D-Lab: Supply Chains

Inventory Management, Part II

October 22, 2014
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Agenda:

- Review: **Economic Order Quantity (EOQ)**
- Single-period: **Newsvendor Model**
- Multi-period:
  - **Base Stock Policy**
  - **(R,Q) Policy**
- Project discussion

Operations Management Seminar

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Data-driven Operations Research Analyses in the Humanitarian Sector

Abstract:

We briefly discuss six projects in the humanitarian sector:

(1) allocating food aid for undernutrutioned children using data from a randomized trial in sub-Saharan Africa,

(2) allocating food aid for undernutritioned children using data from a nutrition project in Guatemala,

(3) analyzing the nutrition-disease nexus in the case of malaria,

(4) allocating aid for health interventions to minimize childhood mortality,

(5) assessing the impact of U.S.'s failure to use local and regional food procurement on childhood mortality, and

(6) deriving individualized biometric identification for India's universal identification program.

Types of inventory models

- **Demand:** constant, deterministic, stochastic
- **Lead times**: "0", ">0", stochastic
- **Horizon**: single period, finite, infinite
- **Products**: one product, multiple products
- **Capacity**: order/inventory limits, no limits
- **Service**: meet all demand, shortages allowed

EOQ

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EOQ **Newsvendor**

Newsvendor Model

- Single order opportunity
- Stochastic demand
- Tradeoff:

Order too little: lost sales

Order too much: wasted investment

- Applications: newsvendors, fashion, seasonal retail, events, flu prevention, etc

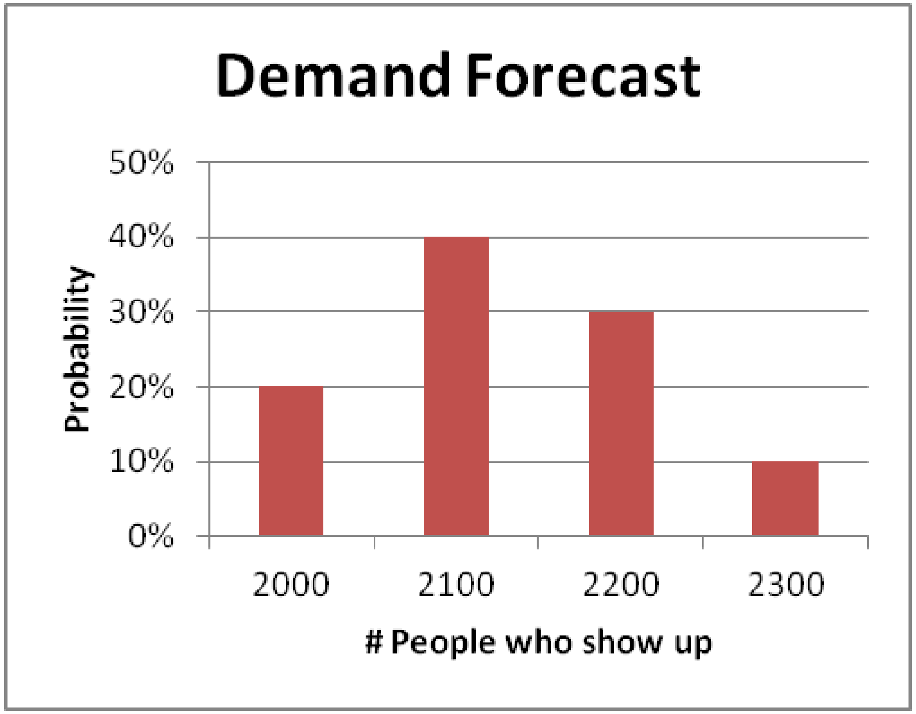
Example: Vaccine Clinic

In preparation for a one-day flu shot clinic, you need to decide the quantity of vaccines to order in advance.

You are given a **probabilistic forecast** based on historical demand data.

Exercise: What is the expected demand?

Sol. $E[D] = \sum_{d} d \cdot Prob(D = d)$



Example: Vaccine Clinic

Optimal order quantity depends on the forecast and the costs:

Expected cost $f(Q) = c_o\, E[\text{overage}(Q)] + c_u\, E[\text{underage}(Q)]$

What is the overage cost c_o and underage cost c_u in the following cases?

