

Chapter 10:

BASIC MICRO-LEVEL VALUATION: "DCF" & "NPV"

THE *famous* DCF VALUATION PROCEDURE...

1. **FORECAST THE EXPECTED FUTURE CASH FLOWS;**
2. **ASCERTAIN THE REQUIRED TOTAL RETURN;**
3. **DISCOUNT THE CASH FLOWS TO PRESENT VALUE AT THE REQUIRED RATE OF RETURN.**

THE VALUE YOU GET TELLS YOU WHAT YOU MUST PAY SO THAT YOUR EXPECTED RETURN WILL EQUAL THE "REQUIRED RETURN" AT WHICH YOU DISCOUNTED THE EXPECTED CASH FLOWS.

$$V_0 = \frac{E_0[CF_1]}{1 + E_0[r]} + \frac{E_0[CF_2]}{(1 + E_0[r])^2} + \dots + \frac{E_0[CF_{T-1}]}{(1 + E_0[r])^{T-1}} + \frac{E_0[CF_T]}{(1 + E_0[r])^T}$$

where:

CF_t = Net cash flow generated by the property in period “t”;

V_t = Property value at the end of period “t”;

E₀[r] = Expected average multi-period return (per period) as of time “zero” (the present), also known as the “going-in IRR”;

T = The terminal period in the expected investment holding period, such that CF_T would include the re-sale value of the property at that time (V_T), in addition to normal operating cash flow.

Numerical example...

	Lease:
<i>Year:</i>	<i>CF:</i>
2001	\$1,000,000
2002	\$1,000,000
2003	\$1,000,000
2004	\$1,500,000
2005	\$1,500,000
2006	\$1,500,000

- *Single-tenant office bldg*
- *6-year “net” lease with a “step-up”...*
- *Expected sale price year 6 = \$15,000,000*
- *Required rate of return (“going-in IRR”) = 10%...*
- *DCF valuation of property is \$15,098,000:*

$$15,098,000 = \frac{1,000,000}{(1.08)} + \frac{1,000,000}{(1.08)^2} + \frac{1,000,000}{(1.08)^3} + \frac{1,500,000}{(1.08)^4} + \frac{1,500,000}{(1.08)^5} + \frac{16,500,000}{(1.08)^6}$$

Why is the DCF procedure *important*?

1. Recognizes asset valuation *fundamentally depends* upon future net *cash flow generation potential* of the asset.
2. Takes *long-term* perspective appropriate for investment decision-making in illiquid markets (multi-period, typically 10 yrs in R.E. applications).
3. Takes the *total return* perspective necessary for successful investment.
4. Due to the above, the *exercise* of going through the DCF procedure, *if taken seriously*, can help to protect the investor from being swept up by an asset market “*bubble*” (either a positive or negative bubble – when asset prices are not related to cash flow generation potential).

Remember:

Investment returns are *inversely related* to the price paid going in for the asset.

e.g., in the previous example, if we could get the asset for \$14,000,000 (instead of \$15,098,000), then our going-in return would be 9.6% (instead of 8%):

$$14,000,000 = \frac{1,000,000}{(1.0962)} + \frac{1,000,000}{(1.0962)^2} + \frac{1,000,000}{(1.0962)^3} + \frac{1,500,000}{(1.0962)^4} + \frac{1,500,000}{(1.0962)^5} + \frac{16,500,000}{(1.0962)^6}$$

vs.

$$15,098,000 = \frac{1,000,000}{(1.08)} + \frac{1,000,000}{(1.08)^2} + \frac{1,000,000}{(1.08)^3} + \frac{1,500,000}{(1.08)^4} + \frac{1,500,000}{(1.08)^5} + \frac{16,500,000}{(1.08)^6}$$

What is the fundamental economic reason for this inverse relationship?

[Hint: What determines fut. CFs?]

Match the discount rate to the risk. . .

$$\mathbf{r = r_f + RP}$$

Disc.Rate = Riskfree Rate + Risk Premium

(Riskfree Rate = US T-Bill Yield.)

10.2.1 Match the Discount Rate to the Risk: Intralease & Interlease Discount Rates

Hypothetical office building net cash flows:										
Year	1	2	3	4	5	6	7	8	9	10
CF _t	\$1	\$1	\$1	\$1.5	\$1.5	\$1.5	\$2	\$2	\$2	\$22

Projected operating CFs will be contractual (covered by leases). 1st 6 yrs in current lease, remainder in a subsequent lease. Prior to signing, lease CFs are more risky (interlease disc rate), once signed, less risky (intralease disc rate). DCF Valuation:

$$\$18,325,000 = \sum_{t=1}^3 \frac{\$1}{(1.07)^t} + \sum_{t=4}^6 \frac{\$1.5}{(1.07)^t} + \left(\frac{1}{(1.09)^6} \right) \left(\sum_{t=1}^4 \frac{\$2}{(1.07)^t} \right) + \frac{\$20}{(1.09)^{10}}$$

Here we have estimated the discount rate at 7% for the relatively low-risk lease CFs (e.g., if T-Bond Yld = 5%, then RP=2%), and at 9% for the relatively high-risk later CFs (→ 4% risk premium).

→ Implied property value = \$18,325,000.

10.2.2 Blended IRR: A single Discount Rate

Current practice usually is not this sophisticated for typical properties. If the lease expiration pattern is typical, a single “*blended*” discount rate is typically used.

Thus, for this building, we would typically observe a “going-in IRR” of 8.57%, applied to *all* the expected future CFs...

$$\$18,325,000 = \sum_{t=1}^3 \frac{\$1}{(1.0857)^t} + \sum_{t=4}^6 \frac{\$1.5}{(1.0857)^t} + \sum_{t=7}^{10} \frac{\$2}{(1.0857)^t} + \frac{\$20}{(1.0857)^{10}}$$

In principle, this can allow “arbitrage” opportunities if investors are not careful (especially as the ABS mkt develops...)

Example:

Suppose property with 7 yrs (instead of 6 yrs) remaining on vintage lease but same expected CFs as before.

Purchase for \$18,325,000, then sell lease for \$7,083,000 and property residual for \$11,319,000, for a profit of \$77,000:

$$\$7,083,000 = \sum_{t=1}^3 \frac{\$1}{(1.07)^t} + \sum_{t=4}^6 \frac{\$1.5}{(1.07)^t} + \frac{\$2}{(1.07)^7}$$

$$\$11,319,000 = \left(\frac{1}{(1.09)^7} \right) \left(\sum_{t=1}^3 \frac{\$2}{(1.07)^t} \right) + \frac{\$20}{(1.09)^{10}}$$

1st 6 CFs in Lease → \$18.325M Value

1st 7 CFs in Lease → \$18.402M Value

Hypothetical office building net cash flows:										
Year	1	2	3	4	5	6	7	8	9	10
CF _t	\$1	\$1	\$1	\$1.5	\$1.5	\$1.5	\$2	\$2	\$2	\$2

10.3 Ratio Valuation Procedures

Valuation shortcuts: “Ratio valuation”...

1) DIRECT CAPITALIZATION:

**A WIDELY-USED SHORTCUT VALUATION
PROCEDURE:**

- **SKIP THE MULTI-YEAR CF FORECAST**
- **DIVIDE CURRENT (UPCOMING YEAR) NET OPERATING INCOME (NOI) BY CURRENT MARKET CAP RATE (YIELD, NOT THE TOTAL RETURN USED IN DCF)**

The idea behind direct capitalization...

IF "CAP RATE" = NOI / V ,

THEN:

V = NOI / CAP RATE

(FORMALLY, NOT CAUSALLY)

**MOST APPROPRIATE FOR BLDGS W SHORT-TERM
LEASES IN LESS CYCLICAL MARKETS, LIKE
APARTMENTS.**

EXAMPLE:

250 UNIT APARTMENT COMPLEX

AVG RENT = \$15,000/unit/yr

5% VACANCY

ANNUAL OPER. EXPENSES = \$6000 / unit

8.82% CAP RATE (KORPACZ SURVEY)

VALUATION BY DIRECT CAPITALIZATION:

POTENTIAL GROSS INCOME (PGI)	= 250*15000	= \$3,750,000
- VACANCY ALLOWANCE (5%)	= 0.5*3750000	= 187,500
- OPERATING EXPENSES	= 250*6000	= 1,500,000
-----		-----
NET OPER. INCOME (NOI)		= \$2,062,500

$$V = 2,062,500 / 0.0882 = \$23,384,354, \text{ say approx. } \$23,400,000$$

2) *GROSS INCOME MULTIPLIER (GIM):*

$$\text{GIM} = \text{V} / \text{GROSS REVENUE}$$

COMMONLY USED FOR SMALL APARTMENTS.

