Queuing Formula: Single Server

Little's law: $L = \lambda W$

Utilization: $\rho = \frac{\lambda}{\mu} = \lambda\tau$

$$W = \text{expected wait time in system} = D + \frac{1}{\mu} = D + \tau$$

$$D = \text{expected wait time in queue for G1/G/1}$$

$$\cong \frac{\rho}{1-\rho}\left(\frac{1}{\mu}\right)\left(\frac{SCV_a + SCV_s}{2}\right)$$

$$\cong \frac{\rho}{1-\rho}(\tau)\left(\frac{SCV_a + SCV_s}{2}\right)$$

Queuing Formula: Multiple Servers

For M/M/k system: $D = \left(\frac{1}{k\mu - \lambda}\right)\left(\frac{(k\rho)^k}{(1-\rho)k!}\right)\pi_0 = \left(\frac{\rho}{1\text{-}\rho}\right)\left(\frac{1}{\mu}\right)\left(\frac{(k\rho)^{k-1}}{(1-\rho)k!}\right)\pi_0$

where $\pi_0 = \dfrac{1}{\dfrac{(k\rho)^k}{(1-\rho)k!} + \sum_{i=0}^{k-1}\dfrac{(k\rho)^i}{i!}}$ and $\rho = \dfrac{\lambda}{k\mu}$

$D =$ expected wait time in queue for M/G/k

$\cong$ (expected wait time in queue for M/M/k) $\left(\frac{1+SCV_s}{2}\right)$

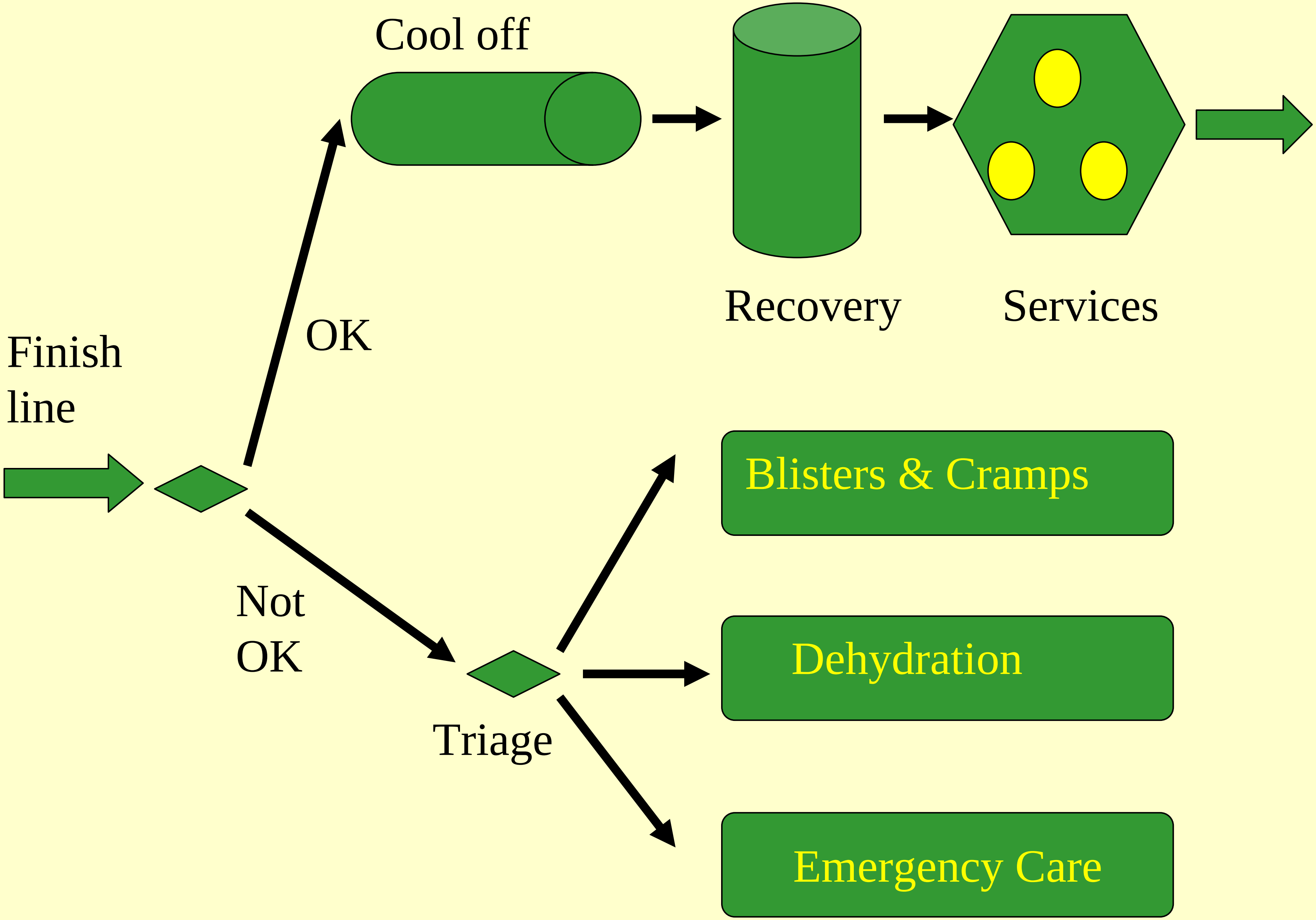
A quick approx. is $D \approx \frac{\rho}{1-\rho}\left(\frac{\tau}{k}\right)\left(\frac{SCV_a + SCV_s}{2}\right)$

