11.433J / 15.021J Real Estate Economics
Fall 2008

Recitation 2

Real Estate Economics:

Rent Gradient and Land Price

Sep 16, 2008

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Outline

- HM1
- Richardian Theory: Rent Gradient and Land Price
  - Summary
  - House Rent
  - House Rent → Land Rent
  - House Rent → House Price
  - Land Rent → Land Price
  - Static vs. Growth
- Full model: power and limitation
  - Power: numerical example by linear regression
  - Assumption and expansion: spatial separation
  - What to read a paper
- Takeaway

Homework 1: quick answer

Q1	Q2a	Q2b	Q2c, part 1	Q2c, part 2
Equilibrium 1 →	Shock 1 →	Equilibrium 2 →	Shock 2 →	Equilibrium 3
P=200	P=400	P=200	P=90	P=200
R=20	R=20	R=10	R=9	R=20
S=2500	S=2500	S=5000	S=5000	S=2250
C=50	C=50	C=100	C=100	C=45

Summary: Price vs. Rent; Land vs. House

	House	Land
Rent	$R(d) = r_a * q + c + k(b-d)$	$r(d) = r_a + k(b-d)/q$
Price	$P_t(d) = \frac{r_a q}{i} + \frac{c}{i} + \frac{k(b_t - d)}{i} + \frac{k b_t g}{i(i-g)}$	$p_t(d) = \frac{r_a}{i} + \frac{k(b_t - d)}{iq} + \frac{k b_t g}{i(i-g)q}$

