Problem Set
Problems, including calculations for Columbia A

1) Consider two plants, A and B, both of which emit carbon, C, (where C is measured in units of carbon per cubic foot per hour.) The two plants have the following marginal cost of abatement schedules.

$$MCA_A = \$10.00 - .25C_A \qquad MCA_B = \$8.00 - .20C_B$$

Where MCA_A is the Marginal Cost of Abatement at Plant A, P_A is the amount of carbon per hour being emitted at Plant A, and so on,

a) If the government sets a carbon tax of $2.00/C, what is the total quantity of carbon emitted per hour?

Answer: Each firm will set emissions where the marginal cost of abatement equals the tax (below that, it is cheaper for the firm to remove the carbon internally.)

For firm A: $2.00 = $10.00 - .25$C_A$ or C_A = 32 units. Similarly C_B = 30 so total carbon emissions = 62 units.

b) What level of emissions fee would have to be imposed to reduce emissions to 44 units per hour?

Answer: We can rework each equation as follows:

$$C_A = (10 - MC_A)/.25 \quad \text{and } C_B = (8 - MCA_B)/.2$$

As we said above, once the tax is imposed, MC_A and MC_B will be adjusted to equal the tax, so we can write:

$(10 - tax)/.25 \; + \; (8 - tax)/.2 \quad = 44$

$40 - 4*tax + 50 \; - 5*tax = 44$

$36 = 9*tax$ or tax = $4.00/ unit of C.

c) Suppose instead that the government wanted to achieve the same goal as (b) using a cap and trade model instead of a carbon tax. Under this cap and trade model, each firm would be given the same number of permits and each permit would allow a firm to emit one unit of pollutant. How many permits would each firm initially receive? For each plant, discuss whether they would initially be a buyer or a seller of permits. When trading begins,

describe the maximum amount the buying firm would be willing to pay for a permit and the minimum amount the selling firm would be willing to receive for a permit.

Answer: As noted in the review session, the last part of this question turned out to be more complicated than we had wanted. We know a 44 permits would be given out, 22 to each firm. We also know that if Firm A emitted 22 units of carbon, its $MC_A = 10-.25*22 = 4.5$ while if Firm B emitted 22 units of carbon, its $MC_B = 8- .20*22 = 3.6$. From this we know that Firm A would like to buy permits from Firm B.

Ignore the last part of the question which gets complicated because the permits are "lumpy" and involve a big change in each firm's position.

2) Consider a community's demand for public television. There are three people in the (small) community and their demands for hours of public television programming are given by the following three demand functions.

$$P_1 = \$150 - H$$

$$P_2 = \$200 - 2H$$

$$P_3 = \$250 - H$$

Where: P_i = the price the i'th individual is willing to pay and H is the number of hours of public television the individual watches.

Hours of public television can be produced at a constant marginal cost of $200 per hour.

a) Consider three varieties of television programming: public television that you can receive with a regular antenna, commercial television (i.e. with advertising) that you can receive with a regular antenna, and cable television. Which, if any, of these varieties of television are pure public goods? Explain your reasoning.

Answer: The first two kinds of television programming – public and commercial television that you can receive with an antenna – are public goods. They are non-rival (your looking at a program doesn't reduce the quantity available for anyone else) and non-excludable – you can't stop particular individuals from receiving the signal.

b) From society's perspective, explain how you would determine the efficient number of hours of public television and calculate what that number is.

Answer: Since the three people are buying the same hours of TV, we have to add the demand curves vertically – i.e. adding together the amount each person is willing to pay for a given quantity of hours.

In doing this, however, we have to be careful not to add negative prices:

For the same amount of hours, H, person 1 will pay $P_1 = \$150 - H$ but only up to 150 hours At anything over 150 hours, his demand is just zero. For example, at 175 hours, the price he is willing to pay is zero – not - $25 which is what the equation would say.

Person 2 will pay $P_2 = \$200 - 2H$ up to 100 hours

Person 3 will pay P_3 $250 – H up to 250 hours

So: at 100 hours or above, all three people are in:

$P_{community} = \$600 - 4H$ (the sum of $P_1 + P_2 + P_3$)

Between 100 and 150 hours, the demand from Person 2 is zero and the demand curve is:

$P_{community} = \$400 - 2H$ (i.e. the sum of $P_1 + P_3$)

 Above 150 hours, the demand from Person 2 and Person 3 = 0 and the demand curve is:

$P_{community} = \$250 - H$ (i.e. just the third person's demand)

One student last year did a good graph of this relationship which I have reproduced below

We know that the social optimum will occur if we set demand equal to marginal cost so we can try to set the three person demand curve = $200.