(OWNER'S MAY NOT RELIABLY REVEAL GOOD EXPENSE RECORDS, SO YOU CAN'T COMPUTE NOI (= Rev - Expense), BUT RENTS CAN BE OBSERVED INDEPENDENTLY IN THE RENTAL MARKET.)

IN PREVIOUS APT EXAMPLE THE GIM IS:

$$23,400,000 / 3,750,000 = 6.2.$$

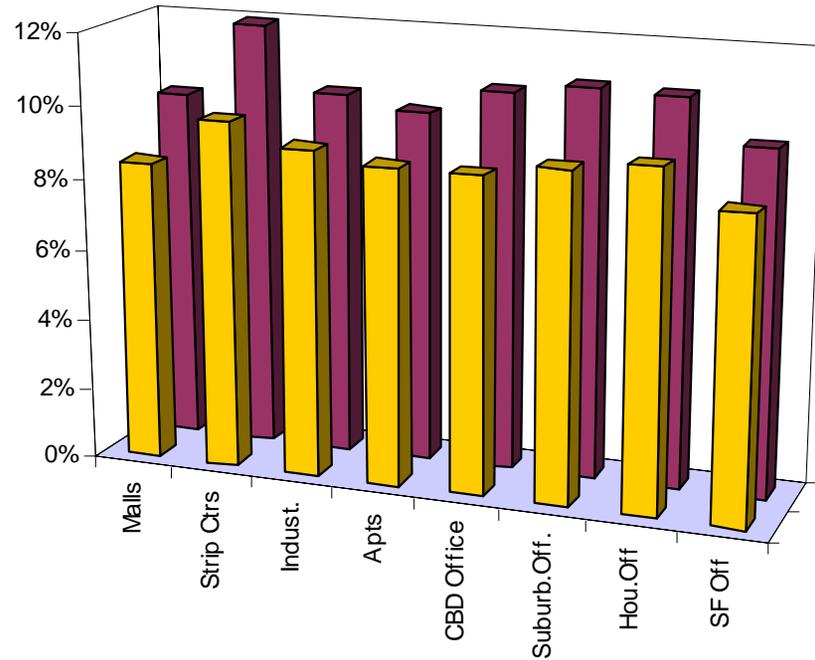
Empirical cap rates and market values. . .

Cap rates are a way of quoting observed market prices for property assets (like bond “yields” are the way bond prices are reported).

E.g., Korpacz Survey



Exh.11-6b: Investor Cap Rate Expectations for Various Property Types*



*Source: Korpacz Investor Survey, 1st quarter 1999

	Malls	Strip Ctrs	Indust.	Apts	CBD Office	Suburb. Off.	Hou. Off	SF Off
■ Institutional	8.41%	9.76%	9.14%	8.83%	8.82%	9.17%	9.43%	8.42%
■ Non-institutional	9.88%	11.97%	10.21%	9.83%	10.56%	10.83%	10.75%	9.58%

DANGERS

IN MKT-BASED RATIO VALUATION...

- 1)** DIRECT CAPITALIZATION CAN BE MISLEADING FOR MARKET VALUE IF PROPERTY DOES NOT HAVE CASH FLOW GROWTH AND RISK PATTERN TYPICAL OF OTHER PROPERTIES FROM WHICH CAP RATE WAS OBTAINED. (WITH GIM IT'S EVEN MORE DANGEROUS: OPERATING EXPENSES MUST ALSO BE TYPICAL.)
- 2)** *Market-based ratio valuation won't protect you from "bubbles"!*

10.4 Typical mistakes in DCF application to commercial property...

CAVEAT!

BEWARE OF “**G.I.G.O.**”

====> Forecasted Cash Flows:

Must Be REALISTIC Expectations
(Neither Optimistic, Nor Pessimistic)

====> Discount Rate should be OCC:

Based on Ex Ante Total Returns in Capital Market
(Including REALISTIC Property Market Expectations)

- *Read the “fine print”.*
- *Look for “hidden assumptions”.*
- *Check realism of assumptions.*

Three most common **mistakes** in R.E. DCF practice:

- 1. Rent & income growth assumption is *too high*—**
aka: “*We all know rents grow with inflation, don’t we!*”?...
Remember: Properties tend to *depreciate* over time in real terms (net of inflation). → Usually, rents & income within a given building do not keep pace with inflation, long run.
- 2. Capital improvement expenditure projection, &/or terminal cap rate projection, are *too low* –**
Remember: Capital improvement expenditures typically average at least *10%-20%* of the NOI (1%-2% of the property value) over the long run.
Going-out cap rate is typically *at least as high* as the going-in cap rate (older properties are more risky and have less growth potential).
- 3. Discount rate (expected return) is *too high* –**
This third mistake may *offset* the first two, resulting in a realistic estimate of property current value, thereby *hiding* all three mistakes!

Numerical example:

Two cash flow streams . . .

Year:									
1	2	3	4	5	6	7	8	9	10
\$1,000,000	\$1,050,000	\$1,102,500	\$1,157,625	\$1,215,506	\$1,276,282	\$1,340,096	\$1,407,100	\$1,477,455	\$17,840,274
\$1,000,000	\$1,010,000	\$1,020,100	\$1,030,301	\$1,040,604	\$1,051,010	\$1,061,520	\$1,072,135	\$1,082,857	\$12,139,907

- First has 5%/yr growth,
- Second has 1%/yr growth.
- Both have same initial cash flow level (\$1,000,000).
- Both have PV = \$10,000,000: First discounted @ 15%, Second discounted @ 11%.

As both streams have same starting value & same PV, both may appear consistent with observable current information in the space and property markets. (*e.g., rents are typically \$1,000,000, and property values are typically \$10,000,000 for properties like this.*)

Suppose *realistic* growth rate is 1%, not 5%. Then the first CF projection gives investors an *unrealistic* return expectation of *15%*.

- ➔ “**Unfair**” comparisons (e.g., bond returns cannot be “fudged” like this).
- ➔ Investor is “**set up**” to be disappointed in long run.

Results of these types of mistakes:

Unrealistic expectations

Long-run undermining credibility of the DCF valuation procedure

Wasted time (why spend time on the exercise if you're not going to try to make it realistic?...)

10.6

*DCF and Investment Decision Rules: the **NPV** Rule...*

DCF → Property value (“V”) . . .

But how do we know whether an investment is a “*good deal*” or not?...

How should we decide whether or not to make a given investment decision?

NPV = PV(Benefit) – PV(Cost)

i.e.: NPV = Value of what you get – Value of what you give up to get it,

All measured in equivalent “apples-to-apples” dollars, because we have discounted all the values to present using discount rates reflecting risk.

“THE NPV INVESTMENT DECISION RULE”:

**1) MAXIMIZE THE NPV ACROSS ALL
MUTUALLY-EXCLUSIVE ALTERNATIVES;
AND**

**2) NEVER CHOOSE AN ALTERNATIVE THAT
HAS: $NPV < 0$.**

The NPV Investment Decision Rule

IF BUYING: $NPV = V - P$

IF SELLING: $NPV = P - V$

Where:

V = Value of property at time-zero (e.g., based on DCF)

P = Selling price of property (in time-zero equivalent \$)

Example:

DCF → V = \$13,000,000

You can buy @ P = \$10,000,000.

NPV = V - P = \$13M - \$10M = +\$3M.

Note: NPV Rule is based directly on the “Wealth Maximization Principle**” . . .**

WEALTH MAXIMIZATION \Rightarrow The NPV Rule

Maximize the current value of the investor's net wealth. Otherwise, you're “leaving money on the table”.

NPV Rule Corollary:

"Zero NPV deals are OK!"

Why? . . .

Zero NPV deals are not zero profit.

(They only lack “super-normal” profit.)

A zero NPV deal is only “bad” if it prevents the investor from undertaking a positive NPV deal.

In fact, on the basis of “market value” (MV)...

