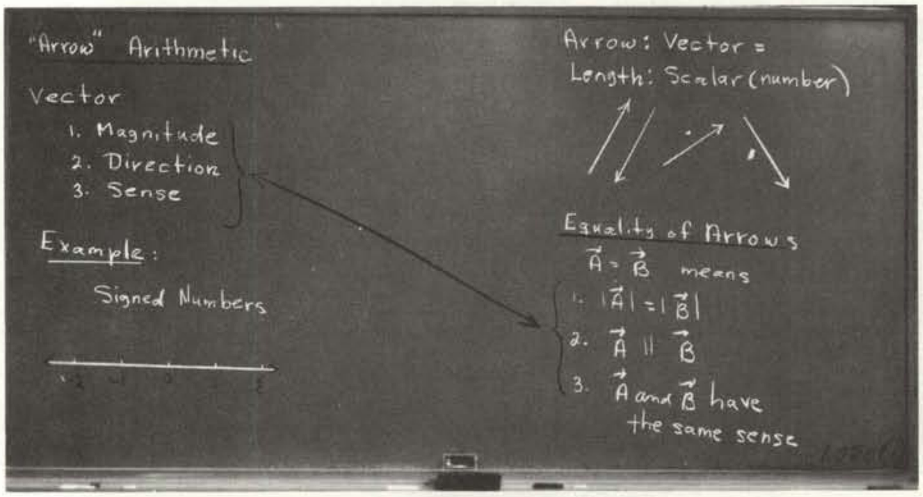
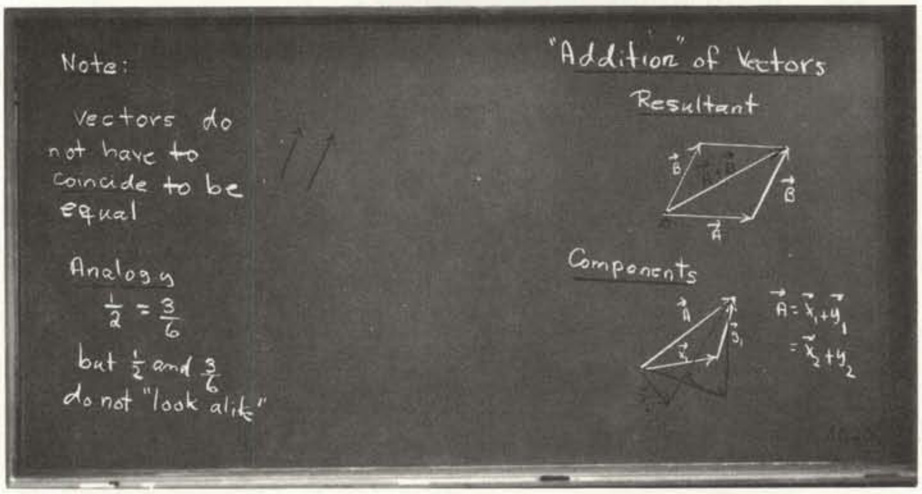
Study Guide
Block 1: Vector Arithmetic

Unit 2: The Structure of Vector Arithmetic

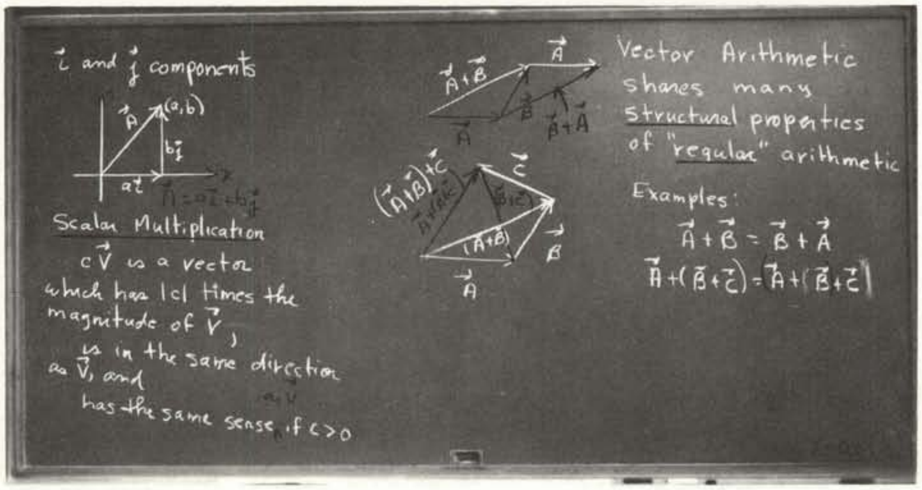
1. Lecture 1.020



a.