$200 = $600 -4H

H = 100 hours.

At 100 hours, the first and third person's demand curves show a positive price and the second person's demand curve shows a zero price – i.e. none of the demand curves is showing s negative price – so this solution is the social optimum (if person 2's demand curve had shown a negative price, we would have had to try the demand curve with just person 1 and person 3, etc.)

Community Demand for Public Television
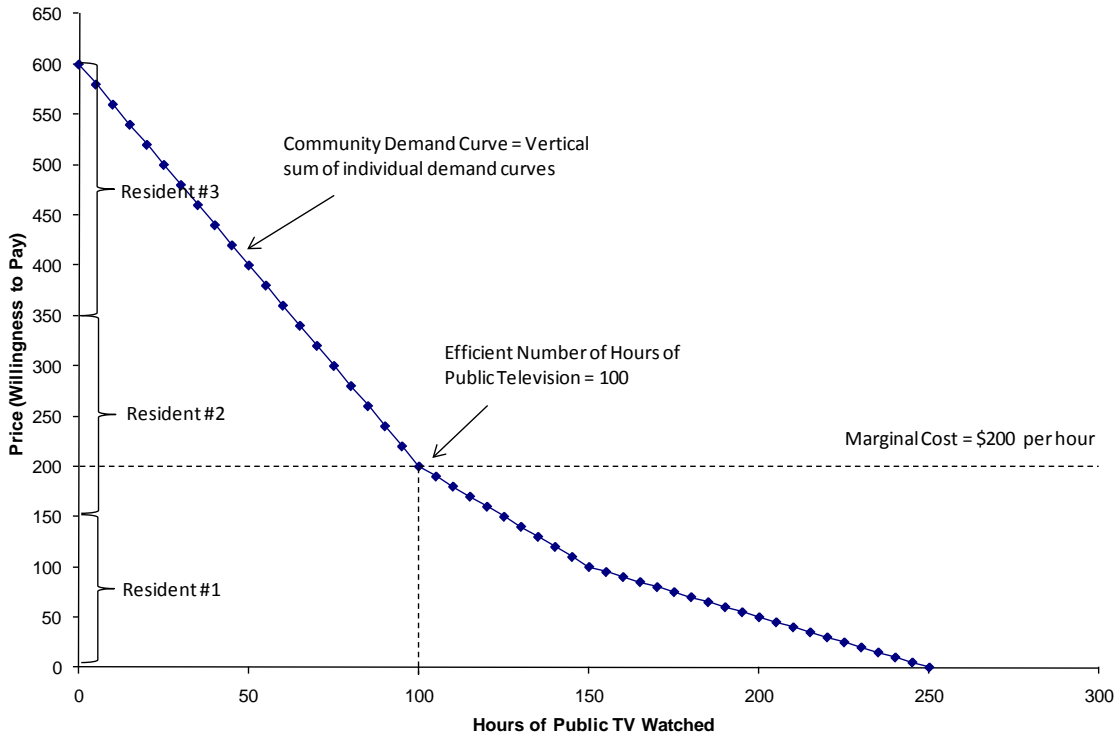
c) If public television was sold to viewers who paid fees to the station, how many hours of public television would a competitive private market provide?

Answer: Assume that it is a competitive market in which each person pays marginal cost, At a price = Marginal Cost of $200 per hour, only the third person would buy and the demand/price relationship would be:

$200 - $250 – H   or H = 50.

3) (**REWRITTEN**) Consider the stream of costs and benefits from a small commercial parking garage. You purchase the parking garage today for a price of $880,000. You immediately begin to operate the parking garage and continue to operate it for ten years. At the end of the tenth year we assume (for simplicity) that the garage instantaneously collapses and there is no salvage value.

End of:	Today	Year 1	Year 2	Year3	Year 4	etc. through year 10
Costs	$880,000	$40,000	$40,000	$40,000	$40,000	$40,000
Revenue	0	$200,000	$200,000	$200,000	$200,000	$200,000

where Year 1 payments occur one year from today, Year 2 payments occur two years from today, etc.

a) Familiarize yourself with the NPV function in Excel. Using this function, determine whether you would undertake the project if the annual interest rate were 5%. Then determine whether you would undertake the project if the annual interest rate were 10%? To use the NPV function, you may find it easier to collapse revenues and costs into a single "net revenue" figure for each year - i.e. -$880,000 for Year 1, $160,000 for Year 2 and so on.

