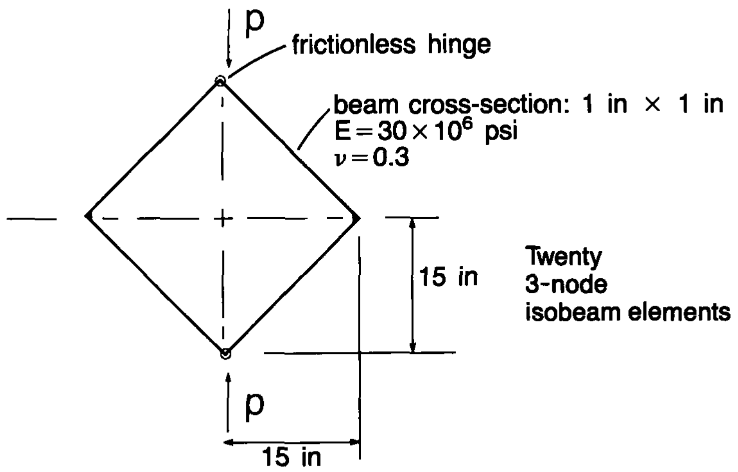
Pictorially, for p = 1.0 MPa,



Example: Diamond-shaped frame



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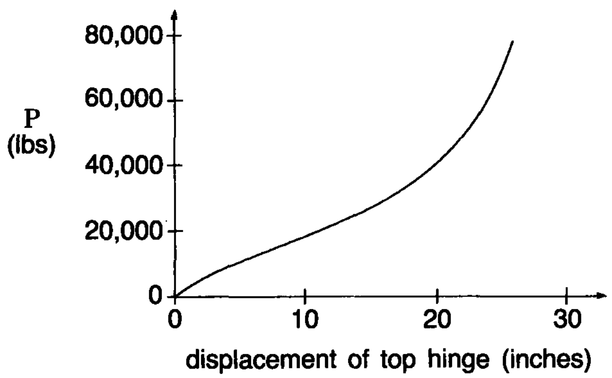
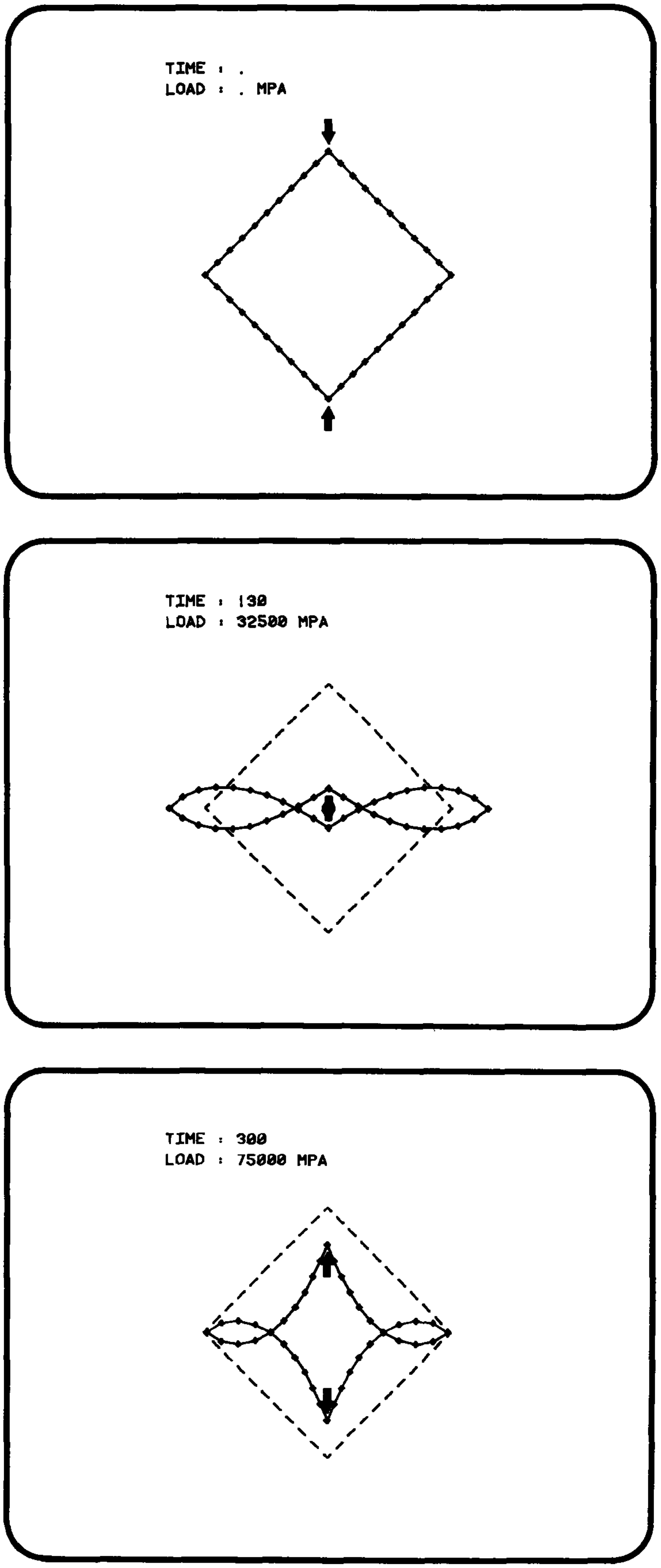
Force-deflection curve, obtained using the T.L formulation:

- A constant load increment of 250 lbs is used.

