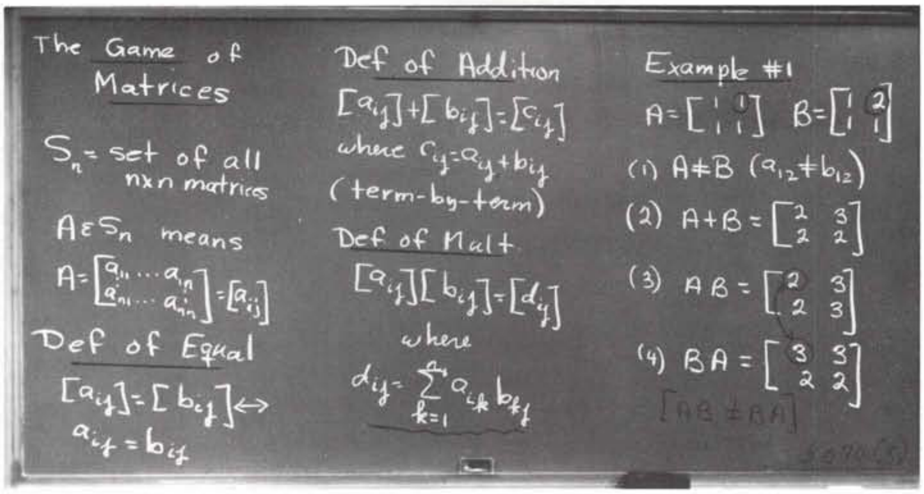
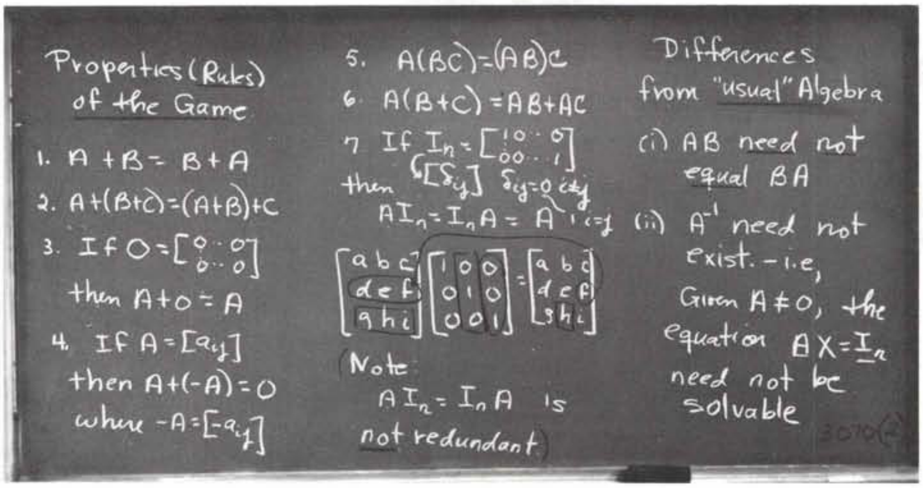
Unit 2: Introduction to Matrix Algebra

1. Lecture 4.020



a.



