Table of Integrals

1 $\int u^n dx = \frac{u^{n+1}}{a(n+1)}$ except for $\int \frac{dx}{u} = \frac{\ln|u|}{a}$ All the integrals 1 - 17 involve $u = ax + b$

2 $\int xu^n\,dx = \frac{u^{n+2}}{a^2(n+2)} - \frac{bu^{n+1}}{a^2(n+1)}$ except for $\int \frac{x\,dx}{u} = \frac{x}{a} - \frac{b\ln|u|}{a^2}$ and $\int \frac{x\,dx}{u^2} = \frac{b}{a^2u} + \frac{\ln|u|}{a^2}$

3 $\int \frac{x^2dx}{u} = \frac{1}{a^3}(\frac{u^2}{2} - 2bu + b^2\ln|u|)$ 4 $\int \frac{x^2\,dx}{u^2} = \frac{1}{a^3}(u - 2b\ln|u| - \frac{b^2}{u})$ 5 $\int \frac{x^2dx}{u^3} = \frac{1}{a^3}(\ln|u| + \frac{2b}{u} - \frac{b^2}{2u^2})$

6 $\int \frac{dx}{xu} = \frac{1}{b}\ln|\frac{x}{u}|$ 7 $\int \frac{dx}{x^2u} = -\frac{1}{bx} + \frac{a}{b^2}\ln|\frac{u}{x}|$ 8 $\int \frac{dx}{xu^2} = \frac{1}{bu} - \frac{1}{b^2}\ln|\frac{u}{x}|$ 9 $\int \frac{dx}{x^2u^2} = -\frac{b+2ax}{b^2xu} + \frac{2a}{b^3}\ln|\frac{u}{x}|$

10 $\int \sqrt{u}\,dx = \frac{2}{3a}u^{3/2}$ 11 $\int x\sqrt{u}\,dx = \frac{2(3ax-2b)}{15a^2}u^{3/2}$ 12 $\int x^2\sqrt{u}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}u^{3/2}$

13 $\int \frac{\sqrt{u}}{x}\,dx = 2\sqrt{u} + b\int \frac{dx}{x\sqrt{u}}$ 14 $\int \frac{x\,dx}{\sqrt{u}} = \frac{2(ax-2b)}{3a^2}\sqrt{u}$ 15 $\int \frac{x^2dx}{\sqrt{u}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{u}$

16 $\int \frac{dx}{x\sqrt{u}} = \frac{1}{\sqrt{b}}\ln|\frac{\sqrt{u}-\sqrt{b}}{\sqrt{u}+\sqrt{b}}|\ (b>0)$ or $\frac{2}{\sqrt{-b}}\tan^{-1}\frac{\sqrt{u}}{\sqrt{-b}}\ (b<0)$ 17 $\int \frac{\sqrt{u}}{x^2}dx = -\frac{\sqrt{u}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{u}}$

18 $\int \frac{dx}{(ax+b)(cx+d)} = \frac{1}{bc-ad}\ln|\frac{cx+d}{ax+b}|$ 19 $\int \frac{x\,dx}{(ax+b)(cx+d)} = \frac{1}{bc-ad}(\frac{b}{a}\ln|ax+b| - \frac{d}{c}\ln|cx+d|)$

20 $\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln|x + \sqrt{x^2 \pm a^2}|$ 21 $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}|$

22 $\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$ 23 $\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\frac{x}{a}$

24 $\int \frac{x^2\,dx}{\sqrt{x^2\pm a^2}} = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln|x + \sqrt{x^2 \pm a^2}|$ 25 $\int \frac{\sqrt{x^2\pm a^2}}{x^2}\,dx = -\frac{\sqrt{x^2\pm a^2}}{x} + \ln|x + \sqrt{x^2 \pm a^2}|$

26 $\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{8}(2x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln|x + \sqrt{x^2 \pm a^2}|$ 27 $\int \frac{dx}{x^2\sqrt{x^2\pm a^2}} = \mp\frac{\sqrt{x^2\pm a^2}}{a^2x}$

28 $\int (x^2 \pm a^2)^{3/2}\,dx = \frac{x}{8}(2x^2 \pm 5a^2)\sqrt{x^2 \pm a^2} + \frac{3a^4}{8}\ln|x + \sqrt{x^2 \pm a^2}|$ 29 $\int \frac{dx}{(x^2\pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2\pm a^2}}$