In using the NPV function, note that Excel expects the first payment entered to occur at the end of the first period so you want to set up the problem as:

Value of the project = -$800,000 + NPV function of (net revenue in year 1, in year 2, etc.)

Answer:

At a 5 percent rate of interest, I get a net present value of $355,477.59
At a 10 percent rate of interest, I get $103,130.74

So you would undertake the project at either interest rate.

When I use Excel to calculate the internal rate of return, you get:12.7% which confirms the judgment above that the IRR must be larger than 10%.

b) The Army Corps of Engineers are responsible for building a significant number of dams and other federally funded construction projects. Before a project is approved, it must be justified by benefit/cost calculations. Historically the Corps used a 3% discount rate when rates 10 year bonds were typically closer to 5%. What might be a justification for the 3% rate? What would be a criticism of this 3% rate.

Answer: The Corps might try to argue that the private market is too present oriented which means the private market discount rate is too high and undervalues future benefits (by discounting them heavily) and that public projects ought to be more future-oriented by using a lower discount rate. The counter argument is that the private market discount rate is correct and that the Corps is using a low discount rate in order to justify doing large numbers of projects.

c) Explain what your answers in (a) tell you about the parking garage's internal rate of return. Use the IRR function in Excel to verify your answer (The IRR function in Excel expects the first entry to be at t=0 so you can the stream of costs and revenues as is).

Answer: The IRR is the discount rate that causes the project's NPV to $= 0$ (i.e. at that rate, the project just breaks even in present value terms). Since the project has a positive NPV at 10% interest, the internal rate of return must be larger than 10%.

Using Excel, the internal rate of return is 12.7% which squares with the argument above.

d) Recall that in Mike Greenstone discussion of estimating the social cost of carbon, the modeling group faced uncertainty over the proper value of the discount rate and ended up using a range of estimates. While this parking garage is not an environmental project, we can use it to examine how you should deal with discount rate uncertainty in calculating net present value. Suppose you are unsure whether the correct interest rate (discount rate) on this project is 5% or 10%? This means that the project's net present value is uncertain. Consider two ways to calculate NPV under this uncertainty:

> i) Take the two values of NPV you calculated in (a) and average them
>   - i.e. average the project's value calculated under each of
>   the two interest rates.

> ii) Average the two interest rates themselves and calculate a single, new
>   NPV value based on this average interest rate.

Using what you have learned in your comparison, briefly discuss the importance of doing this problem correctly in the case of assessing a large environmental cost that occurs 100 years into the future. What does this say about how you should deal with uncertainty over discount rates?

Answer: The first method - taking two separate NPV's and averaging them - is the correct one. The basic argument can be seen in from the graph below.

Discount Factor for Year 100 = 1/(1+r)^100
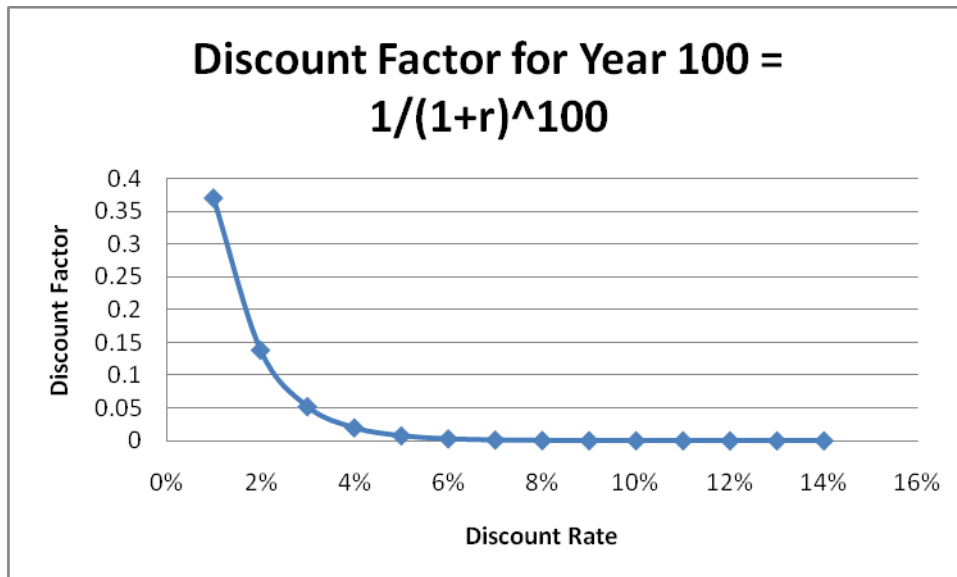
The graph shows how interest rates ranging from 1% to 14% translate into discount factors in year 100 (the discount factor = $1/(1+r)^{100}$ – the term that multiples the costs and benefits in that year). At an interest rate of 1%, the discount factor in year 100 is .37 which means that about two-fifths of costs and benefit in the year 100 are added to your net present value calculation. At 15%, the discount factor in the year 100 is .0000002 which means that almost nothing of the cost or benefit in year 100 is added to your net present value calculation.