1. Suppose vaccines costs you $c = \$1$ per unit if ordered in advance. For every flu shot you give, you are paid $r = \$5$. At the end of the day, leftover vaccines have to be thrown away, so the salvage value is $v = \$0$.

$\Rightarrow$ **$c_o = c\text{-}v = 1$; $c_u = r\text{-}c = 4$ (value of lost sales)**

2. Suppose $c = \$1$, $r = \$5$, but you can sell back leftover vaccines to a recycler for $v = \$0.5$ each.

$\Rightarrow$ **$c_o = c\text{-}v = 0.5$; $c_u = 4$**

3. Suppose again $c = \$1$, $r = \$5$, $v = \$0$. In addition, suppose you cannot turn people away if you run out of pre-ordered vaccines; instead, you can make emergency orders at double the cost, \$2.

$\Rightarrow$ **$c_o = 1$; $c_u = 2\text{-}1 = 1$**

Optimal Order Quantity

Expected cost $f(Q) = c_o\, E[overage(Q)] + c_u\, E[underage(Q)]$

How do you compute the optimal order quantity?

- Sol 1: Brute-force enumeration
  - Calculate f(Q) for all possible Q)
  - See Excel demo
  - May be cumbersome if there are lots of possible Q
- Sol 2: Take derivative of f(Q)
  - See standard inventory textbooks; a bit tedious
- Sol 3: Incremental Analysis

Optimal Order Quantity

- Incremental analysis:
  - What is the cost/benefit of ordering one additional unit ($Q \rightarrow Q+1$)?
    1. Benefit: if the additional unit is used up, you make an extra c_u. This event happens with probability $P(D>Q)$.
       => Expected benefit: $P(D>Q)\, c_u$
    2. Cost: if the additional unit is not used up, you wasted an investment of c_o. This event happens with probability $P(D \leq Q)$.
       => Expected cost: $P(D \leq Q)\, c_o$
  - If the expected benefit outweighs the expected cost, you'd want to continue increasing the order quantity Q. Conversely, if cost outweighs benefit, you'd want to continue decreasing Q.
  - At the optimal Q, the benefit and cost balance each other:

$$P(D>Q)\, c_u = P(D \leq Q)\, c_o$$

  - Collecting the terms, we obtain the optimality condition:

$$P(D \leq Q) = \underbrace{\frac{c_u}{c_u + c_o}}_{\text{Critical ratio}}$$

Example: Vaccine Clinic

Optimal order quantity depends on the forecast and the costs:

Expected cost $f(Q) = c_o\,E[\text{overage}(Q)] + c_u\,E[\text{underage}(Q)]$

What is the critical ratio in the following cases?

1. Suppose vaccines costs you c = \$1 per unit if ordered in advance. For every flu shot you give, you are paid r = \$5. At the end of the day, leftover vaccines have to be thrown away, so the salvage value is v = \$0.

$\Rightarrow$ **$c_o = c - v = 1$; $c_u = r - c = 4$ (value of lost sales)**

$$P(D \leq Q) = \frac{c_u}{c_u + c_o} = 0.8$$

2. Suppose c = \$1, r = \$5, but you can sell back leftover vaccines to a recycler for v = \$0.5 each.

$\Rightarrow$ **$c_o = c - v = 0.5$; $c_u = 4$**

$$P(D \leq Q) = \frac{c_u}{c_u + c_o} = 0.89$$

3. Suppose again c = \$1, r = \$5, v = \$0. In addition, suppose you cannot turn people away if you run out of pre-ordered vaccines; instead, you can make emergency orders at double the cost, \$2.

$\Rightarrow$ **$c_o = 1$; $c_u = 2 - 1 = 1$**

$$P(D \leq Q) = \frac{c_u}{c_u + c_o} = 0.5$$

Note: In this example, since we have a discrete probability, there is no Q that exactly matches the optimality condition; we need to check the two options that bound it. (This is much more efficient than having to check all possible options for Q!)