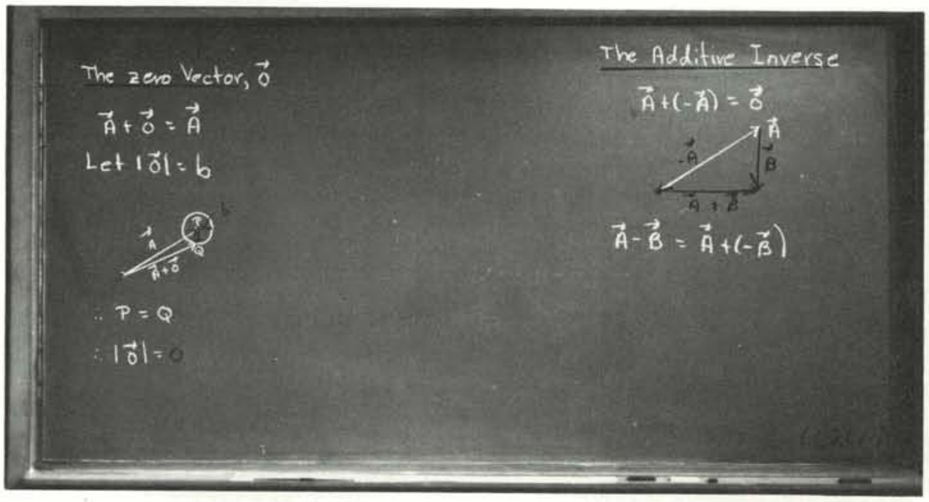
b.



c.

Lecture 1.020 continued

d.



2. Read Supplementary Notes, Chapter 2.

3. Read Thomas 12.3.

4. Exercises:

1.2.1(L)

Use vector methods to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side, and its length is half of that of the third side.

1.2.2(L)

Let O, A, and B be three points not on the same straight line. Let C be chosen on AB so that it divides it into two parts of ratio m:n. That is; $\overline{AC}/\overline{CB} = m/n$.

a. Express OC in terms of $\vec{OA}$, $\vec{OB}$, m, and n.

b. If 0 is the origin (0,0), A is the point (a_1,a_2) and B is the point (b_1,b_2) (where we are using Cartesian coordinates), express the coordinates of C in terms of a_1, a_2, b_1, b_2, m, and n.

c. What are the coordinates of C if A = (1,2), B = (3,5) and C is three-fifths of the way from A to B?

1.2.3

Let A and B be two distinct fixed points in the plane and let O denote an arbitrarily chosen third point. Show that a point P is on the line which joins A and B if and only if OP can be be written in the form: $OP = (1-t)\overline{OA} + t\overline{OB}$.

1.2.4

Use the technique of Exercise 1.2.3 to find the vector equation of the line determined by the points (1,2) and (3,5), and then check your answer by non-vector methods.

1.2.5

Let M denote the point at which the medians of ABC meet. (Recall that a median of a triangle is the line from a vertex to the midpoint of the opposite side and that the medians intersect at a point which is two-thirds of the way from the vertex to the opposite side.) Let O be any other point in the plane determined by A, B, and C.

a. Express $\vec{OM}$ in terms of $\vec{OA}$, $\vec{OB}$, and $\vec{OC}$.

b. Again, using Cartesian coordinates, describe the coordinates of M if $A = (a_1,a_2)$, $B = (b_1,b_2)$, $C = (c_1,c_2)$, and $0 = (0,0)$.

c. If A = (1,2), B = (3,5) and C = (4,9), at what point do the medians of ABC meet?

1.2.6

a. Find a unit vector which originates at (3,9) and is tangent to the curve $y = x^2$ at that point.

b. Find a unit vector which is perpendicular to the vector of part (a). [This vector is said to be a unit normal vector $y = x^2$ at (3,9).]

1.2.7(L)

Let A, B, and C be three points not on the same line.

a. Find a vector which bisects ∡ BAC.

b. Find the vector if A = (1,1), B = (4,5), and C = (6,13).

c. What is the equation of the line which bisects the ∡ BAC as given in (b)?

Comment

The following two exercises are optional. Their purpose is to give you more experience in the playing of the "game of mathematics" in general, and the game of vectors, in particular. It is hoped that those of you who elect to work on these exercises will keep the "game" concept in mind as the primary objective, and relegate the actual steps in the proofs to a secondary role.

1.2.8(L)

Mimic the procedure used in the previous unit to prove that for any scalar, a, $a\vec{0} = \vec{0}$.

1.2.9

Prove that if $a \neq 0$ but $a\vec{v} = \vec{0}$ then $\vec{v} = \vec{0}$.