Based on “market value” (MV),

$$\text{NPV}(\text{Buyer}) = \quad \quad \quad \mathbf{V-P = MV-P}$$

$$\text{NPV}(\text{Seller}) = \mathbf{P-V = P-MV = -NPV(\text{Buyer})}$$

Therefore, if both the buyer and seller apply the NPV Rule ($\text{NPV} \geq 0$), then:

$$\text{(i) } \text{NPV}(\text{Buyer}) \geq 0 \Rightarrow -\text{NPV}(\text{Seller}) \geq 0 \Rightarrow \text{NPV}(\text{Seller}) \leq 0;$$

$$\text{(ii) } \text{NPV}(\text{Seller}) \geq 0 \Rightarrow -\text{NPV}(\text{Buyer}) \geq 0 \Rightarrow \text{NPV}(\text{Buyer}) \leq 0;$$

$$\text{(i)\&(ii) together} \Rightarrow \text{NPV}(\text{Buyer}) = \text{NPV}(\text{Seller}) = 0.$$

Thus, measured on the basis of MV, we actually *expect* that:

$$\mathbf{NPV = 0.}$$

Sources of “*illusions*” of big positive NPVs

- 1) OCC (discount rate) is *not* the cost of borrowed funds (e.g., mortgage interest rate).
- 2) Land value? (not just historical cost)
- 3) Search & Management Costs?
- 4) “Private Info”? (But MV is based on public info.)

However, in Real Estate it is possible to **occasionally** find deals with substantially positive, or negative, NPV, even based on MV.

Real estate asset markets not informationally efficient:

- People make “pricing mistakes” (they can’t observe MV for sure for a given property)
- Your own research may uncover “news” relevant to value (just before the market knows it)

*What about **unique** circumstances or abilities? . . .*

Generally, real uniqueness does not affect MV.

Precisely because you are unique, you can't expect someone else to be willing to pay what you could, or be willing to sell for what you would. (May affect "investment value" – IV, not MV.)

10.6.2 Choosing Among Alternative Zero-NPV Investments

- Alternatives may have different NPVs based on “investment value”, even though they all have equal (zero) NPV based on “market value”. (See Sect. 12.1.)
- One alternative may be preferable for macro or strategic reasons (portfolio target, administrative efficiency, property size preference, etc.).
- Alternatives may present different expected return “*attributes*” (initial yield, cash flow change, yield change). (See Appendix 10B & Sect. 26.1.1.)

10.6.3. Hurdle Rate Version of the Investment Rule:

IRR vs. NPV

SOMETIMES IT IS USEFUL (anyway, it is very common in the real world) TO "INVERT" THE DCF PROCEDURE...

INSTEAD OF CALCULATING THE VALUE ASSOCIATED WITH A GIVEN EXPECTED RETURN,
CALCULATE THE EXPECTED RETURN (IRR) ASSOCIATED WITH A GIVEN PRICE FOR THE PROPERTY.

I.E., WHAT DISCOUNT RATE WILL CAUSE THE EXPECTED FUTURE CASH FLOWS TO BE WORTH THE GIVEN PRICE?...

THEN THE DECISION RULE IS:

1) MAXIMIZE DIFFERENCE BETWEEN:

IRR AND REQUIRED RETURN (ACROSS MUT.EXCLU
ALTS)

2) NEVER DO A DEAL WITH:

IRR < REQ'D RETURN

$$\text{REQ'D RETURN} = \text{"HURDLE RATE"} = r_f + RP$$

*When using the IRR (hurdle) version of the basic investment decision rule:
Watch out for **mutually exclusive alternatives of different scales.***

e.g., \$15M project @ 15% is better than \$5M project @ 20% if cost of capital (hurdle) in both is 10%.

Appendix 10A: A Method to Estimate Interlease Discount Rates

Suppose in a certain property market the typical:

- Lease term is 5 years;
- Cap rate (cash yield) is 7%;
- Long term property value & rental growth rate is 1%/yr (typically equals inflation minus real depreciation rate);
- Leases provide rent step-ups of 1%/yr (per above);
- Tenant borrowing rate (intralease disc rate) is 6%...

Then (assuming pmts in arrears), PV of a lease, per dollar of initial net rent, is:

$$\frac{\$1}{1.06} + \frac{(1.01)\$1}{1.06^2} + \dots + \frac{(1.01)^4 \$1}{1.06^5} = \frac{(\$1/1.06)(1 - \frac{1.01}{1.06})^5}{1 - (\frac{1.01}{1.06})} = \$4.29$$

A stylized space in this market has PV equal to S , as follows (ignoring vacancy between leases), assuming *interlease* discount rate is r :

$$S = \$4.29 + \left(\frac{1.01}{1+r}\right)^5 \$4.29 + \left(\frac{1.01}{1+r}\right)^{10} \$4.29 + \dots$$

This is a constant-growth perpetuity, so (using geometric series formula from Chapter 8) we can shortcut this as:

$$S = \frac{\$4.29}{1 - \left(\frac{1.01}{1+r}\right)^5}$$

From the market's prevailing cap rate we also know that:

$$S = \frac{\$1}{.07}$$

Putting these two together, we have:

$$\frac{\$1}{.07} = \frac{\$4.29}{1 - \left(\frac{1.01}{1+r}\right)^5}$$

\Rightarrow

$$r = 1.01 / \left(1 - .07(\$4.29)\right)^{1/5} - 1 = 8.48\%$$

The implied interlease discount rate is 8.48%.

This is almost 250 bps above the intralease rate of 6%.

But it is only about 50 bps above the blended going-in IRR of 8% (determined based on the same constant-growth perpetuity model, as the cash yield plus the growth rate: 7% + 1% = 8%).

Appendix 10B (& Ch.26 Sect. 26.1.1)

***Property-Level
Investment Performance
Attribution***

Real Estate Investment

“Performance Attribution”

DEFINITION:

The *decomposition* (or “breaking down”, or “parsing”) of the total investment return of a subject property or portfolio of properties (or an investment manager).

PURPOSE:

To assist with the *diagnosis* and understanding of what caused the given investment performance.

USAGE:

By investment managers (agents) and their investor clients (principals).

Two levels

at which performance attribution is performed:

- **Property level**
Pertains to individual properties or static portfolios of multiple properties.
- **Portfolio level**
Pertains to dynamic portfolios or investment manager (or fund) level.

Major *attributes* (return components):

At the **PROPERTY LEVEL:**

Initial Cash Yield

Cash Flow Change

Yield Change

At the **PORTFOLIO LEVEL:**

Allocation

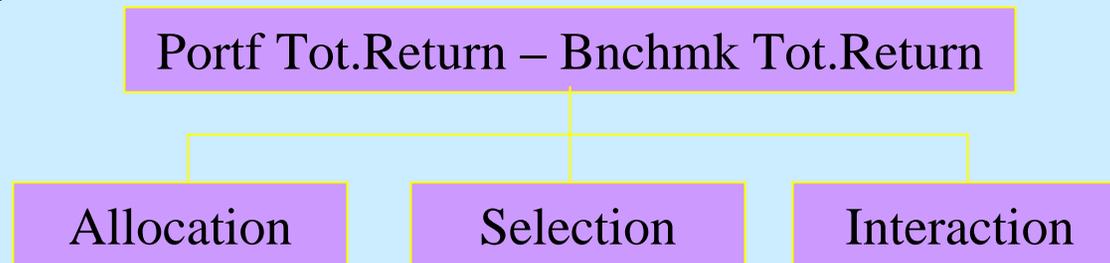
Selection

Interaction

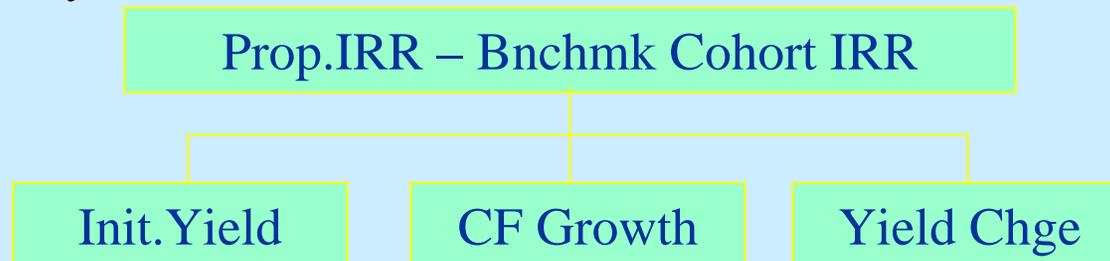
“Performance Attribution”:

- Often useful for *diagnostic* purposes to compare *subject* portfolio or mgr with an appropriate *benchmark* . . .

Portfolio Level:



Property Level:



Property-Level Performance Attribution . . .