Types of inventory models

- **Demand:** constant, deterministic, stochastic
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EOQ

Newsvendor

Continuous review: (R,Q)
Periodic review: (T,S)

Multi-Period & Stochastic Demand

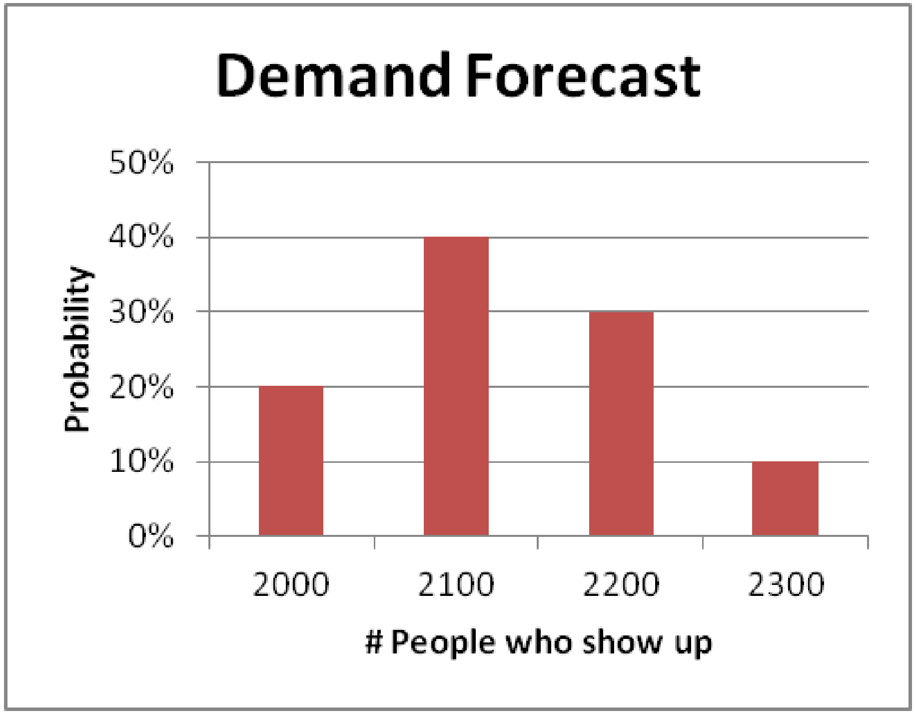
- Overage is no longer a big deal
  - Leftover inventory can be used in the following periods (unlike that in the single-period case)
  - Cost of overage is holding cost
  - Possible economies of scale for fixed ordering cost
- Underage is more serious
  - Performance measure: service level

Service Level α = Prob(no stock-out)