If we first average over uncertain discount rates and then calculate the NPV , we will give too little weight to what could be very high costs in the future. The nonlinear relationship between the discount rate and the discount factor, shown in the graph above, shows why. Suppose we have a choice between doing NPV's at 1% and 10% and averaging them or doing a single NPV at 5.5%. As we move from a discount rate of 1% to a rate of 5.5%%, the discount factor falls from .37 to .0047– i.e. that part of the averaging really downplays future cost. As we move from a discount rate of 10% to 5.5%, the discount factor rises from .0000002 to .0047 – this increases the weight given to future cost but only by a little bit.

In other words, future costs (or benefits) count for much less if we average the discount rates and then do the NPV rather than if we average the two separate NPV's.

4) When a government or private company issues bonds, the bonds are usually sold at auction.

- The bond says it will pay to the holder fixed amounts of money on one or more specific dates – say $10,000 on November 19, 2020. No interest rate is specified

- The bond is sold today at auction for a price based on supply and demand – we can call the price P_{2010}

On November 19, 2010, the city of Lynn, Massachusetts sells bonds at auction that promise to pay the bondholder $1,000 on November 19, 2015 and $5,000 on November 19, 2020. On the date of the auction, similar bonds sold for prices that imply a 5 percent rate of interest.

a) Compute the price these Lynn, Massachusetts bonds will sell for.

Answer: Price = $1,000/(1.05)^5 + $5,000/(1.05)^{10}$ = $3853 (maybe check my math)

b) Financial news stories often contain the explanatory sentence – "Bond prices and interest rates move in opposite directions." Use the kinds of calculations you made in (a) to demonstrate that proposition.

Answer: If you redid the calculations in (a) with the interest rate equal to, say, 8%, the price the bond would sell for = $2,996. One way to think about this is that to reach a given amount in, say, 5 years, the higher the interest rate, the less you have to put away today to reach the future target.

c) Suppose Lynn's bonds were sold with a "call" provision. Under the provision, Lynn has the option of repurchasing the bonds at any future date between now and November 19, 2020 at the price the bond would bring on that date if interest rates remained at 5 percent. If Lynn wanted to exercise this call on November 19, 2016, what price would Lynn pay to repurchase a bond?

Answer: In 2,016, the bond is an asset that will grow at an interest rate of 5% for four years and will pay a value of $5,000 in 2020. This means the value of the bond in 2016 equals:

$$\$5,000/(1.05)^4 = \$4,113$$

d) If Lynn issues its bonds with the call provision, will a bond's initial auction price be higher or lower than if the price if the call provision were omitted? Explain your reasoning.

Answer: Lynn would call the bond if interest rates had fallen below 5 percent. In that case, they could force you to sell your bond to them and then turn around and borrow at the lower interest rate, therefore having a lower interest rate for the reminder of the ten years

From your perspective however, this is a bad deal. We saw above that when interest rates fall, the price of bonds rises. That should benefit you – i.e. you could turn around and sell your bond to another investor. But Lynn won't let you do that. Rather, it will force you to sell the bond at the old price.

To keep the argument simple, suppose interest rate rates fell the day after you bought the bond. As we saw above, the price you paid for the bond equaled:

Price = $1,000/(1.05)^5 + $5,000/(1.05)^{10}$ = $3853

If interest rates fell to 4% the next day, the value of that bond would be:

Price = $1,000/(1.04)^5 + $5,000/(1.04)^{10}$ = $4,200

If you had bought the bond for $3,853, and interest rates fell the next day, you could sell the bond yourself for $4,200 (we are ignoring the passage of one day here). But Lynn won't let you do that – it will exercise the call and buy it back from you at $3,853.

The fact that there is no chance of getting this gain means the bond is worth a little less to you and so the original sale price won't be quite as high.

11.202 Planning Economics
Fall 2010