Problem Set 1

11.126/11.249/14.48 Due February 27, 2007

1. (25 points) Suppose we have a data set with information on N men containing their earnings, their ages, their fathers' earnings, and their fathers' ages. We estimate a regression of the form

$$\ln y_{i}^{s} = \alpha + \beta \ln y_{i}^{f} + \gamma_{1} a_{i}^{s} + \gamma_{2} a_{i}^{s2} + \delta_{1} a_{i}^{f} + \delta_{2} a_{i}^{f2} + \epsilon_{i}$$

where y_i^s is son's earnings averaged over several years, y_i^f is father's earnings averaged over several years, a_i^s is son's age, and a_i^f is father's age.

- (a) (5 points) What is the interpretation of β ? Try to be precise.
- (b) (5 points) Is β likely to be a good estimate of the effects of a pay raise on the earnings of a man's son? Explain why or why not.
- (c) (5 points) If we estimate this regression because we are interested in determining the degree of mobility in the US economy, does the answer to (b) matter? Why or why not?
- (d) (10 points) Suppose we compare two economies with the same β . In the first economy, the standard deviation of ϵ_i is very low (most people's ϵ_i is close to 0), while in the second economy the standard deviation is very high (large negative and large positive values of ϵ_i are common).
 - i. Given the same y_i^f , a_i^s and a_i^f , is our best guess of y_i^s the same or different in the two economies? Explain why.
 - ii. In each economy, we can rank the sons and the fathers by their earnings, with 1 being the lowest rank and N being the highest. Suppose we want to know the probability that a son's rank exceeds his father's. Will this probability be the same or different in the two economies? Explain why.
- 2. (25 points) Write a short essay (a couple of paragraphs) exploring the patterns in Figure 4 of "The Consequences of Increasing the Nation's Supply of College Graduates." In particular, you may want to touch on a) why male enrollment dipped in the 1970's and then grew slowly until the mid-1990's, and b) why the pattern for women is different.
- 3. (20 points) Read the article by Leon Feinstein on the reading list.
 - (a) (6 points) The article shows that cognitive development at young ages predicts educational outcomes in adulthood. Give at least two stories where this does *not* imply that an intervention that improves childhood test scores will have a causal effect on adult education.

- (b) (7 points) A skeptic looks at Figures 2 and 3 and complains, "These figures use too crude a measure of SES by splitting children into only two groups. We can't tell from looking at these figures whether early childhood test scores really do predict later test scores among people with the same family background." Does the skeptic's criticism make sense? Explain why or why not.
- (c) (7 points) Another skeptic complains, "The tests at different ages are not comparable at all—one is measuring things like cube stacking and another is measuring things like reading. We can't infer from Figures 2 and 3 that "ability" at early ages predicts "ability" later in any meaningful way." Does the skeptic's criticism make sense? Explain why or why not.
- 4. (30 points) This problem is designed to help you think about how the rate of return to education is determined. Consider a model where people choose their level of education in order to maximize the present value of their lifetime earnings, i.e., education is a purely an investment. Person i is in school from time t = 0 until time $t = s_i$ (s_i is the level of schooling they achieve), and during this time they earn no income. Assume, however, that there are no tuition costs. After time s_i , a person enters the labor force and earns an amount $y(s_i, t)$ until $t = \infty$ (we abstract away from retirement), where $\ln y(s_i) = \alpha + \beta_1 s_i + \gamma(t)$. All people face the same earnings function $y(s_i, t)$, and they all discount the future at the interest rate r. In continuous time, the present value of a person's lifetime earnings is

$$PV_i = \int_{s_i}^{\infty} e^{-rt} y(s_i) dt$$

- (a) (15 points) How much schooling does each person choose to get in this model as a function of β_1 ? Explain why the result makes sense in words
- (b) (5 points) Suppose that the return to schooling β_1 is a decreasing function of the average level of schooling in the population \overline{s} , $\beta_1(\overline{s})$. Draw a supply and demand diagram with \overline{s} on the horizontal axis and β_1 on the vertical axis.
- (c) (5 points) In this model, what is the equilibrium return to schooling $\beta_1(\overline{s})$? What is the equilibrium effect of an increase in demand on the return to schooling?
- (d) (5 points) Compare your answer to (c) with the college/high school earnings differentials in Table 1 of "The Consequences of Increasing the Nation's Supply of College Graduates." Are the observed earnings differentials higher or lower than you would expect based on this model?