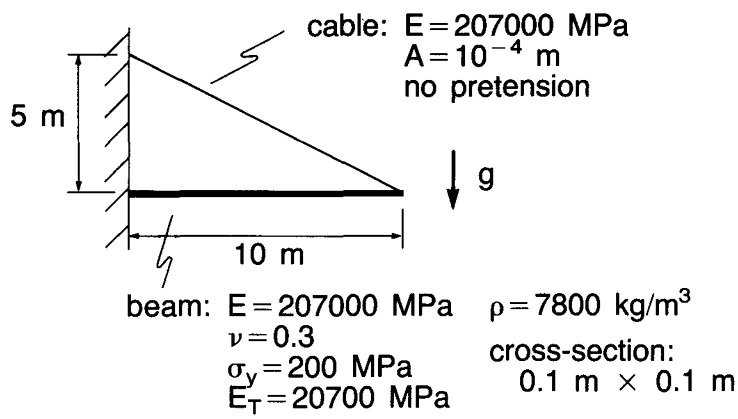


Transparency 12-36

Computer Animation
Diamond shaped frame



Example: Failure and repair of a beam/cable structure



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In this analysis, we simulate the failure and repair of the cable.

Steps in analysis:

Load step	Event
1	Beam sags under its weight, but is supported by cable.
1 to 2	Cable snaps, plastic flow occurs at built-in end of beam.
2 to 4	A new cable is installed, and is tensioned until the tip of the beam returns to its location in load step 1.

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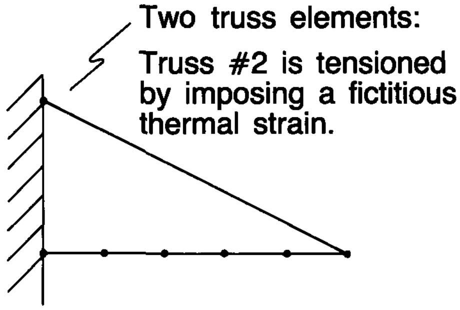
Finite element model:



Load step	Active truss
1	#1
2	none
3	#2
4	#2

Five 2-node Hermitian beam elements

5 Newton-Cotes integration points in r direction
3 Newton-Cotes integration points in s direction

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Solution details: The U.L. formulation is employed for the truss elements and the beam elements.

Convergence tolerances:

$ETOL = 10^{-3}$

$RTOL = 10^{-2}$

$RNORM = 7.6 \times 10^{-3}$ MN

$RMNORM = 3.8 \times 10^{-2}$ MN-m

Transparency 12-41

Comparison of solution algorithms:

Method	Results
Full Newton with line searches	All load steps successful, normalized CPU time = 1.0.
Full Newton	Stiffness matrix not positive definite in load step 2.
BFGS	All load steps successful, normalized CPU time = 2.5.
Modified Newton with or without line searches	No convergence in load step 2.

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Results:

Load step	Disp. of tip	Stress in cable	Moment at built-in end
1	−.008 m	64 MPa	9.7 KN-m
2	−.63 m	—	38 KN-m
3	−.31 m	37 MPa	22 KN-m
4	−.008 m	72 MPa	6.2 KN-m

Note: The elastic limit moment at the built-in end of the beam is 33 KN-m.

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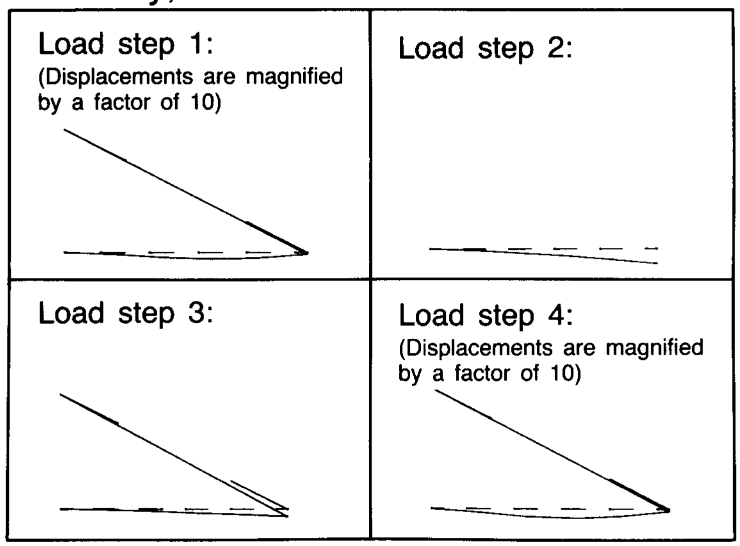
Pictorially,



Resource: Finite Element Procedures for Solids and Structures
Klaus-Jürgen Bathe