  - Need to hold safety stock to achieve service level

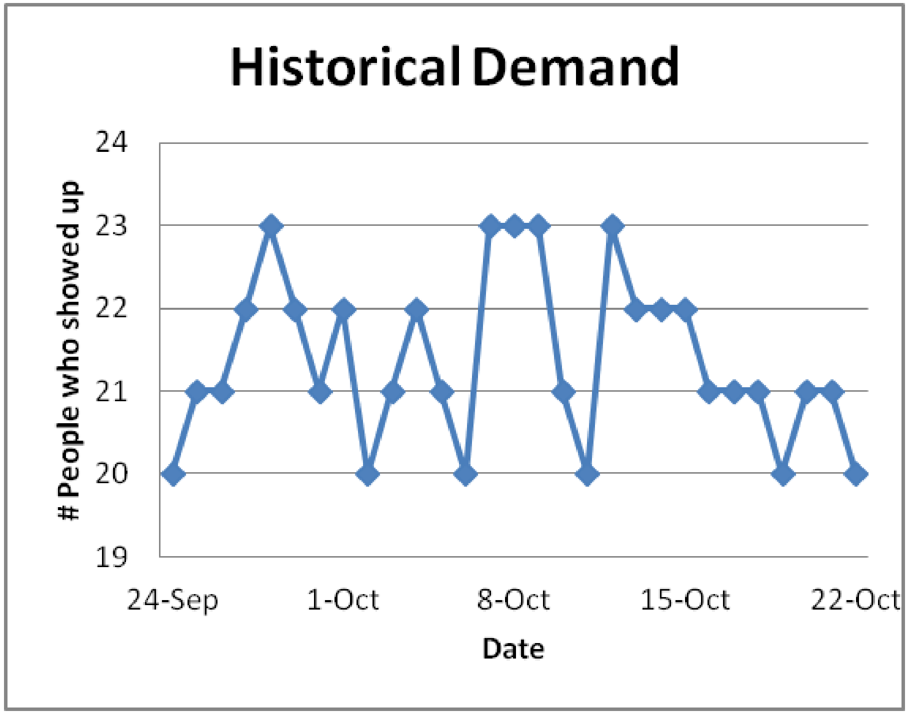
Service Level

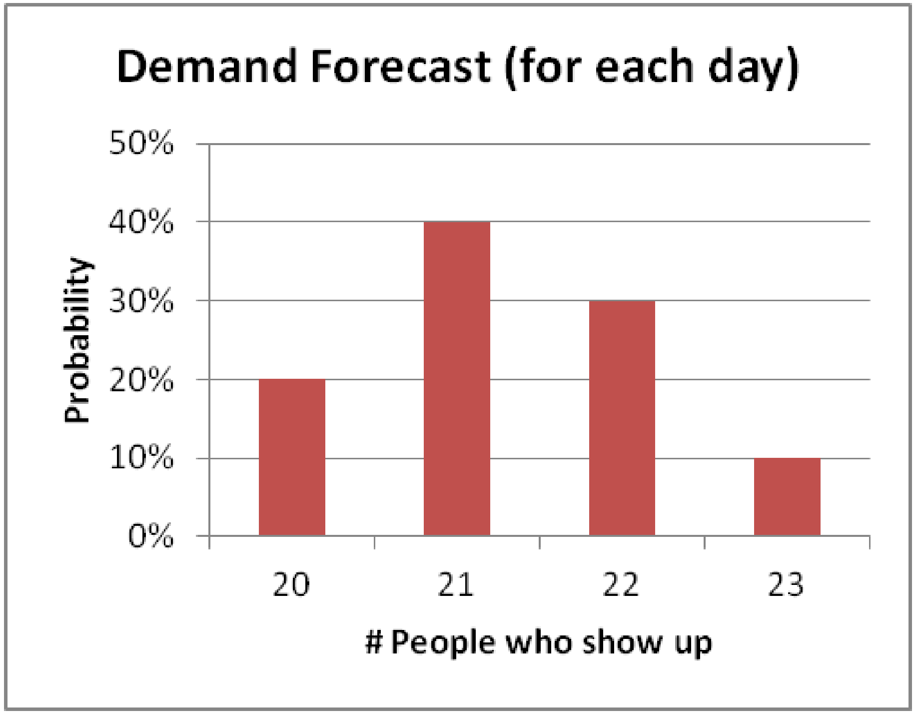
- What is the stock-out probability and service level if you ordered 2100?
- How much should you order to achieve a service level of 90%? 95%?



Example: Vaccine Clinic

Suppose you are now managing a daily, non-seasonal vaccine clinic.





Ordering for Multiple Periods

- Due to economies of scale (e.g. fixed cost per order, as in EOQ), it may be desirable to place one order to cover multiple periods

  => need to know the distribution of demand over multiple periods
- The **Central Limit Theorem** provides an approximation:

  Let D_T = demand over T days, where the daily demand has mean μ_D and std σ_D

  Then by the Central Limit Theorem,

$$D_T \rightarrow \text{Normal}(\mu, \sigma^2)$$

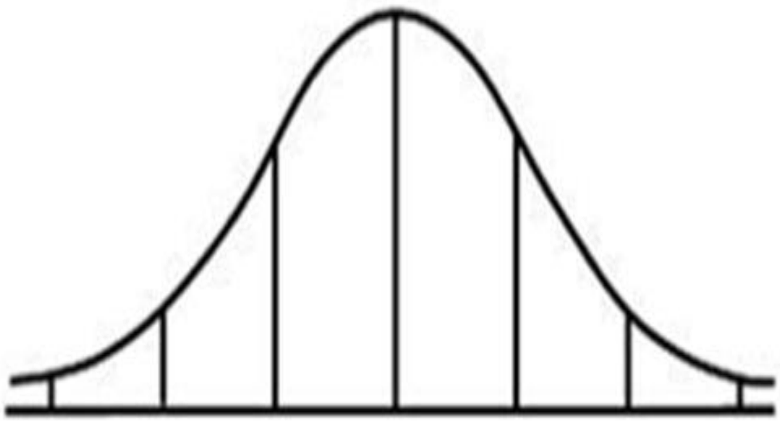
where $\mu = T\mu_D$

$\sigma = \sqrt{T}\sigma_D$ (as a result of summing the variance: $\sigma^2 = T\sigma_D^2$)