30 $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ 31 $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$ 32 $\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}}$

33 $\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln|\frac{a+\sqrt{a^2-x^2}}{x}|$ 34 $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a}$

35 $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$ 36 $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \sin^{-1}\frac{x}{a}$ 37 $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

38 $\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ 39 $\int (a^2-x^2)^{3/2}dx = \frac{x}{8}(5a^2 - 2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a}$

40 $\int \frac{dx}{b+c\sin ax} = \frac{-2}{a\sqrt{b^2-c^2}}\tan^{-1}\left[\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right],\quad b^2 > c^2$ 41 $\int \frac{dx}{1+\sin ax} = -\frac{1}{a}\tan(\frac{\pi}{4} - \frac{ax}{2})$

42 $\int \frac{dx}{b+c\sin ax} = \frac{-1}{a\sqrt{c^2-b^2}}\ln\left|\frac{c+b\sin ax+\sqrt{c^2-b^2}\cos ax}{b+c\sin ax}\right|,\quad b^2 < c^2$ 43 $\int \frac{dx}{1-\sin ax} = \frac{1}{a}\tan(\frac{\pi}{4} + \frac{ax}{2})$

44 $\int \frac{dx}{b+c\cos ax} = \frac{2}{a\sqrt{b^2-c^2}}\tan^{-1}\left[\sqrt{\frac{b-c}{b+c}}\tan\frac{ax}{2}\right],\quad b^2 > c^2$ 45 $\int \frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2}$

46 $\int \frac{dx}{b+c\cos ax} = \frac{1}{a\sqrt{c^2-b^2}}\ln|\frac{c+b\cos ax+\sqrt{c^2-b^2}\sin ax}{b+c\cos ax}|,\quad b^2 < c^2$ 47 $\int \frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2}$

48 $\int \sin^{-1}ax\,dx = x\sin^{-1}ax + \frac{1}{a}\sqrt{1-a^2x^2}$ 49 $\int x^n\sin^{-1}ax\,dx = \frac{x^{n+1}}{n+1}\sin^{-1}ax - \frac{a}{n+1}\int \frac{x^{n+1}dx}{\sqrt{1-a^2x^2}}$

50 $\int \tan^{-1}ax\,dx = x\tan^{-1}ax - \frac{1}{2a}\ln(1+a^2x^2)$ 51 $\int x^n\tan^{-1}ax\,dx = \frac{x^{n+1}}{n+1}\tan^{-1}ax - \frac{a}{n+1}\int \frac{x^{n+1}\,dx}{1+a^2x^2}$

52 $\int e^{ax}dx = \frac{e^{ax}}{a}$ 53 $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$ 54 $\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$ (b^{ax} is $e^{a(\ln b)x}$)

55 $\int \frac{dx}{x\ln ax} = \ln|\ln ax|$ Not elementary: $\int e^{x^2}\,dx$, $\int e^x\ln x\,dx$, $\int \frac{dx}{\ln x}$, $\int \frac{e^x}{x}\,dx$, $\int \frac{\sin x}{x}\,dx$, $\int \frac{\sin^{-1}x}{x}dx$

Exponentials and Logarithms

$y = b^x \leftrightarrow x = \log_b y \quad y = e^x \leftrightarrow x = \ln y$

$e = \lim(1+\frac{1}{n})^n = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828\cdots$

$e^x = \lim(1+\frac{x}{n})^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\ln y = \int_1^y \frac{dx}{x} \qquad \ln 1 = 0 \quad \ln e = 1$

$\ln xy = \ln x + \ln y \quad \ln x^n = n \ln x$

$\log_a y = (\log_a b)(\log_b y) \quad \log_a b = 1/\log_b a$

$e^{x+y} = e^x e^y \quad b^x = e^{x \ln b} \quad e^{\ln y} = y$

Equations and Their Solutions

$y' = cy$	$y_0 e^{ct}$
$y' = cy + s$	$y_0 e^{ct} + \frac{s}{c}(e^{ct} - 1)$
$y' = cy - by^2$	$\frac{c}{b+de^{-ct}} \quad d = \frac{c-by_0}{y_0}$
$y'' = -\lambda^2 y$	$\cos \lambda t$ and $\sin \lambda t$
$my'' + dy' + ky = 0$	$e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ or $te^{\lambda_1 t}$
$y_{n+1} = ay_n$	$a^n y_0$
$y_{n+1} = ay_n + s$	$a^n y_0 + s\frac{a^n - 1}{a-1}$