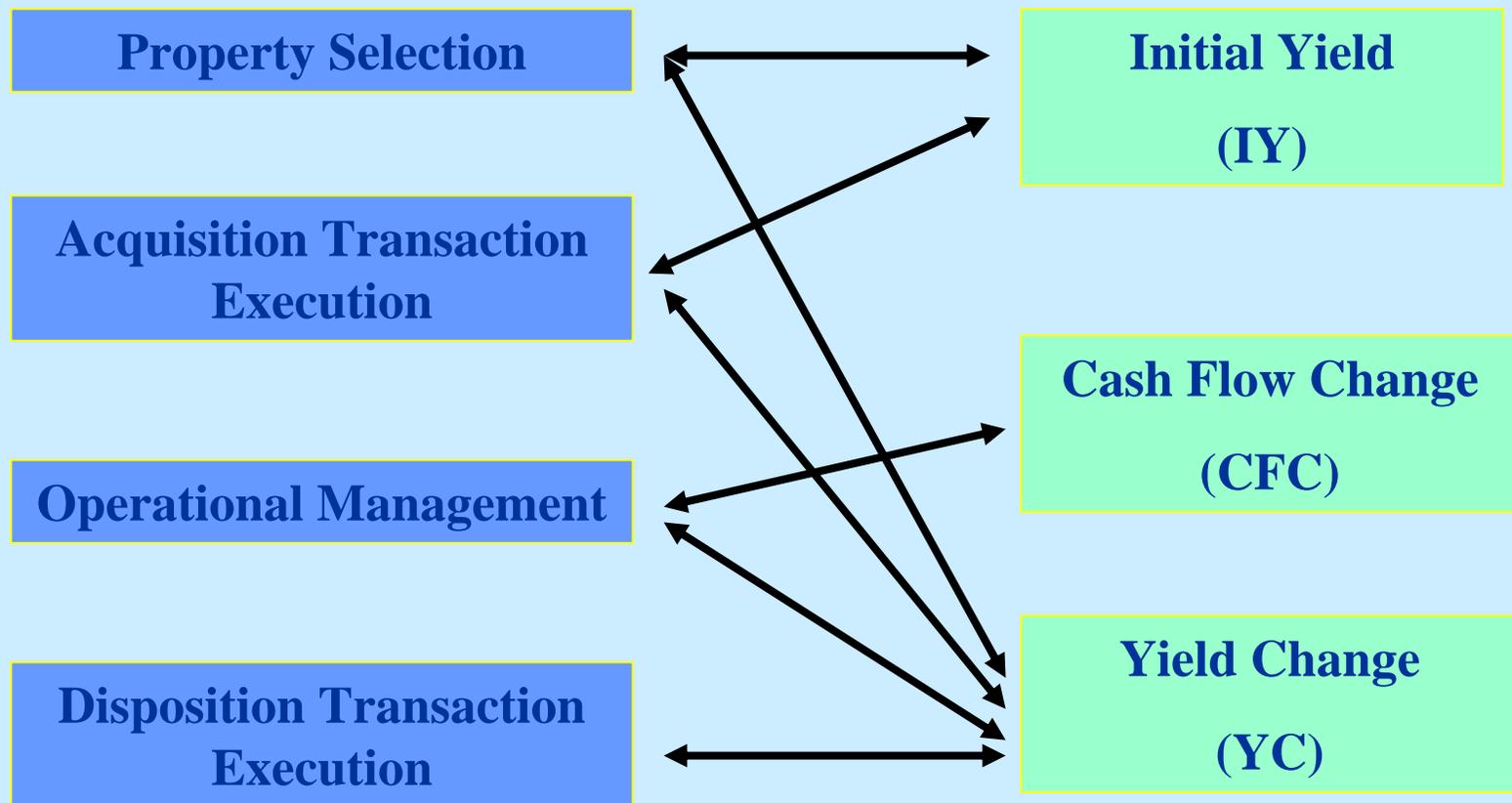
Property level performance attribution focuses on “*property level*” investment performance, i.e., the total return achieved within a given property or a static (fixed) portfolio of properties (that is, apart from the effect of investment allocation decisions, as if holding allocation among categories constant).

Property level attribution should be designed to break out the property level total return performance in a manner useful for shedding light on the four major property level investment management functions:

- **Property selection** (picking “good” properties as found);
- **Acquisition transaction execution;**
- **Operational management** during the holding period (e.g., marketing, leasing, expense mgt, capital expenditure mgt);
- **Disposition transaction execution.**

Property-Level Performance Attribution . . .

These property-level management functions are related generally to three attributes (components) of the property-level **since-acquisition IRR**, essentially as indicated below...



Property-Level Performance Attribution . . .

Conventional property level performance attribution is based on periodic returns, or on time-weighted multi-period returns (TWRRs, e.g., as implemented by IPD in England and PCA in Australia).

But **IRR-based performance attribution is arguably more useful for property level management diagnostic purposes, because:**

- **At the property level, the investment manager is typically **responsible for the major cash flow timing** decisions that can significantly effect property level (static portfolio) returns, e.g., leasing decisions, capital expenditure decisions.**
- **The IRR is sensitive to the effect of cash flow timing, the TWRR is not.**
- **The **IRR is cash flow based** (net of capital improvement expenditures), therefore, more accurately reflecting the investment return effect of capital improvement decisions.**

Property-Level Performance Attribution . . .

Useful IRR-Based property level performance attribution benchmarking requires the use of:

Since-acquisition IRR

- **IRR is computed *since acquisition* of property (or portfolio):**
 - **In order to reflect investment operational performance during entire holding period since acquisition;**
 - **Property investment holding periods are typically multi-year (single period or periodic returns do not reflect effective investment management holding period).**
- **IRR is computed for appropriate *benchmark cohort*, defined as universe of similar investments by competing managers, measured from same inception date (equal to property acquisition date).**

Property-Level Performance Attribution . . .

Simple Numerical Example:

- **Property (or static portfolio) bought at initial cash yield of 9%.**
- **Net cash flow grew at 2% per year.**
- **Property (or properties) sold (or appraised) after 10 years at a terminal yield of 10%, based on yr.11 projected cash flow (also 2% more than yr.10).**
- **IRR is 10.30%.**
- **How much of this IRR is due to 3 components: Initial Yield (**IY**), Cash Flow Change (**CFC**), and Yield Change (**YC**)?...**

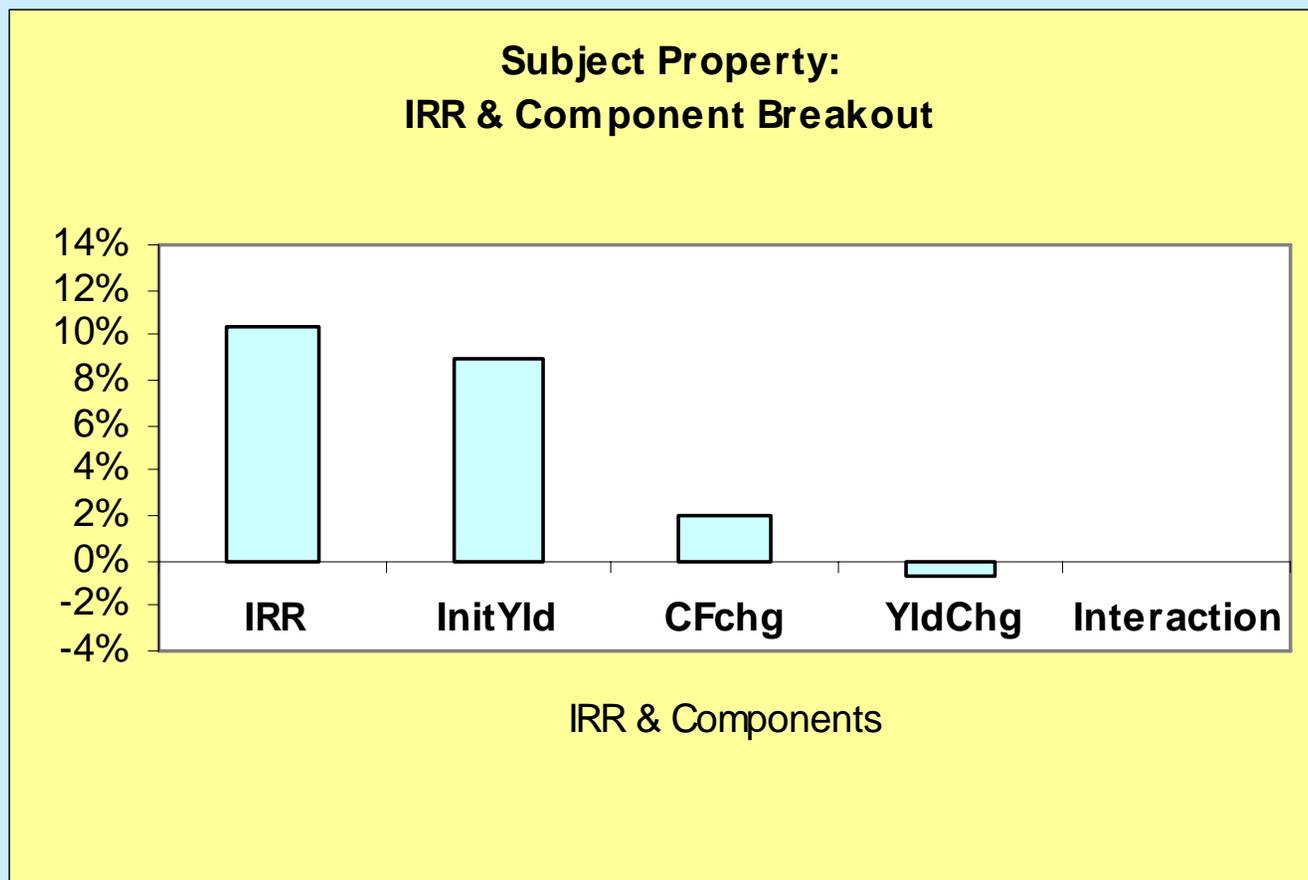
There are several ways one might answer this question. The approach that seems most intuitively related to the 4 basic mgt fcns is presented here...