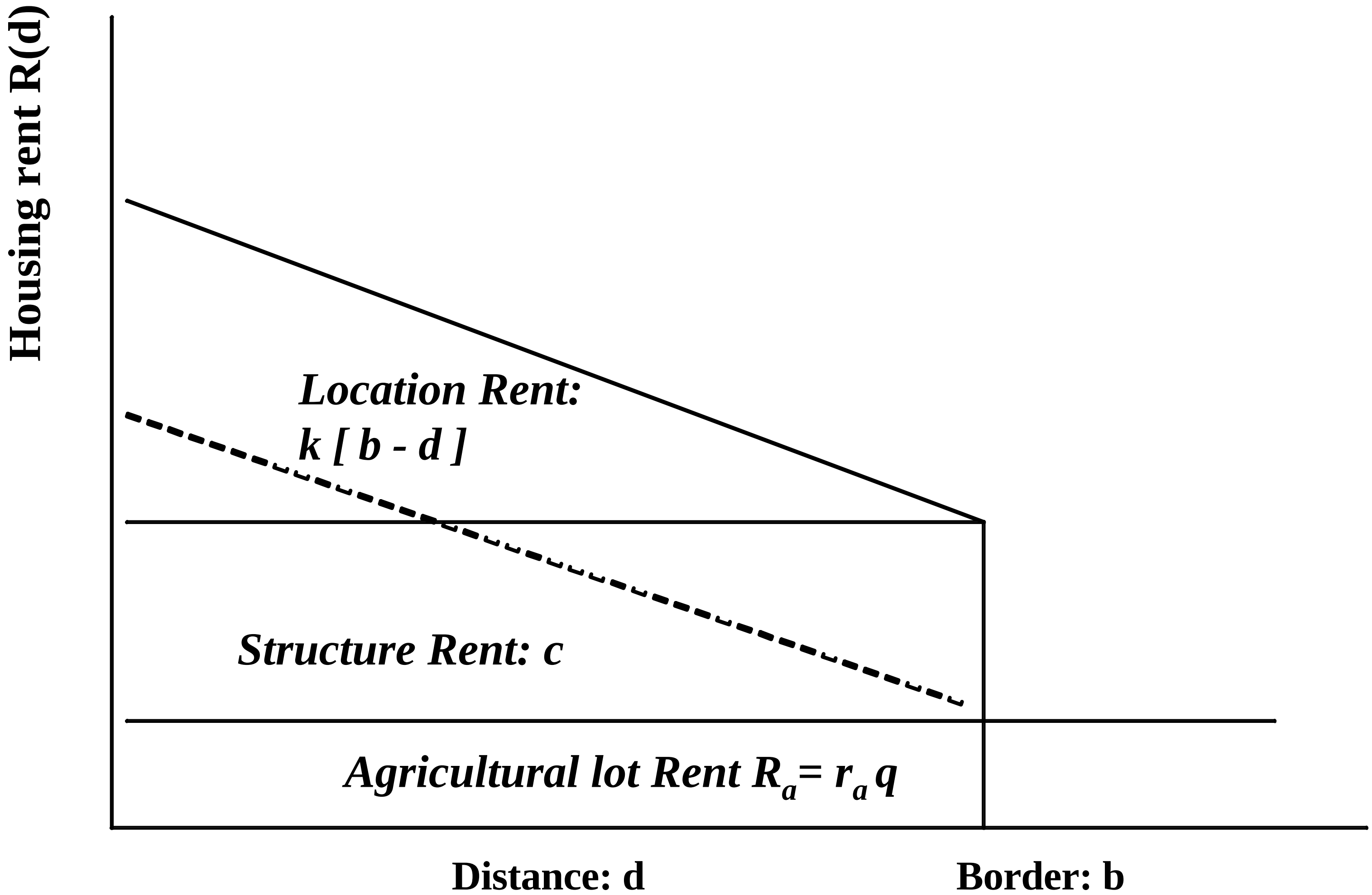
Housing rent gradient

$R(d) = r_a q + c + k(b - d)$

$R(d) - R(b) = k(b - d)$

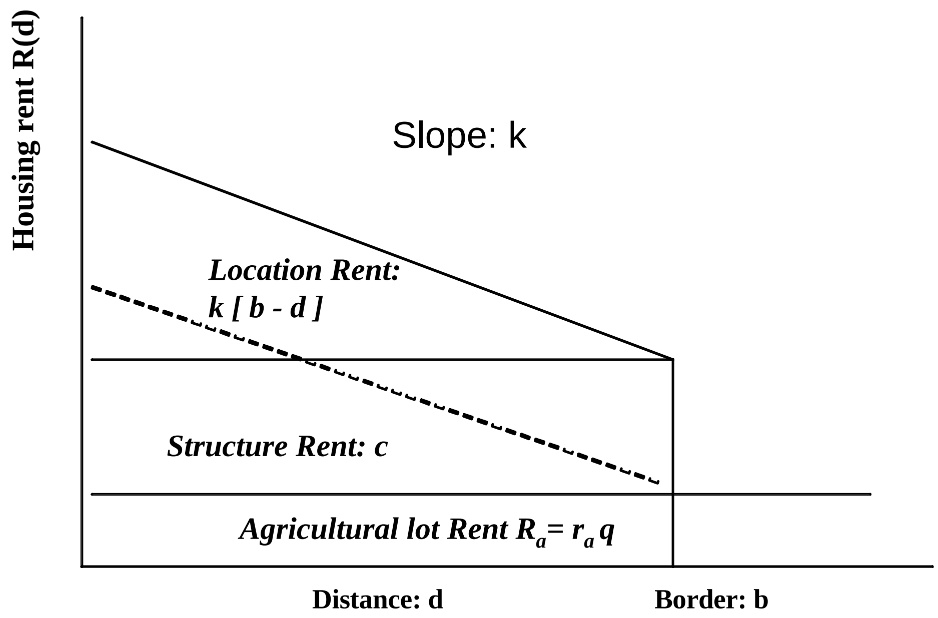
r_a : agriculture cost *per acre*
c: annualized construction cost *per house unit*
q: amount of acre *per house unit*
1/q: unit per acre (density)
d = location
b = boundary, farthest locatin
0 = best location, center
k = annual transport cost per mile (inclusive of all: money, time)

