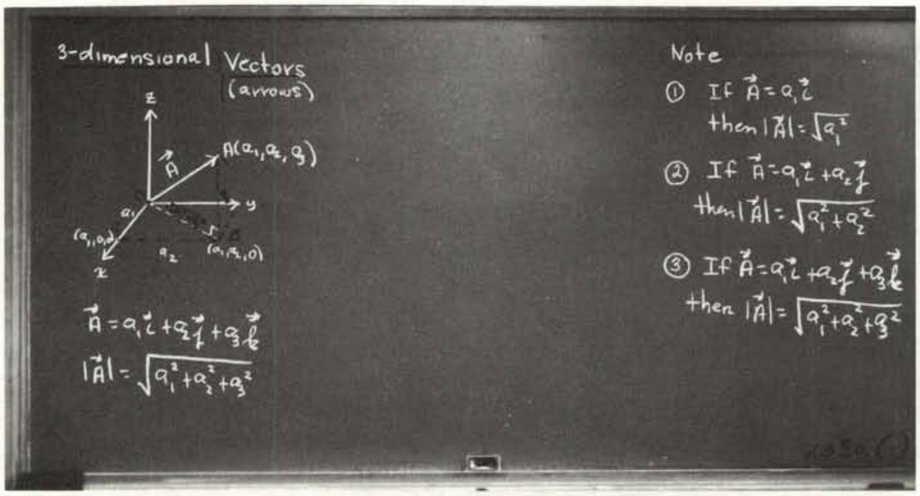
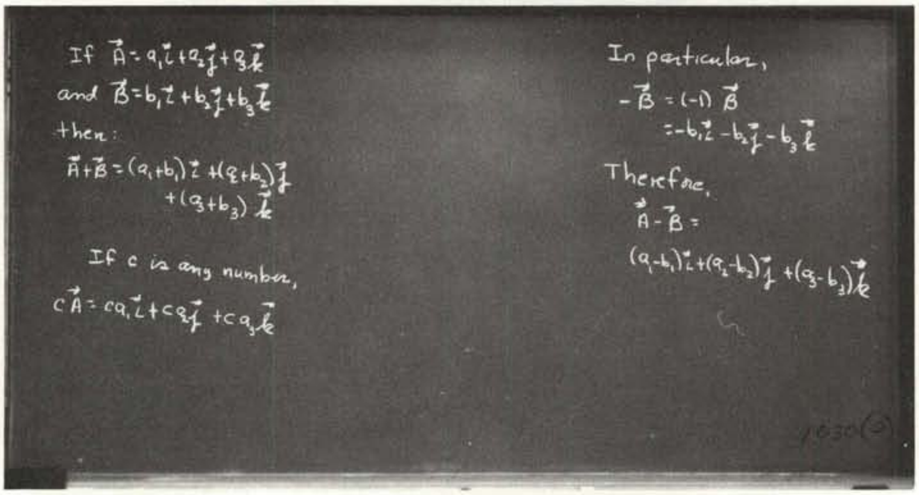
Unit 3: Applications to 3-Dimensional Space

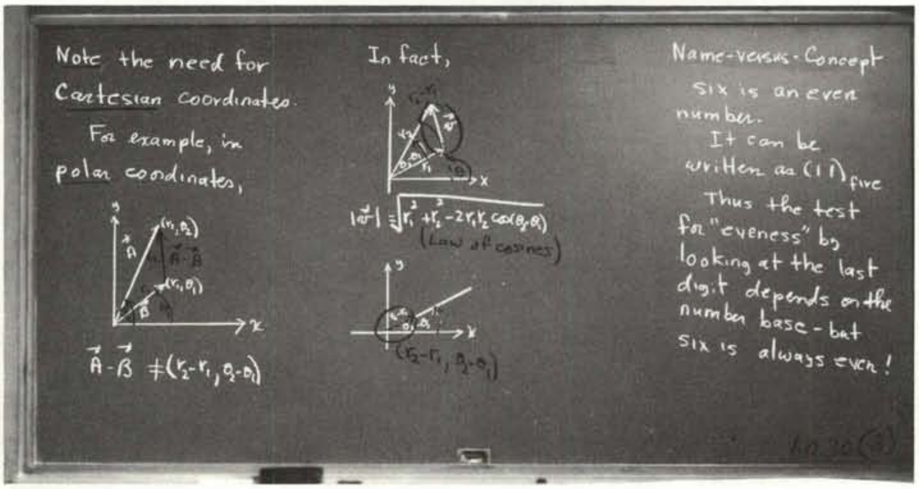
1. Lecture 1.030



a.