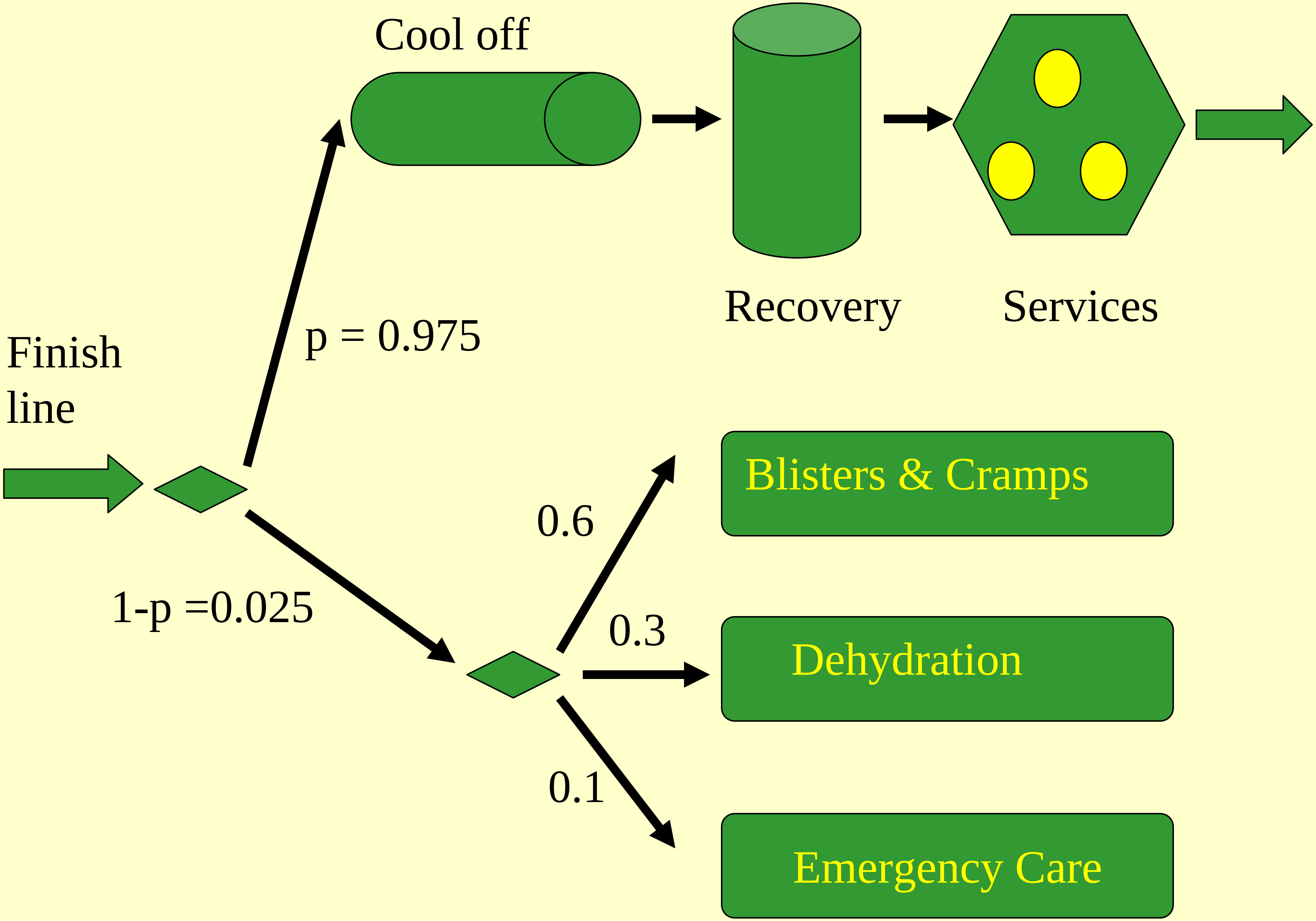
Queuing Formula: Multiple Servers, No Queue