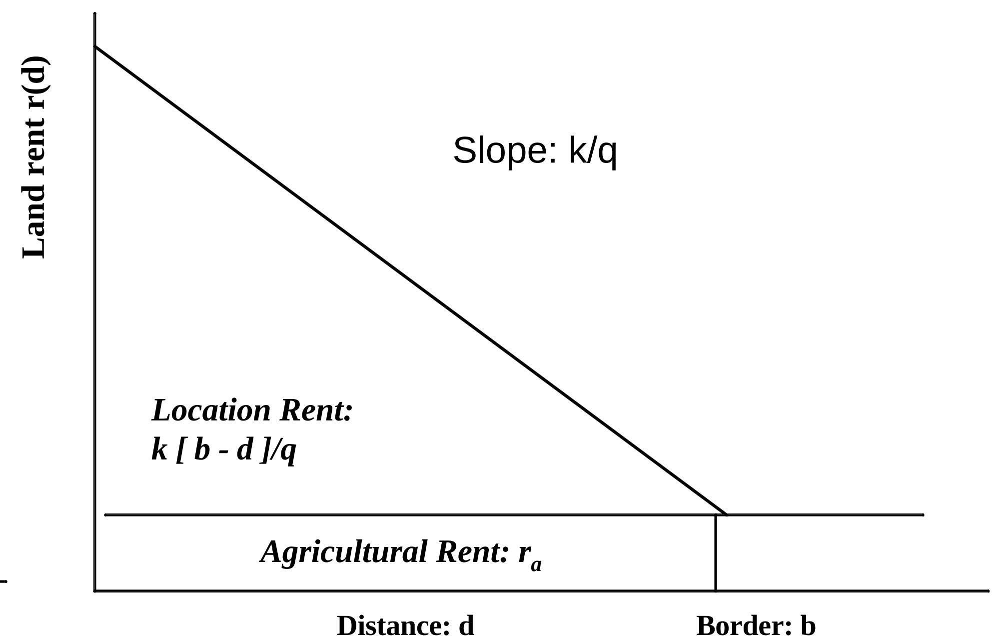
Component of Housing Rent



House rent → Land rent

- House rent (per unit): $$R(d) = r_a * q + c + k(b - d)$$
- Land rent (per acre): a residual $$r(d) = r_a + k(b - d)/q$$





Land Supply

- Land demand = function of population
- In equilibrium: Land supply = Land Demand

$$N * q = \pi * b^2 * V$$

$$b = \sqrt{\frac{N * q}{\pi V}}$$

V: water, wetland, **planning control**

Comparative statistics: effects on house rent and land rent by changing r, k, b, q, v

$$\begin{cases} R(d) = r_a * q + c + k(b-d) \\ r(d) = r_a + k(b-d)/q \\ N * q = \pi * b^2 * V \end{cases}$$

- r_a
- k: oil price changes; transport infrastructure changes
- b
- V
- N → Higher R (for everywhere except the border)
- q given the same population
  - $R(d) = r_a q + c + k(b - d) = r_a q + c + k((nq/\pi v)^{1/2} - d)$
  - $r(d) = r_a + k/q\ (b - d) = r_a + k((nq/\pi v)^{1/2} - d)\ /q$:

    - With higher density (small q),
    - Trade off between smaller city but steeper slope
    - Result: higher r in the center (graphic intuition and algebra)

R or r ? , which location, what time

House: Rent → Price (Spatial Capitalization of Ricardian Rent with growth)

Population growth at rate 2g: $N_t = N_0 * e^{2gt}$

Boundary [b] growth rate of g: $b_t = b_0 * e^{gt}$

Ricardian Rent for existing structures located at (d) in time t:

House rent:

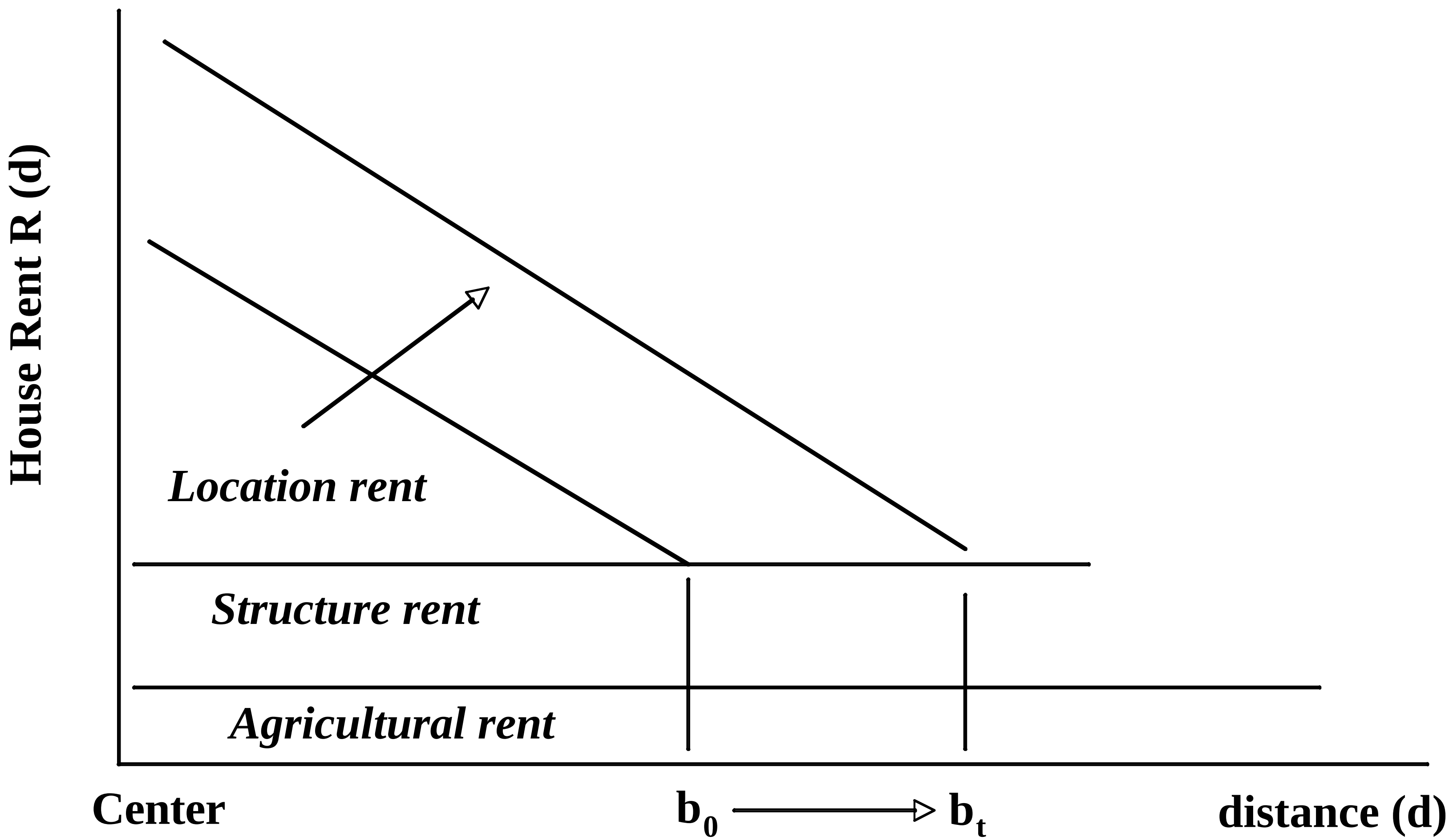
$$R_t(d) = r_a * q + c + k(b_t - d)$$

House price: PDV of future Rent

$$P_t(d) = \underbrace{\frac{r_a q}{i}}_{\text{Agriculture value}} + \underbrace{\frac{c}{i}}_{\text{Structure value}} + \underbrace{\frac{k(b_t - d)}{i}}_{\text{Current location value}} + \underbrace{\frac{k b_t g}{i(i-g)}}_{\text{Future location value}}$$

What if g=0?
What if i=g or i<g?

Expansion of Housing Rent as the city grows in population and the border moves from b0 to bt



Spatial multipliers or capitalization rate

Price / rent ratio: inverse of capitalization rate

$$P_t(d) = \frac{r_a q}{i} + \frac{c}{i} + \frac{k(b_t - d)}{i} + \frac{k b_t g}{i(i-g)}$$

