

Unit 3: Applications to 3-Dimensional Space

1. Lecture 1.030

3-dimensional Vectors (arrows)

$\vec{A} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$   
 $|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Note

- ① If  $\vec{A} = a_1\vec{i}$   
then  $|\vec{A}| = \sqrt{a_1^2}$
- ② If  $\vec{A} = a_1\vec{i} + a_2\vec{j}$   
then  $|\vec{A}| = \sqrt{a_1^2 + a_2^2}$
- ③ If  $\vec{A} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$   
then  $|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

1.030(1)

a.

If  $\vec{A} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$   
and  $\vec{B} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$   
then:

$$\vec{A} + \vec{B} = (a_1 + b_1)\vec{i} + (a_2 + b_2)\vec{j} + (a_3 + b_3)\vec{k}$$

If  $c$  is any number,  
 $c\vec{A} = ca_1\vec{i} + ca_2\vec{j} + ca_3\vec{k}$

In particular,  
 $-\vec{B} = (-1)\vec{B}$   
 $= -b_1\vec{i} - b_2\vec{j} - b_3\vec{k}$

Therefore,  
 $\vec{A} - \vec{B} = (a_1 - b_1)\vec{i} + (a_2 - b_2)\vec{j} + (a_3 - b_3)\vec{k}$

1.030(2)

b.

Note the need for Cartesian coordinates.

For example, in polar coordinates,

$\vec{A} - \vec{B} \neq (r_2 - r_1, \theta_2 - \theta_1)$

In fact,

$|\vec{C}|^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\theta)$   
 (Law of cosines)

Name-versus-Concept  
 Six is an even number.  
 It can be written as (11) five.  
 Thus the test for "evenness" by looking at the last digit depends on the number base - but six is always even!

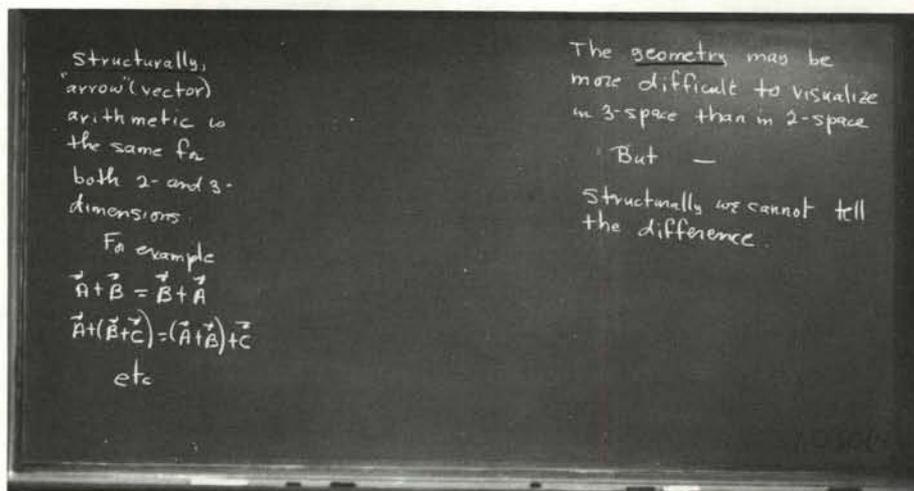
1.030(3)

c.

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Lecture 1.030 continued



d.

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2. Read Thomas, sections 12.4 and 12.5

3. Exercises:

1.3.1(L)

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- a. Let A and B denote two points in space and let O denote any other point. Show that a point P belongs to the line determined by A and B if and only if there exists a scalar t such that  $\vec{OP} = (1 - t)\vec{OA} + t\vec{OB}$ .
- b. If the points A and B of part a. are given in Cartesian coordinates by  $A(a_1, a_2, a_3)$  and  $B = (b_1, b_2, b_3)$ , show that  $P(x, y, z)$  is on the line determined by A and B if and only if  $(x - a_1)/(b_1 - a_1) = (y - a_2)/(b_2 - a_2) = (z - a_3)/(b_3 - a_3)$ .
- c. Find the (Cartesian) equation of the line which passes through  $A(2, -3, 5)$  and  $B = (5, 4, -1)$ .

1.3.2

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- a. Find the (Cartesian) equation of the line which passes through  $A(3, 5, 1)$  and  $B = (7, 2, 4)$ .
- b. At what point does the line described in part a. intersect the xy-plane?

1.3.3

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Find the vector which originates at  $A(1, 2, 3)$  and bisects  $\sphericalangle BAC$ , where  $B = (2, 4, 1)$  and  $C = (4, 8, 5)$ .

1.3.4

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- a. Let A, B, and C be points in space not all on the same straight line, and let M be the point at which the medians of  $\triangle ABC$  intersect. Express  $\vec{OM}$  in terms of  $\vec{OA}$ ,  $\vec{OB}$  and  $\vec{OC}$ .
- b. If in Cartesian coordinates  $A = (a_1, a_2, a_3)$ ,  $B = (b_1, b_2, b_3)$  and  $C = (c_1, c_2, c_3)$ , find the coordinates of M (where A, B, C, and M are as in part a) in terms of the a's, b's and c's.
- c. With A, B, and C as in Exercise 1.3.3 find the coordinates of the points at which the medians of  $\triangle ABC$  meet.

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1.3.4.continued

- d. Given that  $A = (1,2,3)$  and  $B = (2,4,1)$  find the coordinates of  $C$  if the medians of  $\triangle ABC$  meet at  $(0,0,0)$ .

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1.3.5(L)

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Suppose  $A$ ,  $B$ , and  $C$  are three points in space not on the same line. Let  $O$  denote any other point in space. Show that a point  $P$  belongs to the plane determined by  $A$ ,  $B$ , and  $C$  if and only if there exist scalars  $t_1$  and  $t_2$  such that

$$\vec{OP} = (1 - t_1 - t_2)\vec{OA} + t_1 \vec{OB} + t_2 \vec{OC}.$$

The next two exercises are optional. Their purpose is to supply you with extra drill with vector arithmetic, while at the same time, providing you with a better idea of the analytic counterpart of a plane. That is, the geometric notion of degrees of freedom is identified analytically with the number of arbitrary parameters in a formula. In our discussion of a plane, which is obviously 2-dimensional from a geometric point of view, we see that the analytic counterpart involves the two parameters  $t_1$  and  $t_2$ . For those who feel they have had enough drill, these two concepts will be revisited later, at a time when we have the computational tools to simplify the procedure used in these two exercises.

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1.3.6(L)

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- a. Find the equation (using Cartesian coordinates) of the plane determined by  $A(1,2,3)$ ,  $B(2,4,5)$ , and  $C(4,5,7)$ . Express your answer in terms of  $t_1$  and  $t_2$  as in 1.3.5(L).
- b. Find appropriate values for  $t_1$  and  $t_2$  in a. to verify that the given points  $A$ ,  $B$ , and  $C$  satisfy the equation obtained in a.
- c. Let  $t_1 = t_2 = 1$  in your answer to a. to find the point  $P$  if  $ABCP$  is to be a parallelogram.
- d. Determine from your answer in a. whether  $(3,4,5)$  is above, below or in the plane determined by  $A$ ,  $B$ , and  $C$ .

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1.3.7

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- a. Find the equation of the plane which is determined by the points  $A(2,3,4)$ ,  $B(3,1,2)$  and  $C(4,2,5)$ .
- b. Is the point  $(5,6,14)$  in this plane? If not, is it below the plane? Explain.

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**Resource: Calculus Revisited: Multivariable Calculus**  
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