	Yr	IRRs:	0	1	2	3	4	5	6	7	8	9	10	11
(1) Actual Oper.CF				1.0000	1.0200	1.0404	1.0612	1.0824	1.1041	1.1262	1.1487	1.1717	1.1951	1.2190
(2) Actual Capital CF			-11.1111										12.1899	
(3) Actual Total CF (=1+2)		10.30%	-11.1111	1.0000	1.0200	1.0404	1.0612	1.0824	1.1041	1.1262	1.1487	1.1717	13.3850	
(4) Init.Oper.CF constant				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
(5) Capital CF @ Init.Yld.on(4)			-11.1111										11.1111	
(6) Init.CF @ Init.Yld(=4+5)		9.00%	-11.1111	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	12.1111	
(7) Capital CF @ Init.Yld.on(1)			-11.1111										13.5444	
(8) Actual Oper. CF @ Init.Yld(=1+7)		11.00%	-11.1111	1.0000	1.0200	1.0404	1.0612	1.0824	1.1041	1.1262	1.1487	1.1717	14.7395	
(9) Capital CF @ ActualYld.on(4)			-11.1111										10.0000	
(10) Init.CF @ Actual Yld(=4+9)		8.32%	-11.1111	1.0000	11.0000									

- **Initial yield = 9.00%**, computed from line (6) IRR.
- **Cash flow change component = 2.00%** = 11%-9%, computed as the line (8) IRR less the line (6) IRR: = IRR with actual CF – IRR with no CF growth, (with constant yld at initial rate).
- **Yield change component = -0.68%** = 8.32%-9.00%, computed as the line (10) IRR less the line (6) IRR: = IRR with actual yld chg – IRR with no yld chg, (with constant CF at initial level).
 - **Interraction effect = -0.02%**, the difference between the line (3) overall IRR and the sum of the three other attributes [10.3%-(9%+2%-0.68%)].

Property-Level Performance Attribution . . .

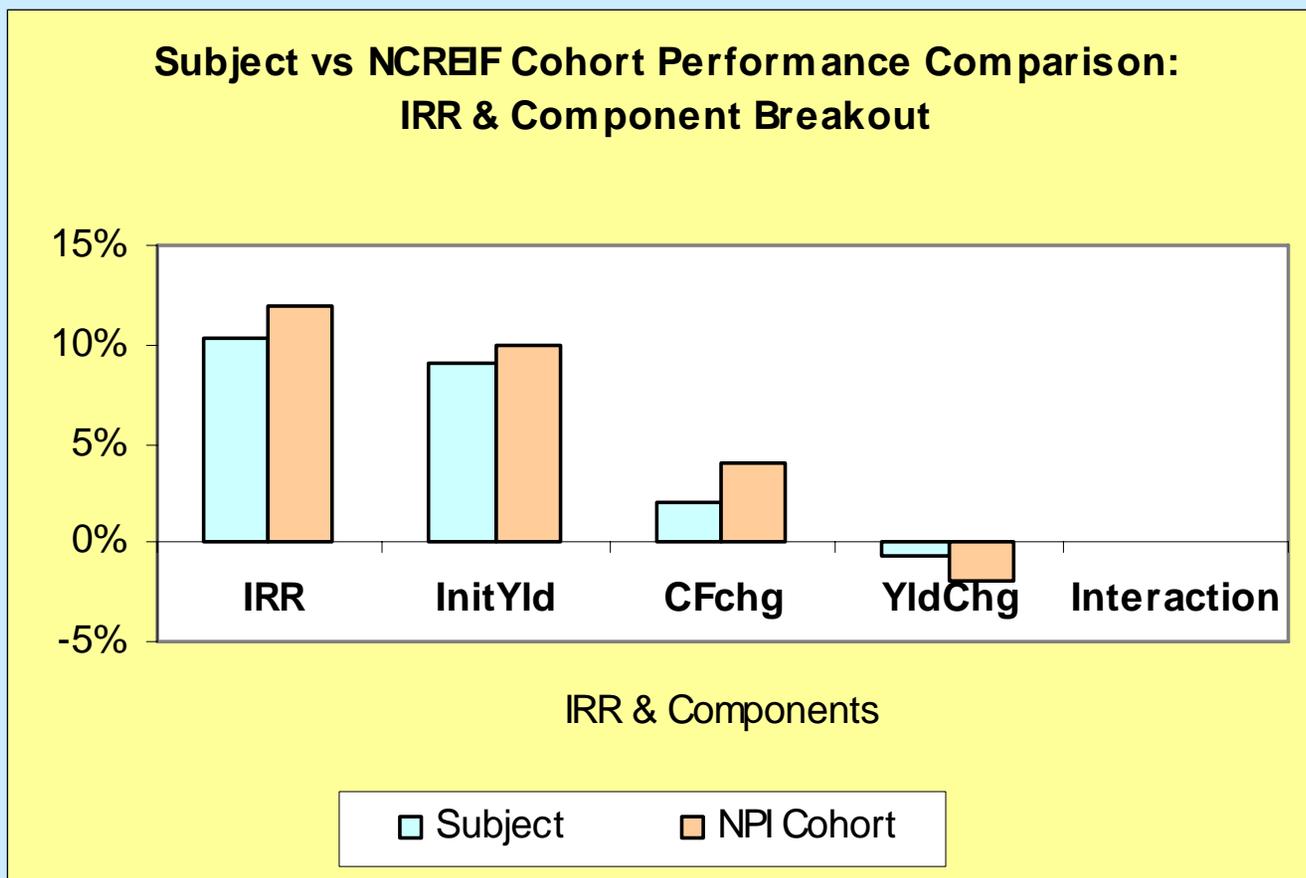
Here is a graphical presentation of the IRR-Based property-level performance attribution we just performed:



Suppose we computed the same type of IRR component breakdown for an appropriate benchmark, that is, a NCREIF sub-index cohort spanning the same period of time...