$$R_t(d) = r_a * q + c + k(b_t - d)$$

$$P_t(d) / R_t(d) = 1/i + \frac{k b_t g}{i(i-g) R_t(d)}$$

w.r.t: g =0; g >0; g<0; g increase; g decrease
w.r.t. d? d=0; d=b

Edge of the city: highest return, highest risk

Land Rent → Price

- Land rent

$$r_t(d) = r_a + k(b_t - d)/q$$

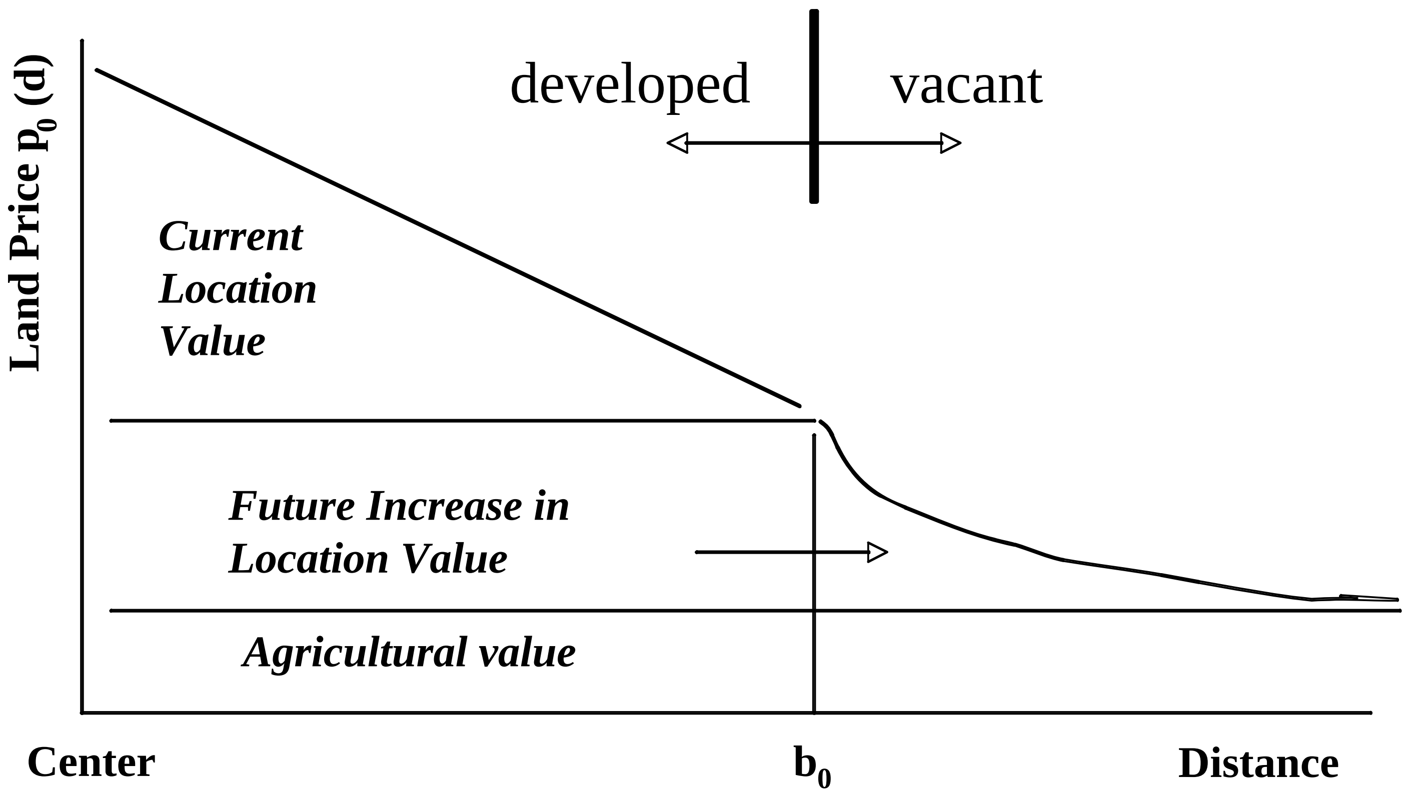
- Land price

$$p_t(d) = \frac{r_a}{i} + \frac{k(b_t - d)}{iq} + \frac{kb_t g}{i(i-g)q}$$