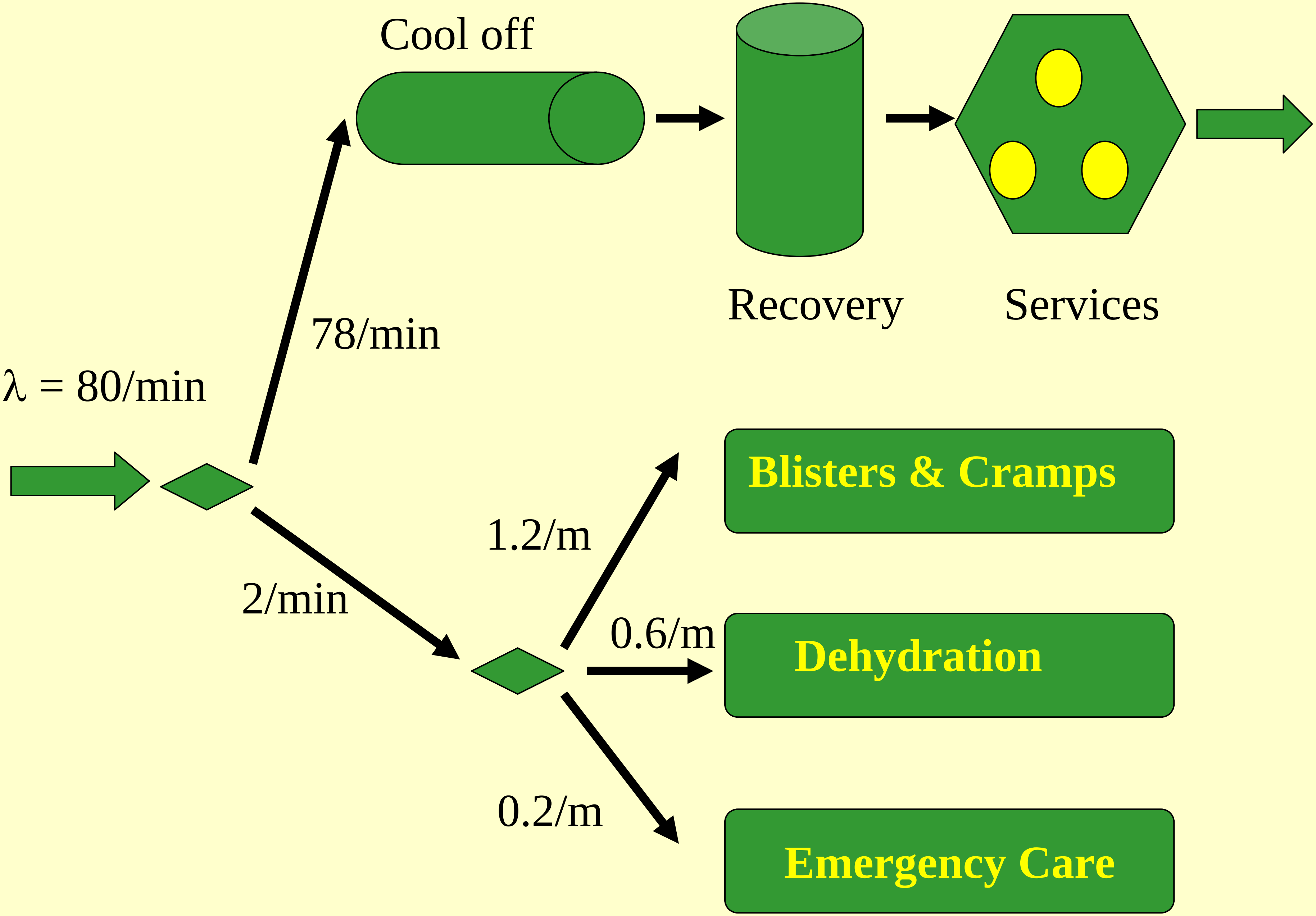
For M/G/k/k: Pr [# in system = n] = $\dfrac{\dfrac{(\lambda\tau)^n}{n!}}{\displaystyle\sum_{i=0}^{k}\dfrac{(\lambda\tau)^i}{i!}}$ and