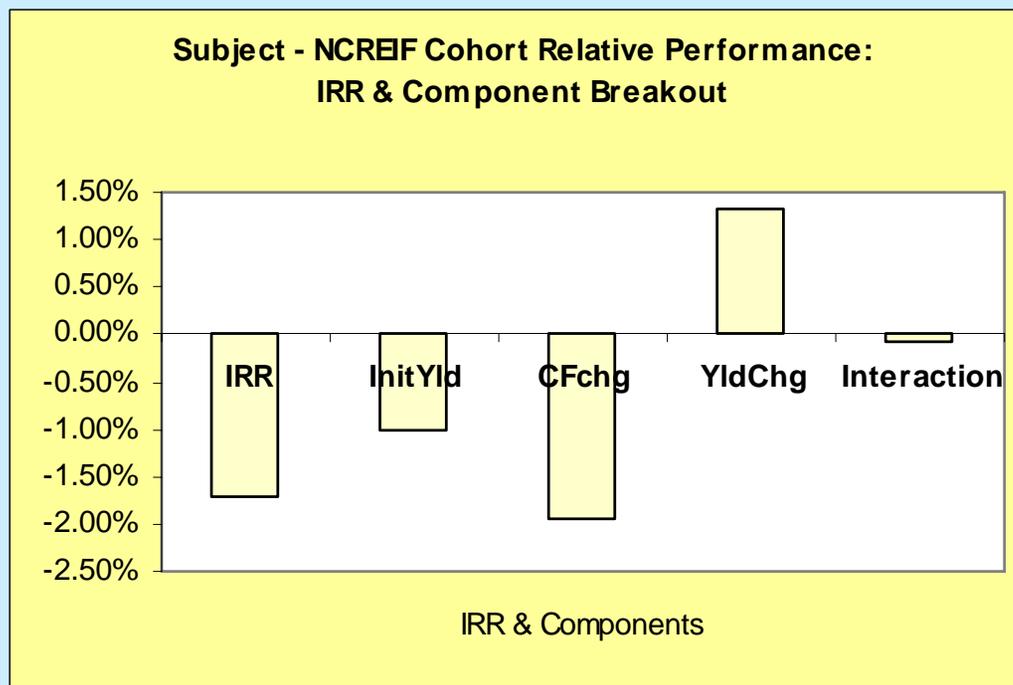
Property-Level Performance Attribution . . .

We could compare our subject performance with that achieved by a peer universe of managers, for similar properties (e.g., Calif. Industr. bldgs):



Property-Level Performance Attribution . . .

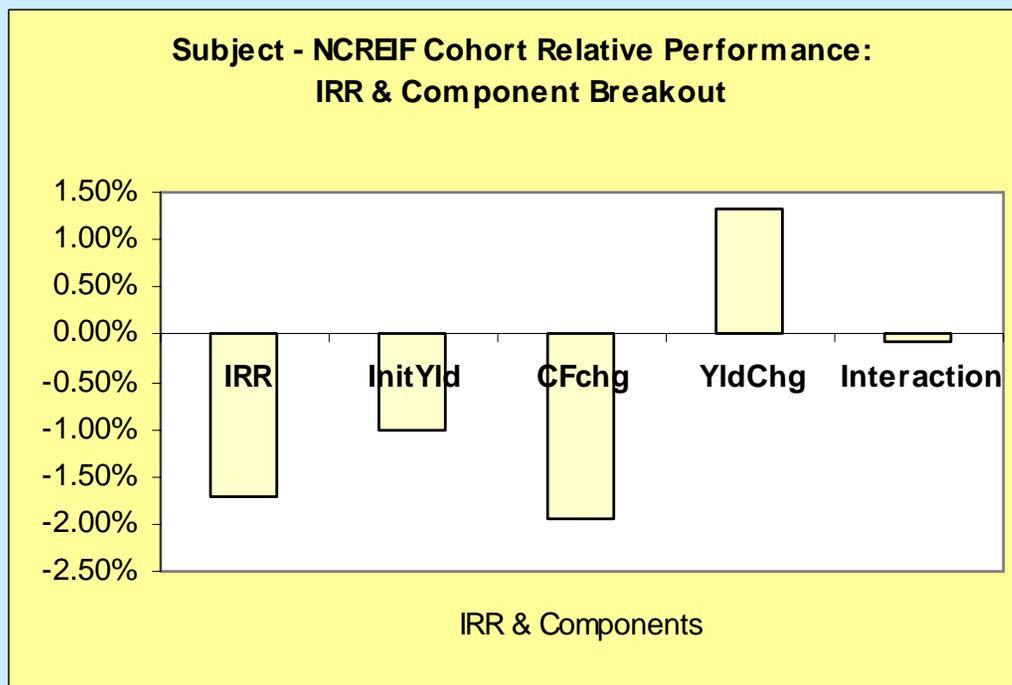
Here is the *relative performance*, the *difference* between our subject property and its benchmark, by attribute:



The above pattern could be plausibly interpreted as tentative evidence for the following hypothesis: Subject performed relatively poorly due largely to some combination of poor selection, acquisition, and operational mgt, partially offset by some combination of good disposition execution (or optimistic terminal appraisal), future-oriented capital improvements, &/or market movements during the holding period.

Property-Level Performance Attribution . . .

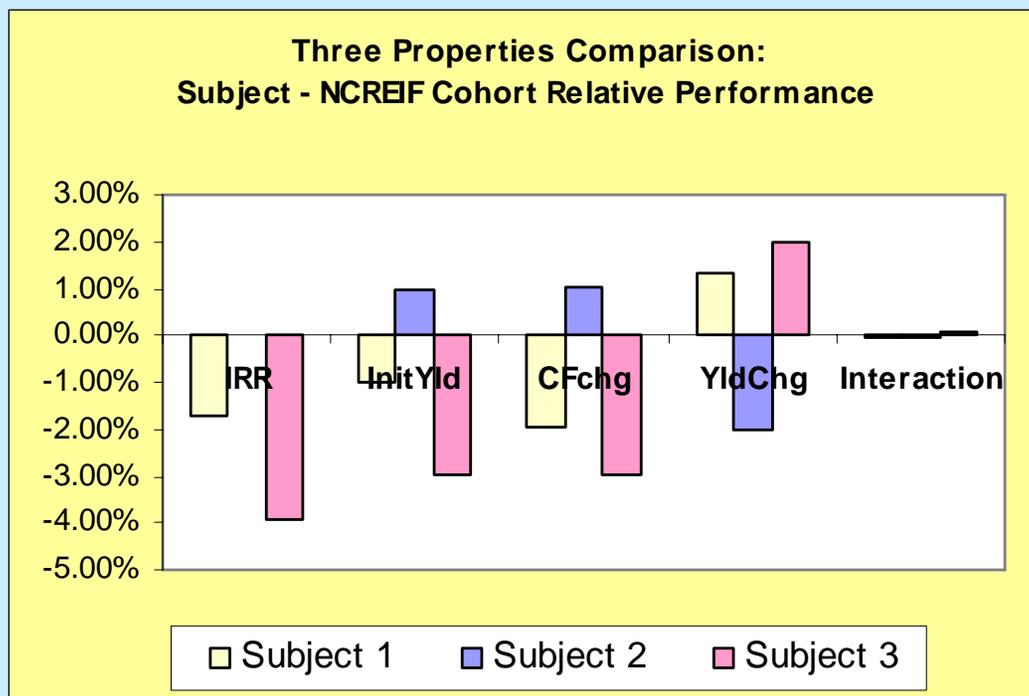
Here is the *relative performance*, the *difference* between our subject property and its benchmark, by attribute:



Now suppose we computed these relative performance differentials across a number of different properties (or portfolios) we have invested in...

Property-Level Performance Attribution . . .

We might gain some insights about our property-level investment and management performance:



In this case Subject Properties #1 & 3 have similarly poor performance (rel to benchmk), due to poor initial yield & poor CF change, suggesting poor acquisition & poor operational mgt. Property #2 did better, with good InitYld & CFchg, but poor YldChg (suggesting good acquisition, but poor disposition or mgt actions that hurt future outlook (e.g., inadequate Cap.Improvement). Mkt movements can also affect these results (less so the longer the holding period).

Consider the fundamental element of our valuation model:

$$V_0 = \frac{E_0[V_1]}{1 + E_0[r]}$$

Here we are accounting for both time and risk in the discount rate in the denominator, as $E_0[r] = r_f + E[RP]$, where r_f is riskless and accounts for the time value of money, and RP is the market’s required risk premium in the expected return for the investment.

But we can easily expand and algebraically manipulate this formula so that the denominator purely reflects the time value of money (the discounting is done *risklessly*) and the risk is completely and purely accounted for in the numerator, by reducing the cash flow amount in the numerator to a “*Certainty Equivalent Value*” . . .

Appendix 10C: “*Certainty-Equivalent Valuation*”...

$$V_0 = \frac{E_0[V_1]}{1 + E_0[r]} = \frac{E_0[V_1]}{1 + r_f + E[RP]}$$

A little algebra . . .

$$(1 + r_f + E[RP])V_0 = E_0[V_1]$$

$$(1 + r_f)V_0 + (E[RP])V_0 = E_0[V_1]$$

$$(1 + r_f)V_0 = E_0[V_1] - (E[RP])V_0$$

$$V_0 = \frac{E_0[V_1] - (E[RP])V_0}{1 + r_f}$$

Appendix 10C: “*Certainty-Equivalent Valuation*”...