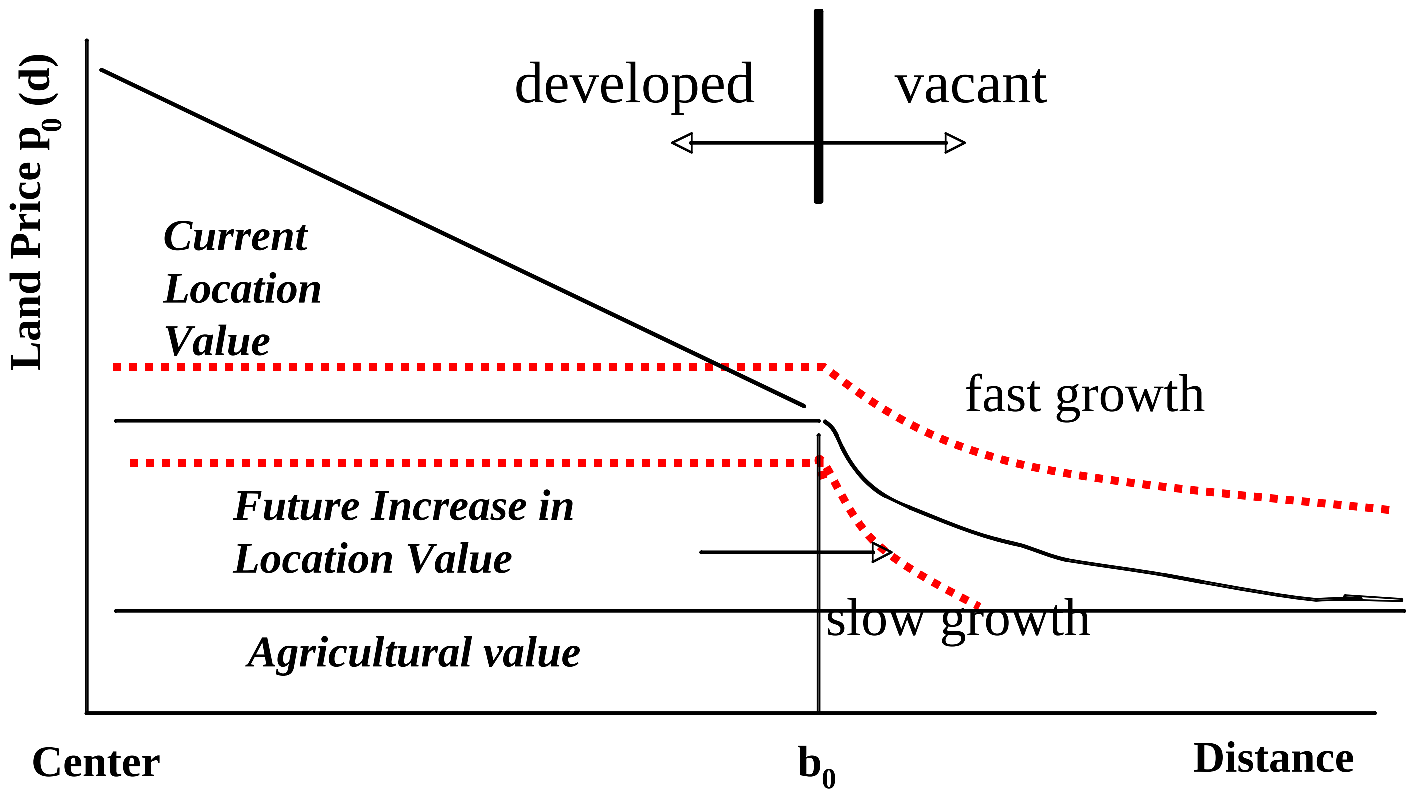
Vacant land price

- The price of land beyond the current border?
  - In t years $b_t = b_0 e^{gT}$
  - Land at distance d> b0 will be developed in $T = \log(\frac{d}{b_0})/g$
  - the value of land at d $$p_0(d) = \frac{r_a}{i} + e^{-iT} \frac{kb_t g}{i(i-g)q}$$
- When g=0 → $p_0(d) = \frac{r_a}{i}$
- Where are the land prices most volatile as g fluctuates?