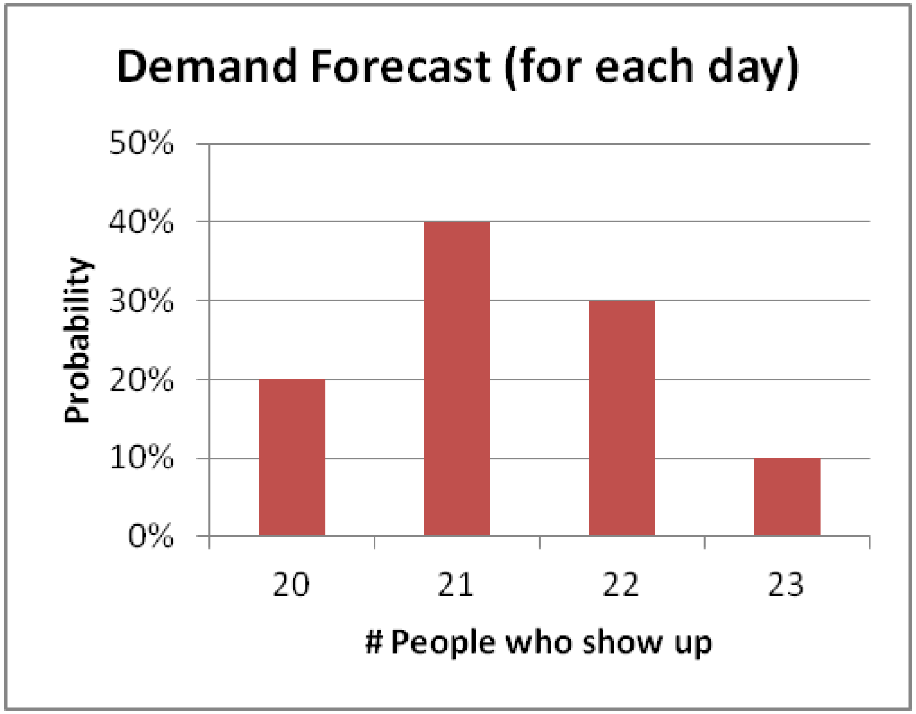
Example: Vaccine Clinic

If you reorder every T=5 days:

Aggregate demand: $D_T \approx N(\mu_T, \sigma_T^2)$

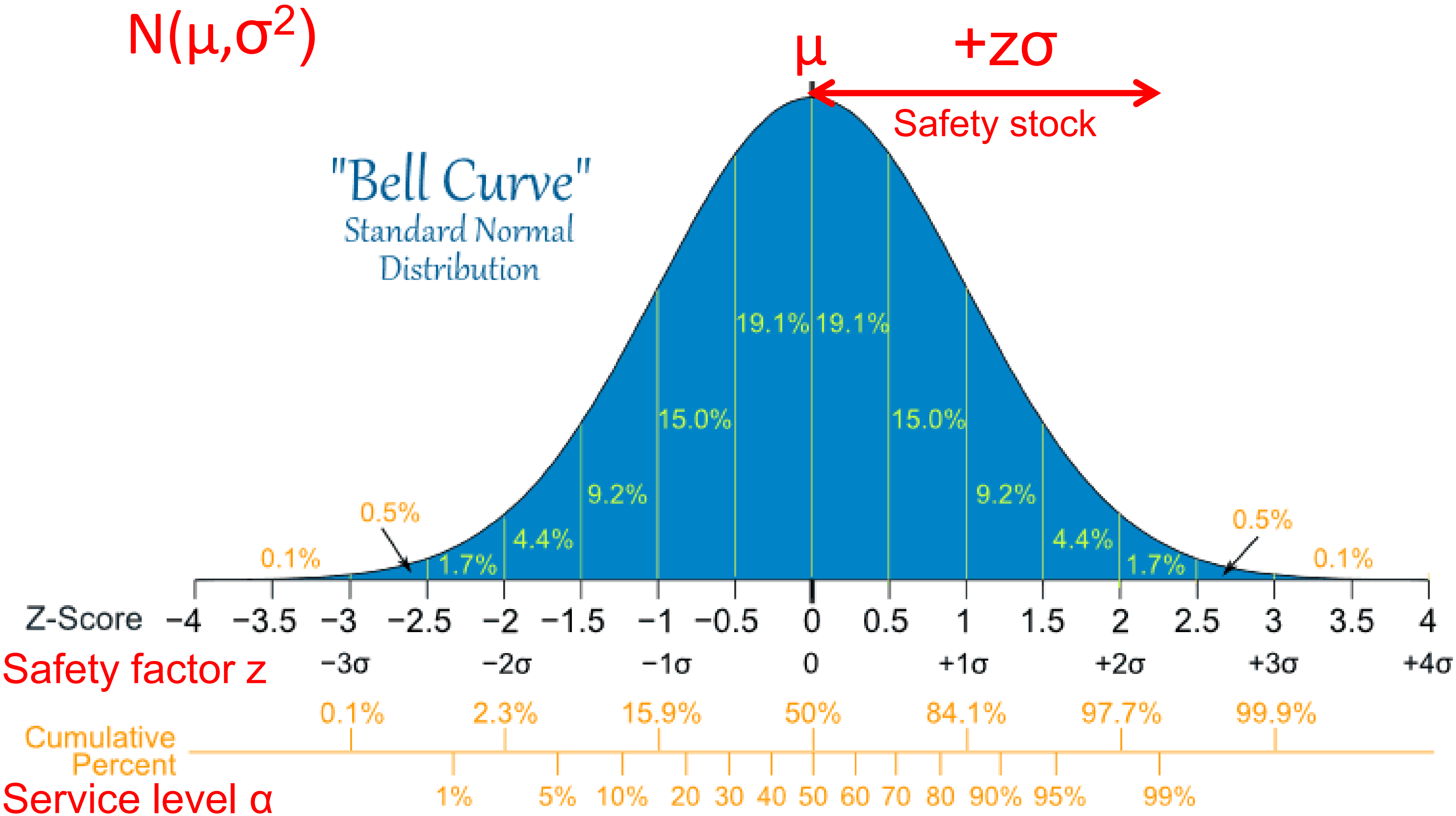


$\mu_T = 5 * 21.3 = 106.5$
$\sigma_T = \sqrt{5} * 0.9 = 2.01$



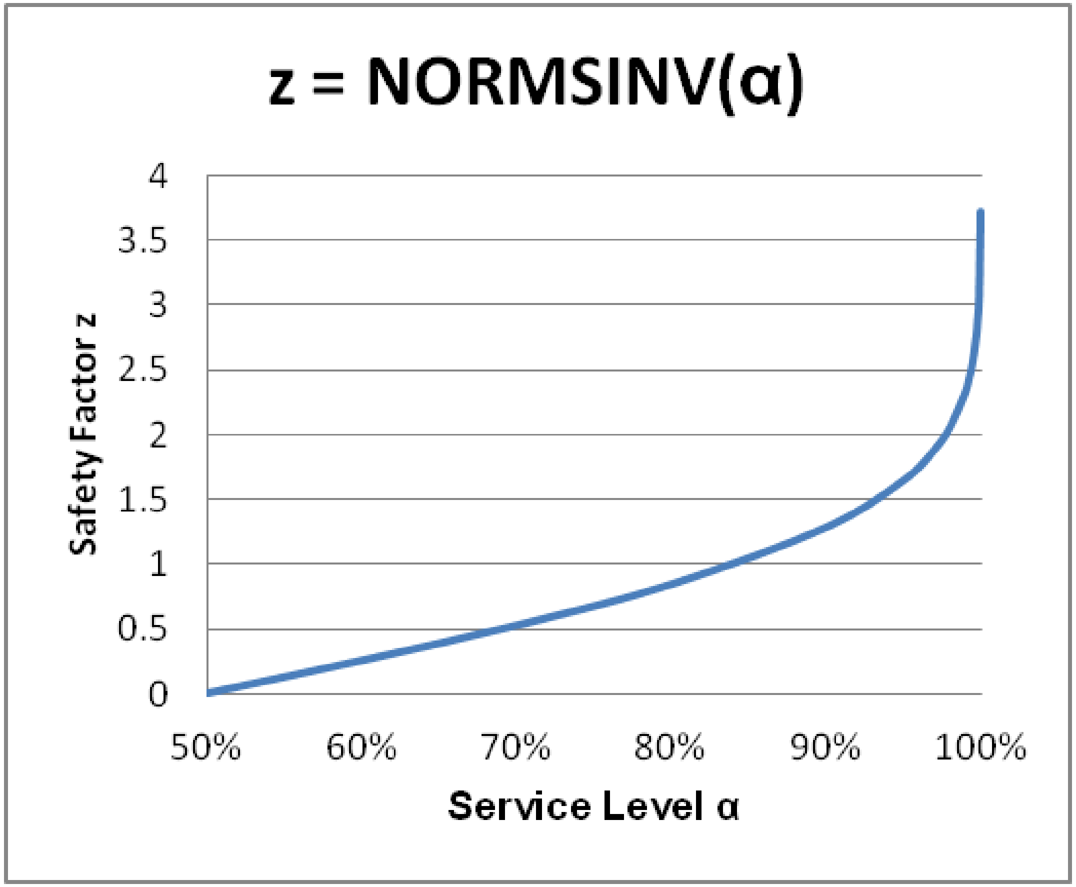
$\mu = 21.3, \sigma = 0.9$

Service Level for Normal Distribution



In Excel: **z = NORMSINV(α)**