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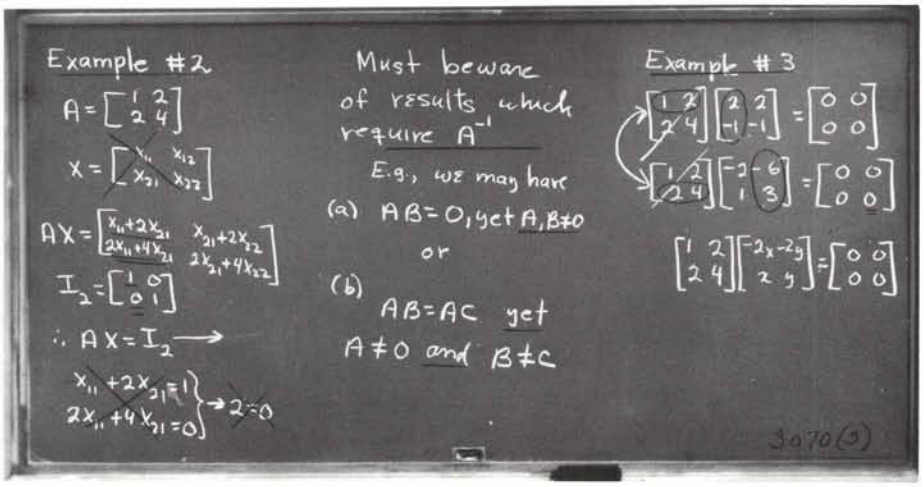
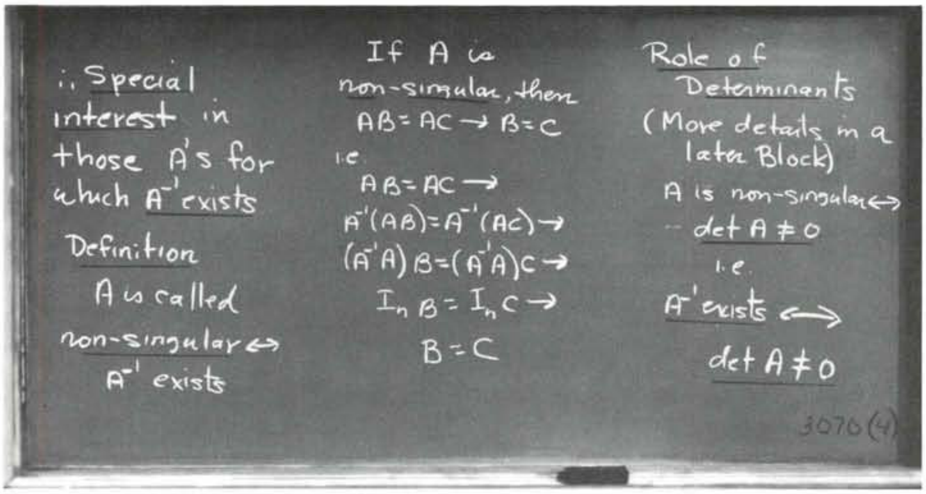
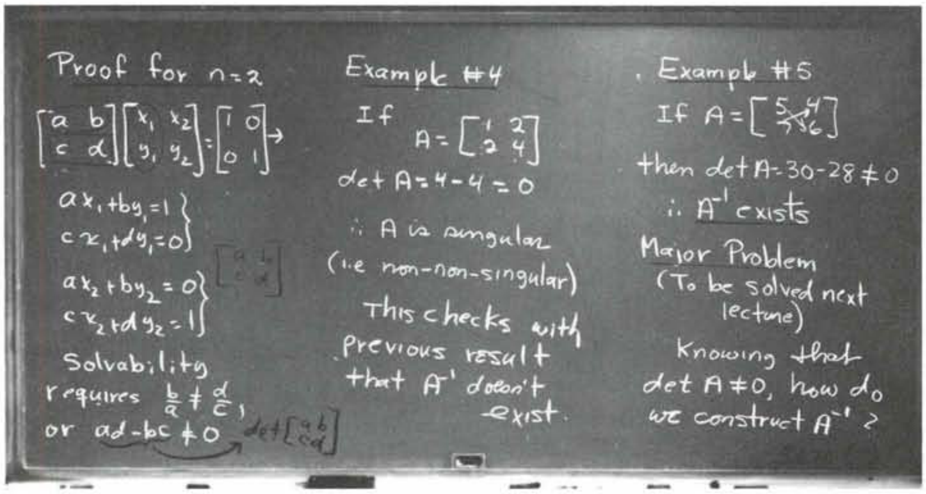


c.

Lecture 4.020 continued



d.



e.

2. Read: Supplementary Notes, Chapter 6, Section D

3. (Optional) Read: Thomas, Section 13.2

4. Exercises

4.2.1

Show that A(B + C) = AB + AC and A(BC) = (AB)C where A,B, and C are each 2 x 2 matrices.

4.2.2

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

a. Find all matrices B such that AB = 0.
b. Find all matrices C such that AC = 0 but CA $\neq$ 0.

4.2.3

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ and $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Find all matrices B such that AB = 0

4.2.4

Let $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

a. Find a matrix X such that AX = I
b. Does there exist a matrix X such that BX = I?

4.2.5

For any square matrix A, we define A^r to mean $\underbrace{AA\ldots A}_{n \text{ times}}$, where r is any positive integer,

a. Compute A^3 if $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

b. Compute A^r if $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(continued on the next page)

4.2.5 (continued)

c. Compute A^2 and A^3 if $A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$

4.2.6

Let $A = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix}$ and let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

a. Compute $A^2 - 2A - 24\,I$

b. Compute A^3 using the result of part (a).

c. Compute A^7

4.2.7

Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$. Find all matrices B such that AB = BA.

4.2.8

If A is any matrix, we define the transpose of A, written A^T, to be the matrix obtained when we interchange the rows and columns of A. That is, the columns of A are the rows of A^T.

a. What is A^T if $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$.

b. If A is any matrix, compute $(A^T)^T$.

c. If A and B are 2 x 2 matrices show that

$(AB)^T = B^T A^T$.

d. Find all 2 x 2 matrices A for which $AA^T = A^T A$.