Components of land prices: agriculture, Current location value, future increase in location value



Land price: with different growth speed



Summary: Price vs. Rent; Land vs. House

	House	Land
Rent	$R(d) = r_a * q + c + k(b-d)$ $\xrightarrow{1/q}$	$r(d) = r_a + k(b-d)/q$
	$\downarrow i$	$\downarrow i$
Price	$P_t(d) = \frac{r_a q}{i} + \frac{c}{i} + \frac{k(b_t - d)}{i} + \frac{k b_t g}{i(i-g)}$	$p_t(d) = \frac{r_a}{i} + \frac{k(b_t - d)}{iq} + \frac{k b_t g}{i(i-g)q}$

Discussion:

1. **Impact of growth: on each of the four**
2. **Slope: (w.r.t: d)**
3. **Compensation differential**
4. **Special case: d=b**
5. **Special case: d=0**
6. **Special case: d>b**
7. **Special case: g=0**

Variation of real estate prices

Question:

Two currently identical cities with A expected to grow 10% a year; B 5% a year

Rent A vs. Rent B?

Price A vs. Price B?

Price accounts for future; Rent does not!

- Much of the variation of real estate prices over time is due to changes in the expected future growth of rental income, rather than to changes in the actual level of current rents.
- Faster growing cities that are otherwise identical to slower growing cities (in population, income, and density) should have similar land and housing rents, but higher land and housing prices.

A full model: simple and powerful but with limitation

$$\begin{cases} R(d) = r_a * q + c + k(b - d) \\ N * q = \pi * b^2 * V \end{cases}$$

Structure

Boundary condition

Static model in equilibrium

Regression: housing price variation among cities (p56)

- A simple model to explain the housing price variation among cities

- Three key factors:
  - Size of the city
  - Growth of the city
  - Construction cost

- Data:
  - 1990, CMSAs in the US

- Variables:
  - Price: median house price in 1990 (PRICE)
  - Size of the city: # of households (HH)
  - Growth of the city: % difference between 1980 and 1990 households (HHGRO)
  - Construction cost: 1990 Construction Cost Index (COST)

- Model:

$$PRICE = \alpha + \beta_1 * HH + \beta_2 * HHGRO + \beta_3 * COST + \varepsilon$$

- Expected results:
  - Size of the city
  - Growth of the city
  - Construction cost

Regression: Housing price variation among cities

- Results:

$$PRICE = -298{,}138 + 0.019 * HH + 152{,}156 * HHGRO + 1622 * COST$$

$$\quad (10.0) \quad (2.4) \quad (2.3) \quad (4.2)$$

R-square=0.76

- Interpretation
  - Betas
    - Constant
    - Size of the city
    - Growth of the city
    - Construction cost
  - t-statistics
  - R2

- Notes:
  - Different scale of the variables → different scale of the betas
  - 3 variables but quite a powerful explanations
  - CMSA as the unit
  - HHGRO as the growth rate proxy

Limitation: assumptions in a stylized city

- Monocentric: one center: all opportunities are in the center
- Location defined by transportation
  - transport cost is the only differential
  - transport infrastructure is evenly distributed and ubiquitous
  - d (distance from the center fully describes location)
  - k per mile cost (linear fashion)
- q is fixed: 1/q density; (no substitution between structure capital and land)
- households identical: same income, same preference
- houses identical: no physical differences except location
- Housing goes to the highest rent → in equilibrium detla rent = delta travel cost → no incentive to move

Rationale (in equilibrium) tradeoff between transport cost vs. housing cost

Expansion of the Ricardian model : relaxation of the assumptions

- Identical households → different household segments
- Identical houses → different density
- Identical houses → different characteristics
- Mono-center → multi-center