$E_0[V_1] - (E[RP])V_0$ is the “*Certainty Equivalent*” value of V_1 .

The risk-adjusted discount rate formulation accounts for both time and risk simultaneously in the denominator:

$$V_0 = \frac{E_0[V_1]}{1 + r_f + E[RP]}$$

The certainty-equivalence formulation accounts for time only in the denominator, and risk only in the numerator, discounting the certainty-equivalent cash flow amount risklessly back to the present:

$$V_0 = \frac{E_0[V_1] - (E[RP])V_0}{1 + r_f}$$

Appendix 10C: “Certainty-Equivalent Valuation”...

The investment market is **indifferent** between a claim on the actual risky amount of V_1 one period in the future (with the actual amount of risk involved in that), versus a claim on the lesser amount of $E_0[V_1] - (E[RP])V_0$ one period in the future with no risk at all.

Why do we care about this? . . .

The certainty-equivalent approach will be quite convenient later on when we develop the “real options” (or “decision tree analysis”) method for evaluating investments in real estate **development** projects.

Consider a simple example . . .

Appendix 10C: “Certainty-Equivalent Valuation”...

Suppose:

Riskfree interest rate = 3%

Stock market total return risk premium = 6%

Stock Mkt expected total return is:

$$E[r_S] = r_f + E[RP_S] = 3\% + 6\% = 9\%$$

“Binomial World”...

A stock mkt total return index that today has value 1.00 will next year have a value of either:

1.13 (with 70% probability), or

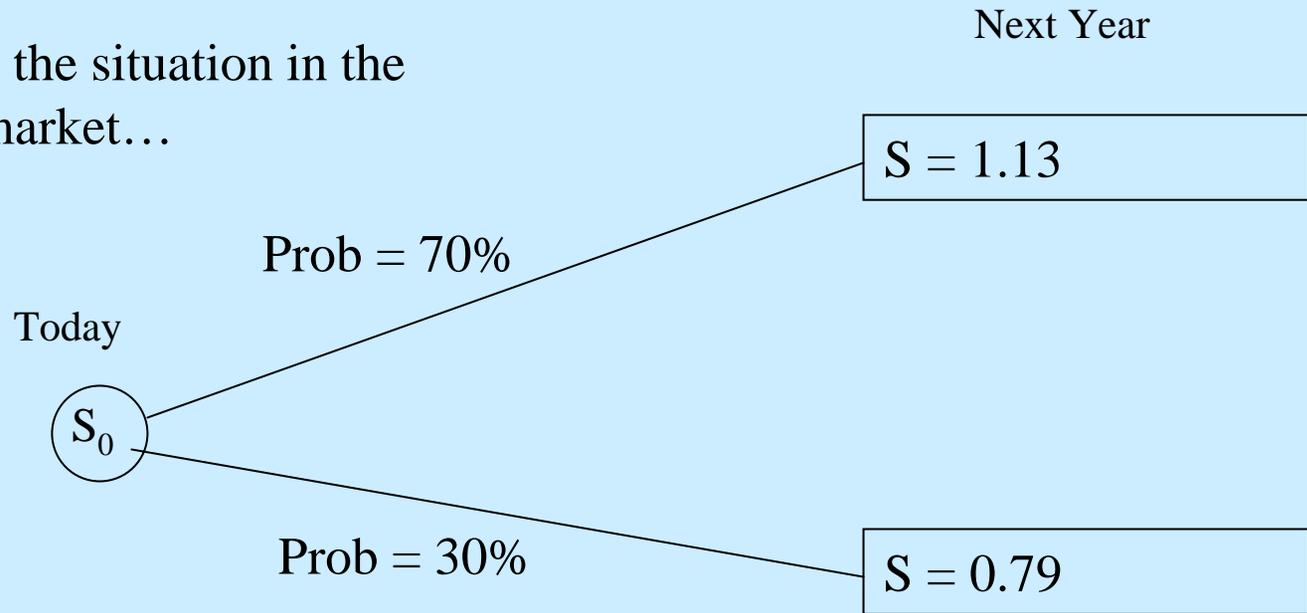
0.79 (with 30% probability).

Therefore, expected value of index next yr is:

$$E[S_1] = (0.70)1.13 + (0.30)0.79 = 1.03$$

Appendix 10C: “Certainty-Equivalent Valuation”...

Here is the situation in the
stock market...



Appendix 10C: “Certainty-Equivalent Valuation”...

Suppose:

We could build an office building that upon completion next year will be worth either \$113 or \$79, correlated with the stock market.

In other words, if the stock market goes up next year, our building will be worth \$113, if the stock mkt goes down next yr, our building will be worth \$79.

How much is this building worth today? . . .

Appendix 10C: “Certainty-Equivalent Valuation”...

Risk-adjusted discount rate valuation:

$$V_0 = \frac{E_0[V_1]}{1 + r_f + E[RP_V]} = \frac{(.70)\$113 + (.30)\$79}{1 + 3\% + 6\%} = \frac{\$103}{1.09} = \$94$$

Certainty-equivalent valuation:

$$V_0 = \frac{E_0[V_1] - E[RP_V]V_0}{1 + r_f} = \frac{\$103 - (.06)\$94}{1 + 3\%} = \frac{\$97}{1.03} = \$94$$

\$97 is the certainty-equivalent value of V_1 whose expected value is \$103.

In general we can determine the certainty-equivalent value if we know the expected future value and the riskfree and risk-adjusted discount rates:

$$\begin{aligned} \frac{E_0[V_1] - E[RP_V]V_0}{1 + r_f} &= \frac{E_0[V_1]}{1 + r_f + E[RP]}, \Rightarrow \\ \frac{E_0[V_1] - E[RP_V]V_0}{E_0[V_1]} &= \frac{1 + r_f}{1 + r_f + E[RP]}, \Rightarrow \\ E_0[V_1] - E[RP_V]V_0 &= \frac{1 + r_f}{1 + r_f + E[RP]} E_0[V_1], e.g. \\ \$97 &= \frac{1 + .03}{1 + .03 + .06} \$103 = \frac{1.03}{1.09} \$103 \end{aligned}$$

Appendix 10C: “Certainty-Equivalent Valuation”...

Notice that the difference between the expected future value and the certainty-equivalent value is $E[RP_V]V_0$:

$$E_0[V_1] - CEQ_1 = E[RP_V]V_0, \text{ e.g. : } \$103 - \$97 = .06(\$100) = \$6$$

The general formula for this difference is:

$$\begin{aligned} E[RP_V]V_0 &= \left(\frac{V \text{ risk in return}\%}{Mkt \text{ risk in return}\%} \right) (Mkt \text{ RP in return}\%)V_0 \\ &= \left(\frac{V \text{ risk in \$}}{Mkt \text{ risk in return}\%} \right) (Mkt \text{ RP in return}\%) \\ &= (V \text{ risk in \$}) \left(\frac{Mkt \text{ RP in return}\%}{Mkt \text{ risk in return}\%} \right) \end{aligned}$$

In the “binomial world” this formula becomes:

$$E[RP_V]V_0 = (V_{up} - V_{down}) \left(\frac{E[r_{Mkt}] - r_f}{Mkt_{up}\% - Mkt_{down}\%} \right)$$

e.g., in our example:

$$E[RP_V]V_0 = (\$113 - \$79) \left(\frac{9\% - 3\%}{113/94 - 79/94} \right) = (\$34) \left(\frac{9\% - 3\%}{120\% - 84\%} \right) = (\$34) \left(\frac{6\%}{36\%} \right) = (\$34)(0.17) = \$6$$

Three asides (see notes for further explanation) . . .

The “*binomial world*” is much more realistic and useful than you might think, in part because we can define the temporal length of a period to be as short as we want, and we can link and branch individual binomial elements together to make a “tree” of asset value possibilities over time.

In the “CAPM”:

$$\frac{E[RP_V]}{E[RP_{Mkt}]} = \left(\frac{V \text{ risk in return\%}}{Mkt \text{ risk in return\%}} \right) = \text{"Beta"} = \left(\frac{COV[r_V, r_{Mkt}]}{VAR[r_{Mkt}]} \right), \text{ Thus:}$$

$$\begin{aligned} V_0 &= (E_0[V_1] - \beta(E[r_{Mkt}] - r_f)V_0) / (1 + r_f) = \left(E_0[V_1] - \frac{COV[V_1, r_{Mkt}]}{VAR[r_{Mkt}]} (E[r_{Mkt}] - r_f) \right) / (1 + r_f) \\ &= E_0[V_1] / (1 + r_f + \beta(E[r_{Mkt}] - r_f)) \end{aligned}$$

(Don't worry about this in this class.)