Tradeoff: Service Level vs. Safety Stock

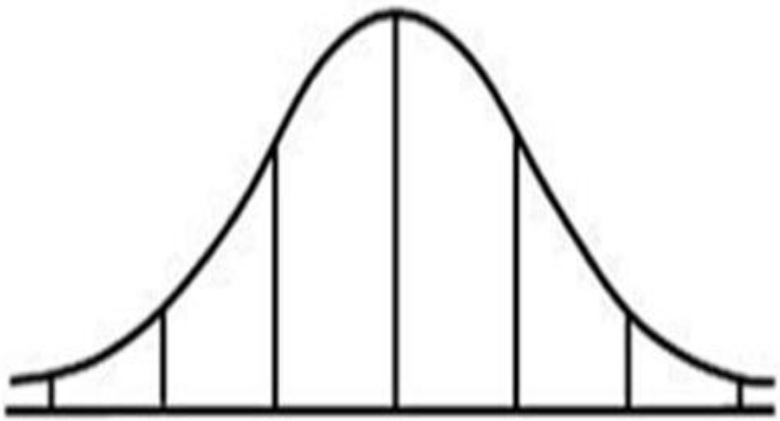


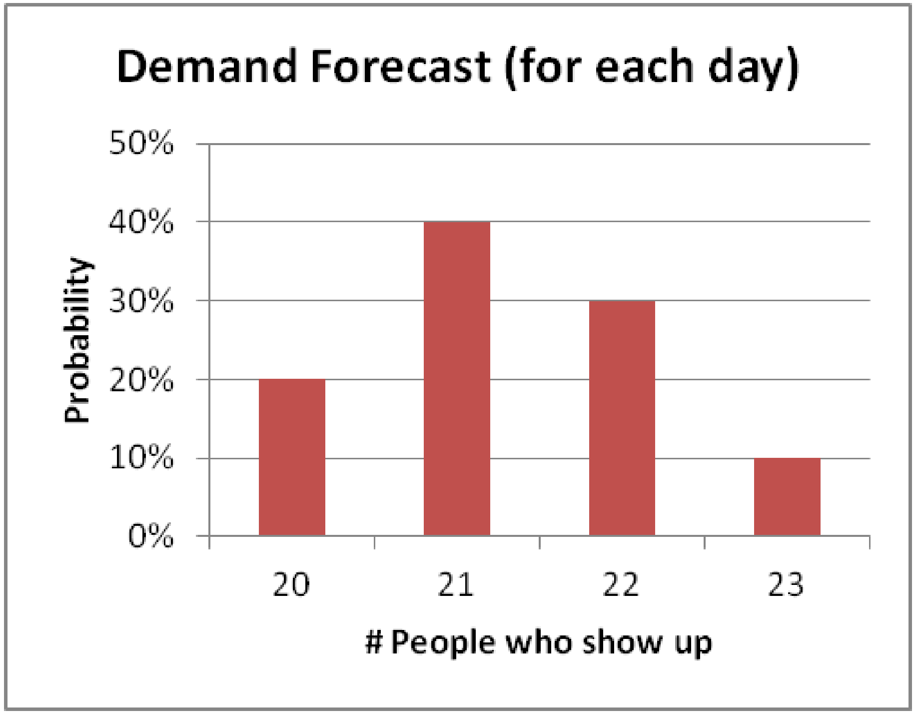
⇒ Relationship is nonlinear when the service level is close to 1; i.e., need disproportionately high safety stock to achieve very high service level

