

Study Guide
Block 2: Vector Calculus

Unit 4: Polar Coordinates I

1. Lecture 2.030

Polar Coordinates

$x = r \cos \theta$
 $y = r \sin \theta$

$r^2 = x^2 + y^2$
 $\tan \theta = \frac{y}{x}$

"Complication"

$(r_1, \theta_1) = (r_2, \theta_2) \rightarrow$
 $r_1 = r_2 \text{ and } \theta_1 = \theta_2$

For example,

① $(r, \theta) = (r, \theta + 2k\pi)$

[Note: $\sin \theta_1 = \sin(\theta_1 + 2\pi k)$
but $\theta_1 \neq \theta_1 + 2\pi k$
 $k \neq 0$]

② r and/or θ may be negative

$(r, \theta) = (-r, \theta + \pi)$

[e.g., $r = \sin^2 \theta$ is satisfied by $(\frac{1}{4}, \frac{\pi}{6})$ but not by $(-\frac{1}{4}, \frac{7\pi}{6})$
 $\therefore (\frac{1}{4}, \frac{\pi}{6}) = (-\frac{1}{4}, \frac{7\pi}{6})$]

a.

Nomenclature

If the polar equation for C is $r = a \sin \theta$, we don't mean

$r = a \sin \theta \rightarrow r^2 = r a \sin \theta \rightarrow x^2 + y^2 = ay$

$\therefore \frac{dr}{d\theta}$ does not give the slope of C .

We do mean

$m = \tan \psi = \frac{dy}{dx}$

$\tan \psi = \frac{r}{r \frac{dr}{d\theta}}$

$r = f(\theta)$

b.

Area

$\frac{\Delta A}{\Delta \theta} \approx \frac{\pi r_m^2}{2\pi} \leq \Delta A \leq \frac{\pi R_M^2}{2\pi}$

$\frac{1}{2} r_m^2 \Delta \theta \leq \Delta A \leq \frac{1}{2} R_M^2 \Delta \theta$

$\therefore \Delta \theta > 0 \rightarrow$

$\frac{1}{2} r_m^2 \leq \frac{\Delta A}{\Delta \theta} \leq \frac{1}{2} R_M^2$

As $\Delta \theta \rightarrow 0^+$
 r_m and $R_M \rightarrow r$
 provided r is a continuous function of θ .

In this event

$\frac{dA}{d\theta} = \lim_{\Delta \theta \rightarrow 0} \frac{\Delta A}{\Delta \theta} = \frac{1}{2} r^2$

$\therefore A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$

$= \int_{\theta_1}^{\theta_2} \frac{1}{2} [g(\theta)]^2 d\theta$

$A_R = \int_a^b [f_1(x) - f_2(x)] dx$

$= \frac{1}{2} \int_{\theta_1}^{\theta_2} [g(\theta)]^2 d\theta$

Same answer - different expressions

c.

2. Read Thomas, Sections 11.1, 11.2, and 11.3.

3. Exercises:

2.4.1(L)

Describe the curve C if its polar equation is $r = \cos \theta$, $0 \leq \theta \leq \pi$.

2.4.2

The curve C is given by the polar equation

$$\frac{1}{r^2} = 4 \cos^2 \theta + 9 \sin^2 \theta.$$

Sketch C by converting its polar equation into the equivalent Cartesian form.

2.4.3(L)

- Plot the curve C if its polar equation is $r = \sec \theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.
- Plot the curve whose polar equation is $r = \theta$, and then write the equation of this curve in Cartesian coordinates.

2.4.4(L)

- The curve C is given by the polar equation $r = f(\theta)$. What can we conclude about the symmetry of C if we know that whenever (r_0, θ_0) belongs to C so also does $(-r_0, -\theta_0)$?
 - With C as above, what can we conclude about its symmetry if we know instead that whenever (r_0, θ_0) is on C so is $(-r_0, \pi - \theta_0)$?
- Use the result of (a) together with the information contained in $\frac{dr}{d\theta}$ to sketch the curve whose polar equation is $r = \sin 2\theta$.
- What is the Cartesian equation of the curve in (b)?

2.4.5(L)

- a. Let C_1 and C_2 be defined by the polar equations $r = \cos \theta + 1$ and $r = \cos \theta - 1$, respectively. Show that C_1 and C_2 have no simultaneous points of intersection.
- b. With C_1 and C_2 as in part (a), sketch these two curves.
- c. Explain why the results of (a) and (b) are not contradictory.

2.4.6(L)

The curve C_1 is defined by the polar equation $r = \cos 2\theta$, while C_2 is defined by $r = 1 + \cos \theta$. Find all points at which C_1 and C_2 intersect.

2.4.7(L)

Let C denote the curve whose polar equation is $r = \sin \frac{\theta}{4}$, $0^\circ \leq \theta \leq 720^\circ$. If P denotes the point $(\frac{1}{2}, 240^\circ)$, does P belong to C ? Explain.

2.4.8

Find all points of intersection of the curves C_1 and C_2 if the polar equation for C_1 is $r = 1 + \cos \theta$ and the polar equation for C_2 is $r = 1 + \sin \theta$.

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Resource: Calculus Revisited: Multivariable Calculus
Prof. Herbert Gross

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