Vectors and Determinants

$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

$|\mathbf{A}|^2 = \mathbf{A}\cdot\mathbf{A} = a_1^2 + a_2^2 + a_3^2$ (length squared)

$\mathbf{A}\cdot\mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{A}||\mathbf{B}|\cos\theta$

$|\mathbf{A}\cdot\mathbf{B}| \le |\mathbf{A}||\mathbf{B}|$ (Schwarz inequality: $|\cos\theta| \le 1$)

$|\mathbf{A}+\mathbf{B}| \le |\mathbf{A}| + |\mathbf{B}|$ (triangle inequality)

$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}||\sin\theta|$ (cross product)

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{matrix} \mathbf{i}(a_2b_3 - a_3b_2) \\ +\mathbf{j}(a_3b_1 - a_1b_3) \\ +\mathbf{k}(a_1b_2 - a_2b_1) \end{matrix}$$

Right hand rule $\mathbf{i}\times\mathbf{j} = \mathbf{k}$, $\mathbf{j}\times\mathbf{k} = \mathbf{i}$, $\mathbf{k}\times\mathbf{i} = \mathbf{j}$

Parallelogram area $= |a_1b_2 - a_2b_1| = |\text{Det}|$

Triangle area $= \frac{1}{2}|a_1b_2 - a_2b_1| = \frac{1}{2}|\text{Det}|$

Box volume $= |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| = |\text{Determinant}|$

Matrices and Inverses

$Ax =$ combination of columns $= b$

Solution $x = A^{-1}b$ if $A^{-1}A = I$

Least squares $A^TA\bar{x} = A^Tb$

$Ax = \lambda x$ (λ is an eigenvalue)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$(AB)^{-1} = B^{-1}A^{-1}$, $(AB)^T = B^TA^T$

$$\begin{bmatrix} & & \\ \mathbf{a} & \mathbf{b} & \mathbf{c} \\ & & \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} \mathbf{b}\times\mathbf{c} \\ \mathbf{c}\times\mathbf{a} \\ \mathbf{a}\times\mathbf{b} \end{bmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{matrix} +\,a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 \\ -\,a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1 \end{matrix}$$

	SI Units	Symbols
length	meter	m
mass	kilogram	kg
time	second	s
current	ampere	A
frequency	hertz	Hz ~ 1/s
force	newton	N ~ kg·m/s²
pressure	pascal	Pa ~ N/m²
energy, work	joule	J ~ N·m
power	watt	W ~ J/s
charge	coulomb	C ~ A·s
temperature	kelvin	K

From	To	Multiply by
degrees	radians	.01745
calories	joules	4.1868
BTU	joules	1055.1
foot-pounds	joules	1.3558
feet	meters	.3048
miles	km	1.609
feet/sec	km/hr	1.0973
pounds	kg	.45359
ounces	kg	.02835
gallons	liters	3.785
horsepower	watts	745.7

Speed of light $c = 2.9979 \times 10^8$ m/s

Gravity $G = 6.6720 \times 10^{-11}$ Nm²/kg²

Radius at Equator $R = 6378$ km $= 3964$ miles

Acceleration $g = 9.8067$ m/s² $= 32.174$ ft/s²

Sums and Infinite Series

$1 + x + \cdots + x^{n-1} = \frac{1-x^n}{1-x}$

$1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots + x^n = (1+x)^n$

$1 + 2 + \cdots + n = \frac{1}{2}n(n+1) \approx \frac{n^2}{2}$

$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3}$

$1 + \frac{1}{2} + \cdots + \frac{1}{n} \approx \ln n \to \infty$ (harmonic)

$1 - \frac{1}{2} + \frac{1}{3} - \cdots = \ln 2$ (alternating)

$1 - \frac{1}{3} + \frac{1}{5} - \cdots = \frac{\pi}{4} \quad \sum \frac{1}{n^2} = \frac{\pi^2}{6} \quad \sum \frac{1}{n^4} = \frac{\pi^4}{90}$