More generally for a cash flow T periods in the future with expected value $E_0[V_T]$, define the certainty-equivalent amount $CEQ[V_T]$. We have:

$$V_0 = \frac{E_0[V_T]}{(1 + r_f + E[RP_V])^T} = \frac{CEQ[V_T]}{(1 + r_f)^T}, \Rightarrow CEQ[V_T] = \left(\frac{(1 + r_f)}{(1 + r_f + E[RP_V])} \right)^T E_0[V_T]$$

Appendix 10C: “Certainty-Equivalent Valuation”...

Let’s go back to our office building example...

Suppose the office building does not exist yet,

but we could build it in 1 year

by paying \$90 of construction cost upon completion of construction next year. (Construction is instantaneous: We can decide next year whether to build or not.)

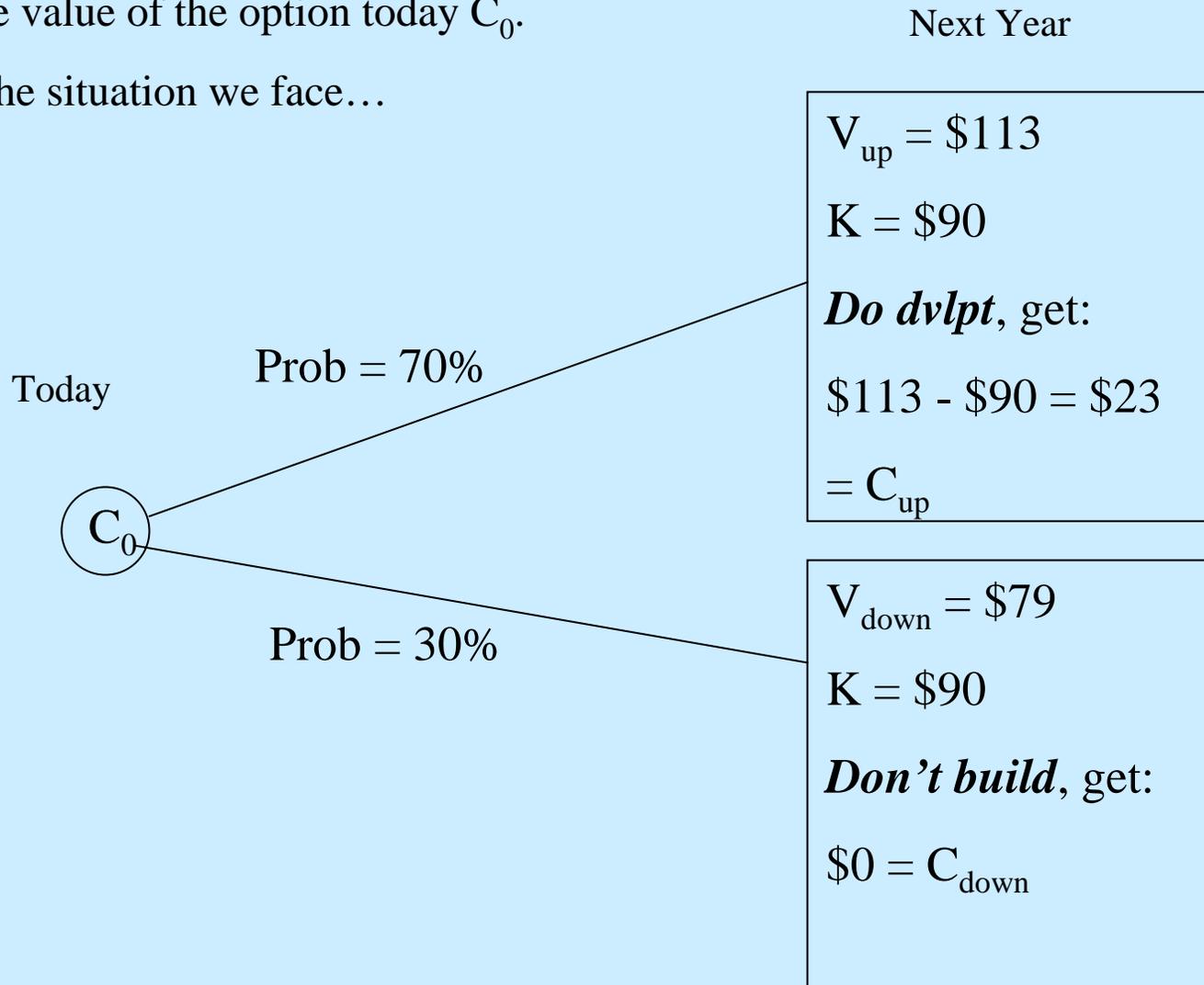
Suppose our *option* to build this project expires in 1 year: We either build then or we lose the right to ever build.

How much is this option worth?

Appendix 10C: “Certainty-Equivalent Valuation”...

Label the value of the option today C_0 .

Here is the situation we face...



In general: $C_{up} = \text{MAX}[V_{up} - K, 0]$, $C_{down} = \text{MAX}[V_{down} - K, 0]$.

Appendix 10C: “Certainty-Equivalent Valuation”...

Use our certainty-equivalent valuation formula to value the option...

$$C_0 = \frac{E_0[C_1] - (C_{up} - C_{down}) \left(\frac{E[r_{Mkt}] - r_f}{Mkt_{up} \% - Mkt_{down} \%} \right)}{1 + r_f}$$

e.g., in our example:

$$\begin{aligned} C_0 &= \frac{((.70)\$23 + (.30)0) - (\$23 - \$0) \left(\frac{9\% - 3\%}{120\% - 84\%} \right)}{1 + 3\%} \\ &= \frac{\$16.1 - (\$23) \left(\frac{6\%}{36\%} \right)}{1.03} = \frac{\$16.1 - (\$23)(.17)}{1.03} \\ &= \frac{\$16.1 - \$3.9}{1.03} = \frac{\$12.2}{1.03} = \$12 \end{aligned}$$

Appendix 10C: “Certainty-Equivalent Valuation”...

The equivalent valuation using the risk-adjusted discount rate approach is:

$$C_0 = \frac{E_0[C_1]}{1 + r_f + E[RP_C]} = \frac{(.70)\$23 + (.30)\$0}{1 + 3\% + 31\%} = \frac{\$16.1}{1.34} = \$12$$

Which implies that the option has $31/6 = 5.2$ *times* the investment risk that the office building has (and that the stock market has), as:

$$RP_C = 31\% = 5.2 * 6\% = 5.2 * RP_V (= 5.2 * RP_S).$$

However, if we didn't already know the option's risk premium (31%), or hadn't already done the certainty-equivalent approach, we wouldn't be able to use the risk-adjusted discount rate approach.

The certainty-equivalent approach does not require prior knowledge of RP_C . It only requires knowledge of C_{up} , C_{down} , and the “*up*” probability (70% in our example).

This makes the certainty-equivalent approach very useful for option valuation. Most development projects are essentially “options”.

Appendix 10C: “Certainty-Equivalent Valuation”...

Fundamentally, what we are doing here is making sure that the expected return risk premium *per unit of risk* is the same between an investment in the development option, and an investment in already-build (stabilized) property. For example, for stabilized property:

$$\frac{9\% - 3\%}{(\$113 - \$79)/\$94} = \frac{RP_V}{(V_1^{up} \$ - V_1^{down} \$)/PV[V_1]} = \frac{RP_V}{V_1^{up} \% - V_1^{down} \%} = \frac{9\% - 3\%}{(+20\%) - (-16\%)} = \frac{6\%}{36\%} = 0.17$$

For the development option:

$$\frac{34\% - 3\%}{(\$23 - \$0)/\$12} = \frac{RP_C}{(C_1^{up} \$ - C_1^{down} \$)/PV[C_1]} = \frac{RP_C}{C_1^{up} \% - C_1^{down} \%} = \frac{34\% - 3\%}{(+90\%) - (-100\%)} = \frac{31\%}{190\%} = 0.17$$

Thus, this procedure for the valuation of the development option is equivalent to a cross-market equilibrium condition: that the expected return risk premium per unit of risk (or the “risk-adjusted return”) presented by the investments should be the same between the market for development options and the market for stabilized built property. Otherwise, investors will buy one type of asset and sell the other type until prices equilibrate the risk-adjusted returns across the two markets.