Pr[# in system = k] = "loss probability"

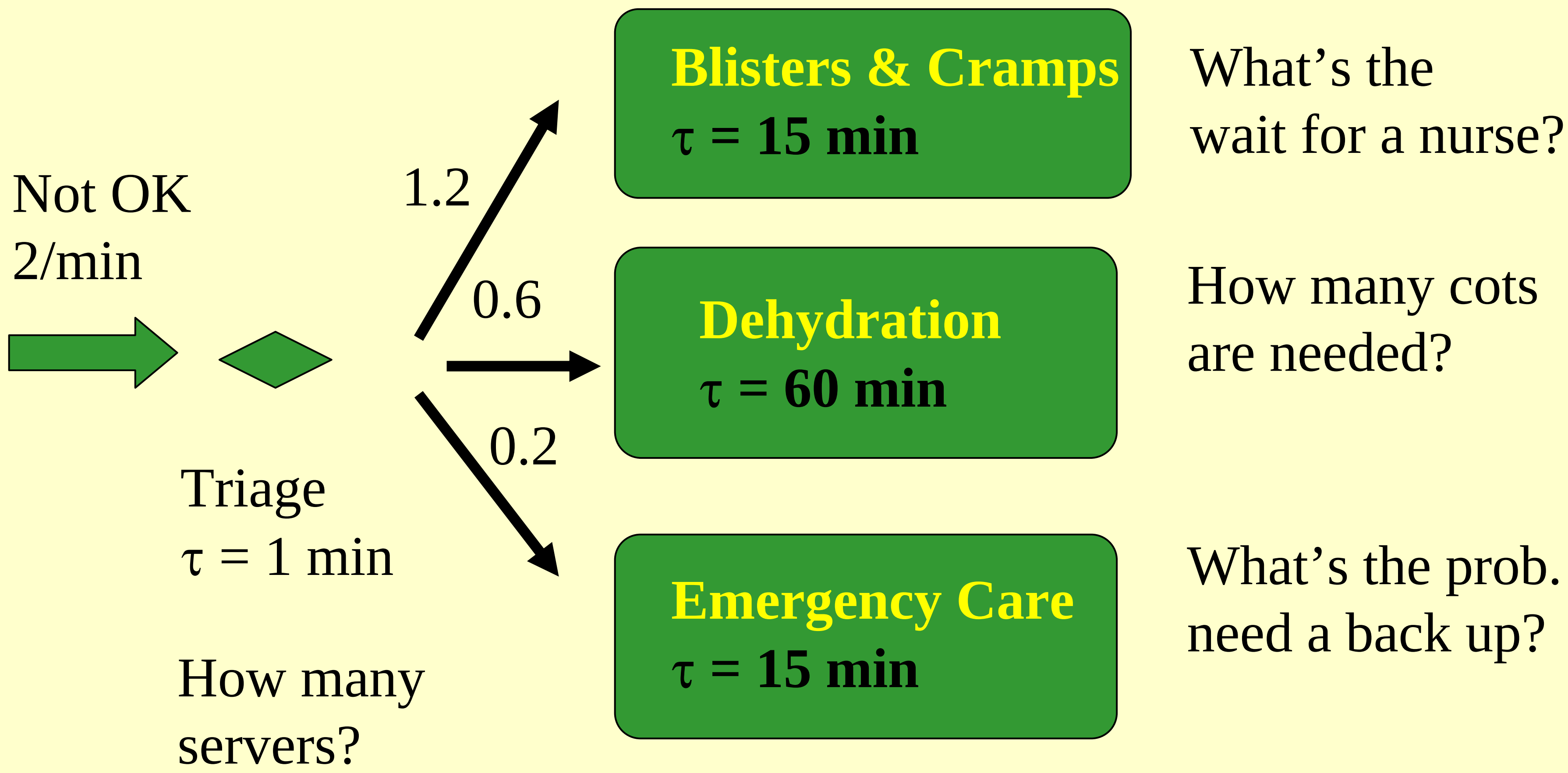
For M/G/∞, number in the system is Poisson with mean = $\lambda\tau$ and with variance = $\lambda\tau$


