Unit 4: Matrices as Linear Functions

1. Read Supplementary Notes, Chapter 6, Section F.

2. Exercises:

4.4.1

Define $\underline{f}:E^2 \to E^2$ by $\underline{f}(x,y) = (u,v)$ where $u = 6x + 5y$ and $v = x + y$.

a. In terms of the determinant of the matrix of coefficients, show how we may conclude that $\underline{f}^{-1}$ exists.

b. Letting $A = \begin{bmatrix} 6 & 5 \\ 1 & 1 \end{bmatrix}$, compute A^{-1} and then describe the mapping $\underline{f}^{-1}:E^2 \to E^2$ explicitly.

c. In particular, compute $\underline{f}^{-1}(16,3)$.

d. Compute $\underline{f}(L)$ where L is the line $y = 2x$ [i.e., $L = \{(x,y):y = 2x\}$].

4.4.2

Define $\underline{f}:E^2 \to E^2$ by $\underline{f}(x,y) = (u,v)$ where $u = x + 4y$, $v = 3x + 12y$.

a. Using determinants, show that $\underline{f}^{-1}$ does not exist.

b. Describe the set $\underline{f}(E^2)$.

c. Assuming that we view $\underline{f}$ geometrically, find the locus of all points (x,y) in the xy-plane such that $\underline{f}(x,y) = (8,24)$.

d. Use (c) to show a geometric construction for finding the point (x,y) on the line $2x + 9y = 17$ for which $\underline{f}(x,y) = (8,24)$.

e. Show that no other point on $2x + 9y = 17$ can be mapped into (8,24) by $\underline{f}$.

4.4.3

Define $\underline{f}:E^3 \to E^3$ by $\underline{f}(x,y,z) = (u,v,w)$ where

$$\begin{cases} u = x + y + z \\ v = 2x + 3y + 2z \\ w = x + 3y + z \end{cases}$$

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4.4.3 continued

a. Show that $\underline{f}(E^3)$ is contained in $\{(u,v,w): 3u - 2v + w = 0\}$.

b. Interpret part (a) geometrically.

c. Describe the set S of all elements of E^3 for which $\underline{f}(x,y,z) = (0,0,0)$.

d. The points (0,0,0) and (1,1,-1) lie in the plane defined by $f(E^3)$ [i.e., in $w = -3u + 2v$]. Describe the locus of all points (x,y,z) such that $\underline{f}(x,y,z)$ is on the line L determined by (0,0,0) and (1,1,-1).

4.4.4

Given the system of equations

$$x_1 + 2x_2 + x_4 + x_4 = b_1$$

$$2x_1 + 5x_2 + 3x_3 + 4x_4 = b_2$$

$$3x_1 + 5x_2 + 2x_3 + x_4 = b_3$$

$$3x_1 + 4x_2 + x_3 - x_4 = b_4$$

a. Use the augmented-matrix technique to determine the constraints under which the above equations have a solution.

b. In particular, show that if the constraints are met, x_3 and x_4 may be chosen at random, after which x_1 and x_2 are uniquely determined.

c. Let $\underline{f}: E^4 \to E^4$ be defined by $f(x_1, x_2, x_3, x_4) = (b_1, b_2, b_3, b_4)$, where b_1, b_2, b_3, and b_4 are as above.

(i) Show that there is no $\underline{x} \varepsilon E^4$ such that $\underline{f}(\underline{x}) = (1,1,1,1)$.

(ii) Find all $\underline{x} \varepsilon E^4$ such that $\underline{f}(\underline{x}) = (1,1,4,5)$.

4.4.5

Find the constraints under which the system

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4.4.5 continued

$$\left.\begin{aligned}
x_1 + x_2 + x_3 + 2x_4 + x_5 &= b_1 \\
2x_1 + 3x_2 + 2x_3 + 3x_4 + 3x_5 &= b_2 \\
3x_1 + 3x_2 + 4x_3 + 5x_4 + 2x_5 &= b_3 \\
x_1 + 3x_2 - x_3 + 2x_4 + 5x_5 &= b_4 \\
-2x_1 + x_2 - 6x_3 - 3x_4 + 5x_5 &= b_5
\end{aligned}\right\} \qquad (1)$$

has solutions. In particular, discuss the function $\underline{f}:E^5 \to E^5$ defined by $\underline{f}(\underline{x}) = \underline{f}(x_1,x_2,x_3,x_4,x_5) = (b_1,b_2,b_3,b_4,b_5)$, where b_1, b_2, b_3, b_4, and b_5 are as defined in (1).

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