$\frac{1}{1-x} = 1 + x + x^2 + \cdots$ (geometric: $|x| < 1$)

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots = \frac{d}{dx}\left(\frac{1}{1-x}\right)$

$\frac{1}{1+x} = 1 - x + x^2 - \cdots$ (geometric for $-x$)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots = \int \frac{dx}{1+x}$

$\sin x = x - x^3/6 + x^5/120 - \cdots$ (all x)

$\cos x = 1 - x^2/2 + x^4/24 - \cdots$ (all x)

$e^x = 1 + x + \frac{x^2}{2!} + \cdots (e = 1 + 1 + \frac{1}{2!} + \cdots)$

$e^{ix} = \cos x + i \sin x$ (Euler's formula)

$\cosh x = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \cdots$

$\sinh x = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \cdots$

$(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \cdots$ (Taylor)

$f(x,y) = f + xf_x + yf_y + \frac{x^2}{2!}f_{xx} + xyf_{xy} + \cdots$

Polar and Spherical

$x = r\cos\theta$ and $y = r\sin\theta$

$r = \sqrt{x^2+y^2}$ and $\tan\theta = y/x$

$x + iy = r(\cos\theta + i \sin\theta) = re^{i\theta}$

Area $\int \frac{1}{2}r^2 d\theta$ Length $\int \sqrt{r_\theta^2 + r^2}\, d\theta$

$x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$

Area $dA = dx\,dy = r\,dr\,d\theta = J\,du\,dv$

Volume $r\,dr\,d\theta\,dz = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

Stretching factor $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$

Area - Volume - Length - Mass - Moment

Circle πr^2 Ellipse πab Wedge of circle $r^2\theta/2$

Cylinder side $2\pi rh$ Volume $\pi r^2 h$ Shell $dV = 2\pi rh\,dr$

Sphere surface $4\pi r^2$ Volume $\frac{4}{3}\pi r^3$ Shell $dV = 4\pi r^2 dr$

Cone or pyramid Volume $\frac{1}{3}$ (base area) (height)

Length of curve $\int ds = \int \sqrt{1 + (dy/dx)^2}\, dx$

Area between curves $\int (v(x) - w(x))dx$

Surface area of revolution $\int 2\pi r\, ds (r = x$ or $r = y)$

Volume of revolution: Slices $\int \pi y^2 dx$ Shells $\int 2\pi xh\, dx$

Area of surface $z(x,y) : \iint \sqrt{1 + z_x^2 + z_y^2}\, dx\, dy$

Mass $M = \iint \rho\, dA$ Moment $M_y = \iint \rho x\, dA$

$\overline{x} = M_y/M, \overline{y} = M_x/M$ Moment of Inertia $I_y = \iint \rho x^2 dA$

Work $W = \int_a^b F(x)dx = V(b) - V(a)$ Force $F = dV/dx$

Partial Derivatives of $z = f(x,y)$

Tangent plane $z - z_0 = (\frac{\partial f}{\partial x})(x - x_0) + (\frac{\partial f}{\partial y})(y - y_0)$

Approximation $\Delta z \approx (\frac{\partial f}{\partial x})\Delta x + (\frac{\partial f}{\partial y})\Delta y$

Normal $\mathbf{N} = (f_x, f_y, -1)$ or (F_x, F_y, F_z)

Gradient $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$

Directional derivative: $D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u} = f_x u_1 + f_y u_2$

Chain rule: $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

Vector field $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$

Work $\int \mathbf{F} \cdot d\mathbf{R}$ Flux $\int M\, dy - N dx$

Divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl of $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M & N & P \end{vmatrix}$

Conservative $\mathbf{F} = \nabla f =$ gradient of f if curl $\mathbf{F} = \mathbf{O}$

Green's Theorem $\oint M\, dx + N\, dy = \iint (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y})\, dx\, dy$

Divergence Theorem $\iint \mathbf{F} \cdot \mathbf{n}\, dS = \iiint \text{div}\, \mathbf{F}\, dV$

Stokes' Theorem $\oint \mathbf{F} \cdot d\mathbf{R} = \iint (\text{curl}\, \mathbf{F}) \cdot \mathbf{n}\, dS$

An additional table of integrals is included just after the index.