Cool off
Recovery
Services
OK
Finish
line
Not
OK
Triage
Blisters & Cramps
Dehydration
Emergency Care

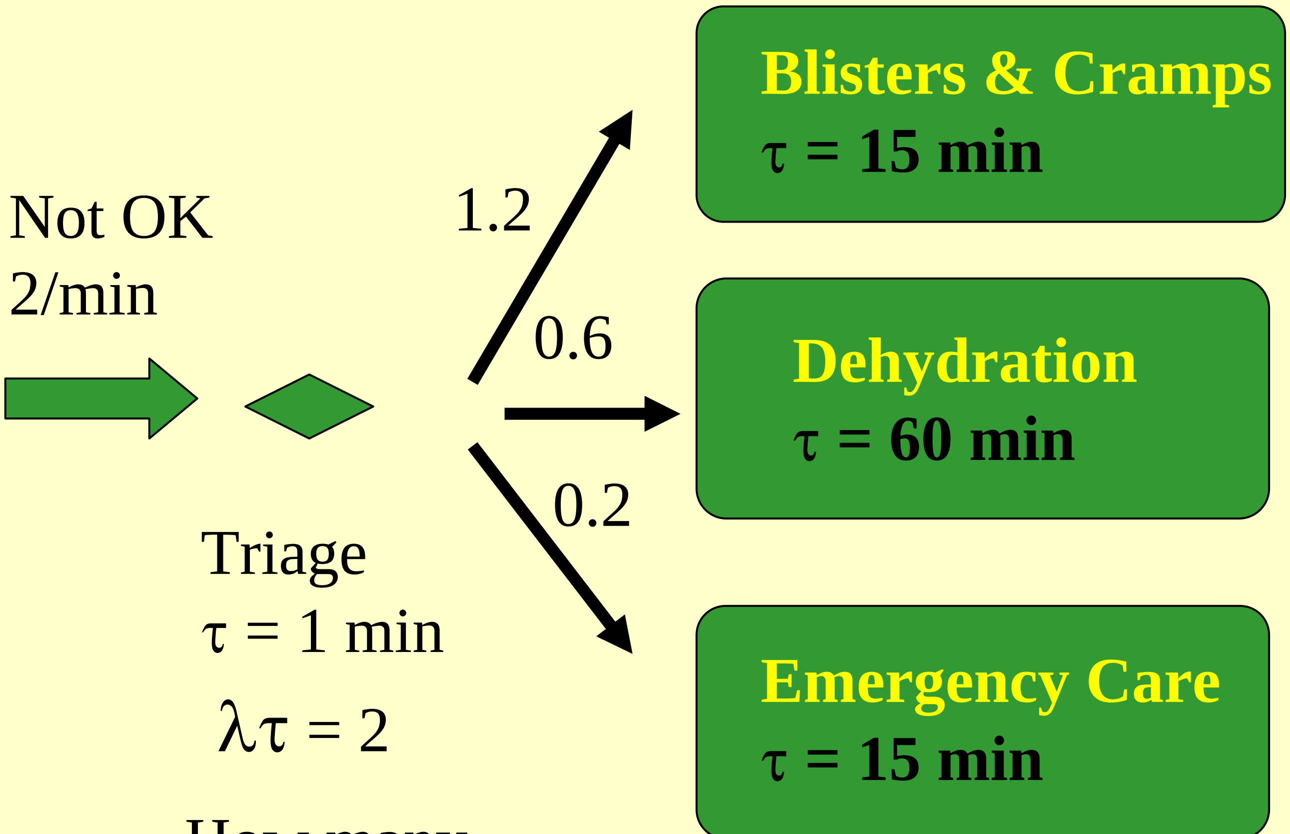

Cool off
Recovery
Services
Finish line
p = 0.975
1-p =0.025
0.6
0.3
0.1
Blisters & Cramps
Dehydration
Emergency Care



Cool off
Recovery
Services
78/min
λ = 80/min
Blisters & Cramps
1.2/m
2/min
0.6/m
Dehydration
0.2/m
Emergency Care


Not OK
2/min
Triage
τ = 1 min
How many
servers?
1.2
0.6
0.2
Blisters & Cramps
τ = 15 min
Dehydration
τ = 60 min
Emergency Care
τ = 15 min
What's the
wait for a nurse?
How many cots
are needed?
What's the prob.
need a back up?

$\lambda\tau$ = workload per minute



$\lambda\tau = 2$

How many servers?

What's the wait for a nurse?

$\lambda\tau = 18$

How many cots are needed?