Spatial separation: relaxing the assumption of identical households

Short run: compete over house rent

$R_1(d) = R(b) + k_1(b - d)$

$R_2(d) = R(b) + k_2(b - d)$,

slope: $k_1 > k_2$

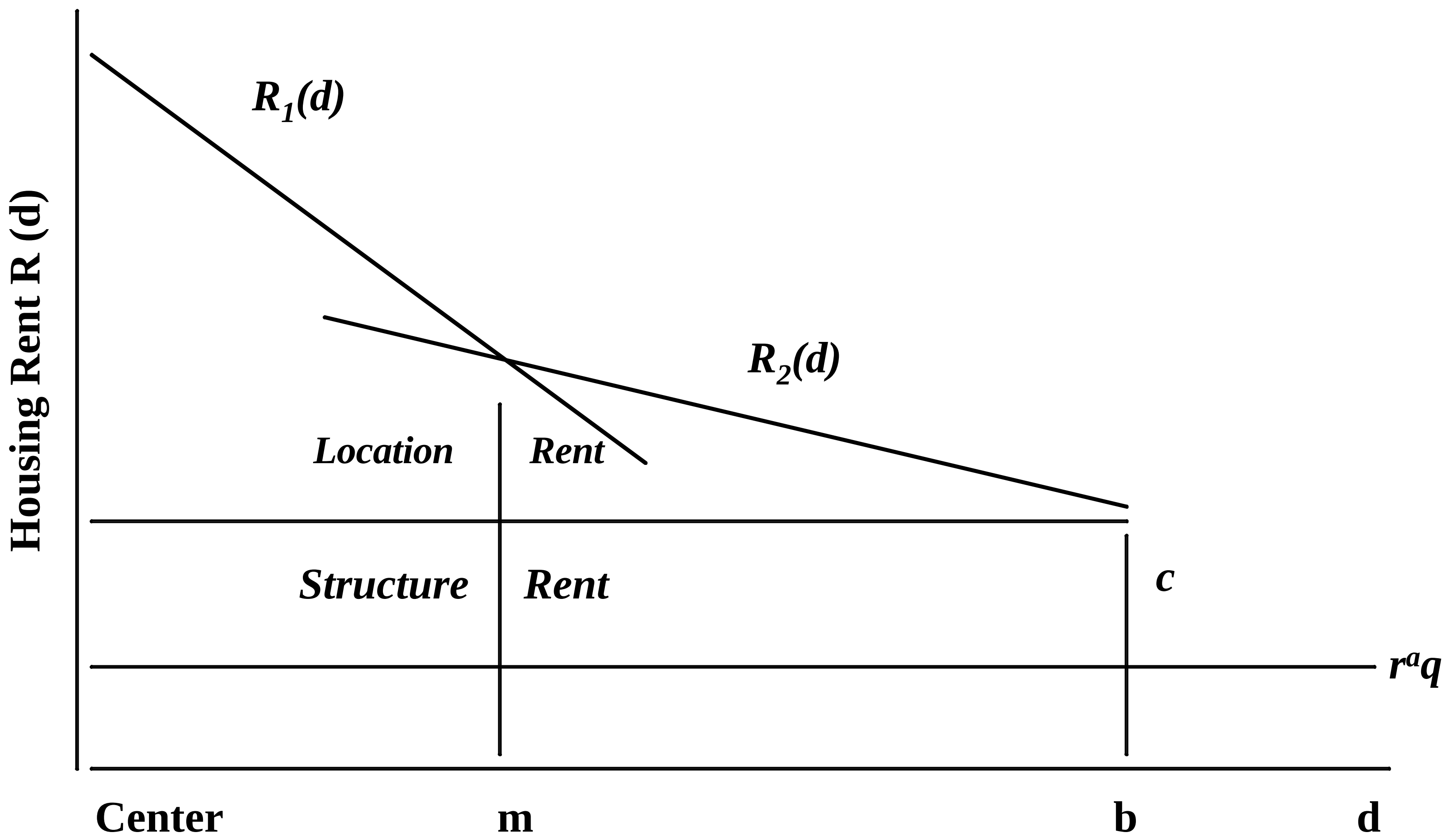
Long run: compete over land rent

$r_1(d) = r_a + k_1[b – d]/q_1$

$r_2(d) = r_a + k_2[b – d]/q_2$

slope: k_1/q_1 vs. k_2/q_2

Housing Rents and Land Use Competition with 2 Household types



Income effect

Two factors here:

- cost of commuting, k
- Land consumption, q

$k_1 > k_2$

$q_1 > q_2$ then what?

Income elasticity of land demand: elastic or inelastic

Income elasticity of commuting costs: elastic or inelastic

Which effect dominates the other? – Paris vs. U.S.

Takeaway

- Rent vs. price; land vs. housing
  - Rents vary by location within cities to offset the location value
  - Population growth expands the city horizontally and increase rent at all locations
  - Price accounts for future; Rent does not!
- Locations are occupied by the best user (highest rent payer)
- Segregation
  - Income growth effect depends upon the elasticity on land demand vs. commuting costs
  - Segregation could be a natural result of market competition ; not necessarily discrimination
- Ricardian theory:
  - simple but powerful model
  - theory development: starting from restrictive assumptions and gradually relaxing them