Example: Vaccine Clinic

If you reorder every T=5 days:

Aggregate demand: $D_T \approx N(\mu_T, \sigma_T^2)$





$\mu_T = 5 * 21.3 = 106.5$

$\sigma_T = \sqrt{5} * 0.9 = 2.01$

Service level α = 97.7% (safety factor z = 2)

$\Rightarrow$ Base stock level S = 106.5+4.02 = 110.5

$\mu = 21.3$, $\sigma = 0.9$

Base Stock Policy

1. Determine review period T
   - EOQ: $T = \sqrt{\frac{2K}{h\mu}}$
2. Find aggregate demand over T
   - Use daily demand data & approximate with Central Limit Theorem => $N(\mu_T , \sigma_T)$
3. Find safety factor z
   - Given service level α, z = NORMSINV(α)
4. Compute base stock level $S = \mu_T + z\ \sigma_T$

Base Stock Policy with Lead Time

1. Determine review period T
   - EOQ: $T = \sqrt{\frac{2K}{h\mu}}$
2. Find aggregate demand over T+L
   - Use daily demand data & approximate with Central Limit Theorem => $N(\mu_{T+L}, \sigma_{T+L})$
3. Find safety factor z
   - Given service level α, z = NORMSINV(α)
4. Compute base stock level $S = \mu_{T+L} + z\,\sigma_{T+L}$

Periodic vs. Continuous Review

- Period review:
  Base stock policy
- At each review period T, order up to base stock level S
- $$T = \sqrt{\frac{2K}{h\mu}}$$
  $$S = \mu_{T+L} + z\sigma_{T+L}$$

- Continuous Review: (R,Q) policy
- When inventory dropps below reorder point R, place new order with order quantity Q
- $$R = z\sigma_{T+L}$$
  $$Q = \sqrt{\frac{2K\mu}{h}}$$

Summary: Inventory Models

Model	EOQ	Newsvendor	Base Stock	(R,Q)
Decision variable	•Order quantity Q •(Order period T)	•Order quantity Q	•Review period T •Order-up-to level S	•Reorder point R •Order quantity Q
Demand	Constant	Stochastic	Stochastic	Stochastic
Lead time	0	0	L>0	L>0
Horizon	Infinite	Single	Infinite (periodic review)	Infinite (con-tinuous review)
Optimal solution	$Q = \sqrt{\frac{2K\mu}{h}}$ $\left(T = \frac{Q}{\mu} = \sqrt{\frac{2K}{h\mu}}\right)$	$P(D \leq Q) = \frac{c_u}{c_u + c_o}$	$T = \sqrt{\frac{2K}{h\mu}}$ $S = \mu_{T+L} + z\sigma_{T+L}$	$R = z\sigma_{T+L}$ $Q = \sqrt{\frac{2K\mu}{h}}$

Project discussion

Things to consider:

- What is the inventory?
  - Could be raw material, finished goods, workforce, etc.
  - E.g., oxygen, manure/flies, recyclable material/fleet of collectors, charcoal…
- What is the setup? Which inventory model might be appropriate for this setup?
  - Demand pattern: Constant or stochastic?
  - Order policy: One-shot or multi-period?
- What demand data is available?
  - If not available, what data should be collected?