

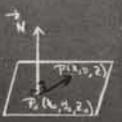
Unit 6: Equations of Lines and Planes

1. Lecture 1.060

Equations of Lines and Planes

Planes: Surfaces =
Lines: Curves

Cartesian Coordinates
 $P_0(x_0, y_0, z_0)$ is in plane
 $\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$ is \perp to plane



$\vec{N} \cdot \vec{P} = 0$
 $(A, B, C) \cdot (x - x_0, y - y_0, z - z_0) = 0$
 $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$\therefore A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
is equation of plane through (x_0, y_0, z_0) with (A, B, C) as a normal.

Example
 $2(x - 1) + 3(y + 2) + 4(z - 5) = 0$
passes through $(1, -2, 5)$
and has $2\vec{i} + 3\vec{j} + 4\vec{k}$ as a normal.

a.

Note:

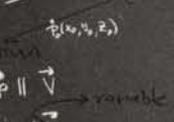
① For fixed A, B, C
 $Ax + By + Cz = D$
is a family of parallel planes.
Each has $\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$ as a normal.

② Equation of plane is linear.
 $ax + by + cz = d$,
which "generalizes"
 $ax + by = d$ (line)

③ Plane has 2 degrees of freedom
Example
 $x + 2y + 3z = 6$
May pick two of three unknowns at random, and solve for the third.
 $(0, 0, 2)$
 $(x - 0) + 2(y - 0) + 3(z - 0) = 0$
 $(0, 0, 2)$

b.

Equation of a Line
 $\vec{v} = (A, B, C)$



$\vec{P}_0 \vec{P} \parallel \vec{v}$
 $\vec{P}_0 \vec{P} = t\vec{v}$
 $(x - x_0, y - y_0, z - z_0) = (tA, tB, tC)$
 $\left. \begin{aligned} x - x_0 &= tA \\ y - y_0 &= tB \\ z - z_0 &= tC \end{aligned} \right\}$

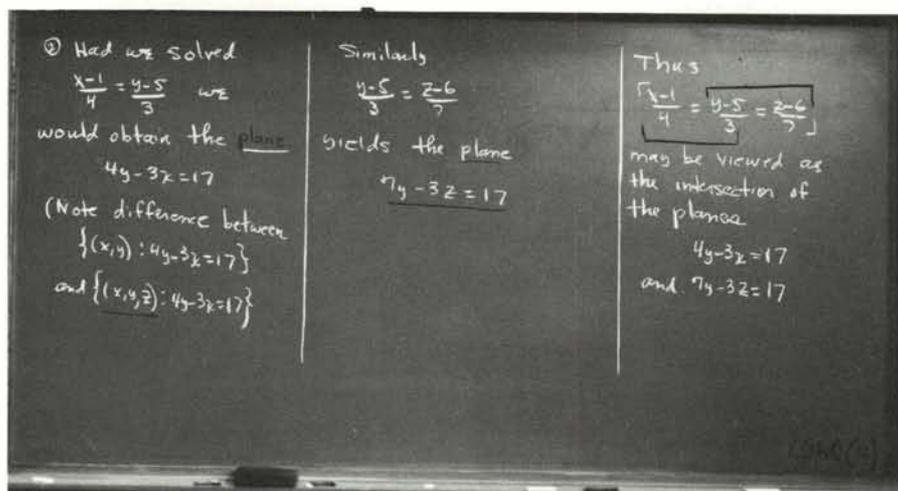
$\frac{x - x_0}{A} = \frac{y - y_0}{B} = \frac{z - z_0}{C} (=t)$

Example
 $\frac{x - 1}{4} = \frac{y - 5}{3} = \frac{z - 6}{7}$
passes through $(1, 5, 6)$ and is \parallel to $4\vec{i} + 3\vec{j} + 7\vec{k}$
 $4(x - 1) + 3(y - 5) + 7(z - 6) = 0$

Note
① The line has one degree of freedom.
Example
If $\frac{x - 1}{4} = \frac{y - 5}{3} = \frac{z - 6}{7}$
let $x = 9$
then $z = \frac{9 - 5}{3} = \frac{z - 6}{7}$
 $\therefore y = 11, z = 20$
 $(9, 11, 20)$ is a point on the line

c.

Lecture 1.060 continued



Study Guide
Block 1: Vector Arithmetic
Unit 6: Equations of Lines and Planes

2. Read Thomas, section 12.8

3. Exercises:

1.6.1(L)

Find the equation of the plane determined by the points $A(1,2,3)$, $B(3,3,5)$, and $C(4,8,1)$. (see Exercise 1.5.2)

1.6.2

Show that $y = 2x$ is the equation of the plane which passes through $(0,0,0)$ and has $2\vec{i} - \vec{j}$ as its normal.

1.6.3(L)

- a. What is the equation of the line which is parallel to $3\vec{i} + 4\vec{j} + 2\vec{k}$ and passes through $(-2,5,-1)$?
- b. At what point does this line intersect the xy -plane?

1.6.4

Find the directional cosines for the line

$$\frac{x-1}{6} = \frac{y+2}{3} = \frac{z-4}{2}.$$

1.6.5

At what point does the line which is parallel to $2\vec{i} + 3\vec{j} + 5\vec{k}$ and which passes through $(-1,3,-2)$ intersect the plane given in Exercise 1.6.1?

1.6.6(L)

- a. Find the distance from $(7,8,9)$ to the plane $2x + 3y + 6z = 8$.
- b. At what point does the line through $(7,8,9)$ perpendicular to the plane in a. intersect this plane?
- c. Find the distance between the planes $2x + 3y + 6z = 22$ and $2x + 3y + 6z = 8$.

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1.6.7

The points $A(1,1,4)$ and $B(3,4,5)$ are on the line L , while the points $C(3,4,-1)$, $D(4,6,2)$, and $E(8,9,7)$ are in the plane M . At what point does the line L intersect the plane M ?

1.6.8

- a. Find the angle between the planes whose Cartesian equations are $3x + 2y + 6z = 8$ and $2x + 2y - z = 5$.
- b. Find the Cartesian equation of the line which is the intersection of the two planes given in part a.

Quiz

1. By computing $(-1)(1 - 1)$ in two different ways, use the rules and theorems of arithmetic to prove that $(-1)(-1) = 1$.
2. Find a unit vector which is normal to the curve $y = x^3 + x$ at the point $(1,2)$.
3. Let $A(1,3,5)$, $B(3,4,7)$ and $C(-1,0,-1)$ be points in space. Find a unit vector such that it originates at A ; lies in the plane determined by A , B , and C ; and bisects $\sphericalangle BAC$.
4. Let P denote the plane determined by the points $A(1,2,3)$, $B(3,3,5)$, and $(4,4,9)$.
 - (a) Determine the measure of $\sphericalangle BAC$.
 - (b) Find the equation of the plane P .
 - (c) What is the area of $\triangle ABC$?
 - (d) At what point do the medians of $\triangle ABC$ intersect?
 - (e) What is the equation of the line which passes through C and is parallel to AB ?
5. Find the distance of the point $P_0(2,3,4)$ from the plane whose Cartesian equation is $4x + 5y + 2z = 6$. Also determine the point at which the line through P_0 , perpendicular to the plane $4x + 5y + 2z = 6$, intersects this plane.

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Resource: Calculus Revisited: Multivariable Calculus
Prof. Herbert Gross

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