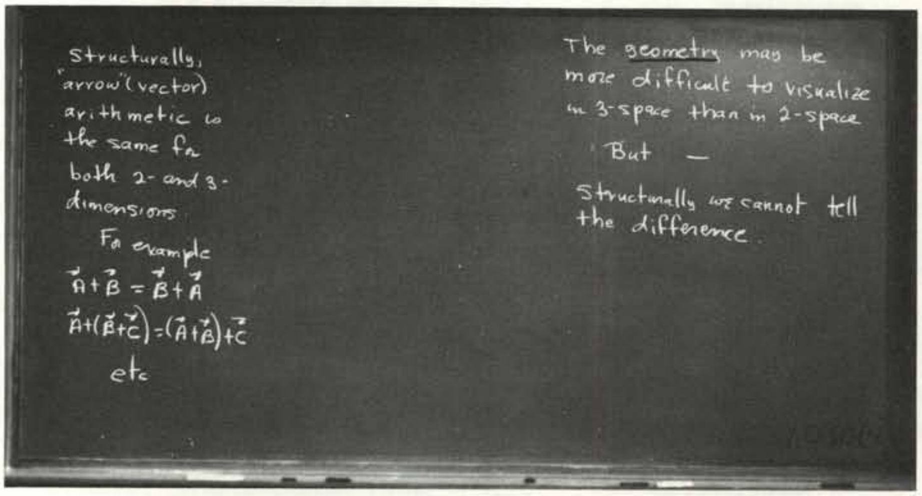


b.



c.

Lecture 1.030 continued

d. 

2. Read Thomas, sections 12.4 and 12.5

3. Exercises:

1.3.1(L)

a. Let A and B denote two points in space and let O denote any other point Show that a point P belongs to the line determined by A and B if and only if there exists a scalar t such that $\vec{OP} = (1 - t)\vec{OA} + t\,\vec{OB}$.

b. If the points A and B of part a. are given in Cartesian coordinates by $A(a_1,a_2,a_3)$ and $B = (b_1,b_2,b_3)$, show that $P(x,y,z)$ is on the line determined by A and B if and only if $(x - a_1)/(b_1 - a_1) = (y - a_2)/(b_2 - a_2) = (z - a_3)/(b_3 - a_3)$.

c. Find the (Cartesian) equation of the line which passes through A(2,-3,5) and B = (5,4,-1).

1.3.2

a. Find the (Cartesian) equation of the line which passes through A(3,5,1) and B = (7,2,4).

b. At what point does the line described in part a. intersect the xy-plane?

1.3.3

Find the vector which originates at A(1,2,3) and bisects ∢BAC, where B = (2,4,1) and C = (4,8,5).

1.3.4

a. Let A, B, and C be points in space not all on the same straight line, and let M be the point at which the medians of △ABC intersect. Express $\vec{OM}$ in terms of $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$.

b. If in Cartesian coordinates $A = (a_1,a_2,a_3)$, $B = (b_1,b_2,b_3)$ and $C = (c_1,c_2,c_3)$, find the coordinates of M (where A, B, C, and M are as in part a) in terms of the a's, b's and c's.

c. With A, B, and C as in Exercise 1.3.3 find the coordinates of the points at which the medians of △ABC meet.

(continued on next page)

1.3.4.continued

d. Given that A = (1,2,3) and B = (2,4,1) find the coordinates of C if the medians of ΔABC meet at (0,0,0).

1.3.5(L)

Suppose A, B, and C are three points in space not on the same line. Let O denote any other point in space. Show that a point P belongs to the plane determined by A, B, and C if and only if there exist scalars t_1 and t_2 such that

$$\vec{OP} = (1 - t_1 - t_2)\vec{OA} + t_1\,\vec{OB} + t_2\,\vec{OC}.$$

The next two exercises are optional. Their purpose is to supply you with extra drill with vector arithmetic, while at the same time, providing you with a better idea of the analytic counterpart of a plane. That is, the geometric notion of degrees of freedom is identified analytically with the number of arbitrary parameters in a formula. In our discussion of a plane, which is obviously 2-dimensional from a geometric point of view, we see that the analytic counterpart involves the two parameters t_1 and t_2. For those who feel thay have had enough drill, these two concepts will be revisited later, at a time when we have the computational tools to simplify the procedure used in these two exercises.

1.3.6(L)

a. Find the equation (using Cartesian coordinates) of the plane determined by A(1,2,3), B(2,4,5), and C(4,5,7). Express your answer in terms of t_1 and t_2 as in 1.3.5(L).

b. Find appropriate values for t_1 and t_2 in a. to verify that the given points A, B, and C satisfy the equation obtained in a.

c. Let $t_1 = t_2 = 1$ in your answer to a. to find the point P if ABCP is to be a parallelogram.

d. Determine from your answer in a. whether (3,4,5) is above, below or in the plane determined by A, B, and C.

1.3.7

a. Find the equation of the plane which is determined by the points A(2,3,4), B(3,1,2) and C(4,2,5).

b. Is the point (5,6,14) in this plane? If not, is it below the plane? Explain.