$\lambda\tau = 36$

What's the prob. need a back up?

$\lambda\tau = 3$

$\lambda\tau$ = workload per minute

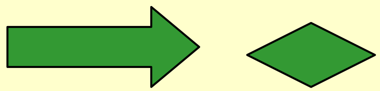
Triage

τ = 1 min

How many servers?

Not OK

2/min



M/G/k queue

$\lambda\tau = 2$

$k = 3$

$\rho = 0.66$

expected wait time in queue

$$\approx \frac{\rho}{1-\rho}\left(\frac{\tau}{k}\right)\left(\frac{SCV_a + SCV_s}{2}\right)$$

$\lambda\tau$ = workload per minute

1.2/min

Blisters & Cramps

τ = 15 min

What's the wait for a nurse?

M/G/k queue

$\lambda\tau = 18$

$k > 18$

$\rho = \lambda\tau/k$

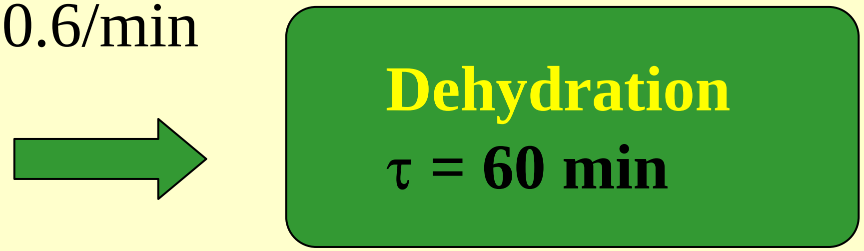
expected wait time in queue

$$\approx \frac{\rho}{1-\rho}\left(\frac{\tau}{k}\right)\left(\frac{SCV_a + SCV_s}{2}\right)$$

$\lambda\tau$ = workload per minute

How many cots are needed?

0.6/min



Model as
M/G/k/k or
M/G/inf queue
$\lambda\tau = 36$
E[# runners] = 36
V[# runners] = 36
σ [# runners] = 6

$\lambda\tau$ = workload per minute

What's the prob. need a back up?

0.2/min → **Emergency Care**

$\tau = 15$ min

Model as M/G/k/k

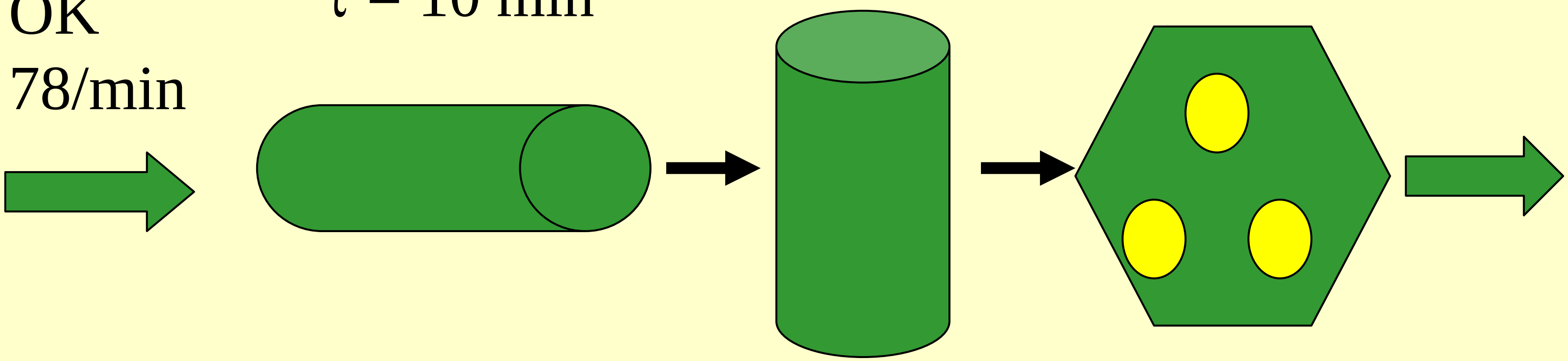
$\lambda\tau = 3$

Loss probability = Pr [# in system = k] = $$\frac{(\lambda\tau)^k}{k!} \Bigg/ \sum_{i=0}^{k} \frac{(\lambda\tau)^i}{i!}$$

k	1	2	3	4	5	6	7
Pr[#=k]	0.75	0.53	0.35	0.21	0.11	0.05	0.02

OK
78/min

Cool off
τ = 10 min



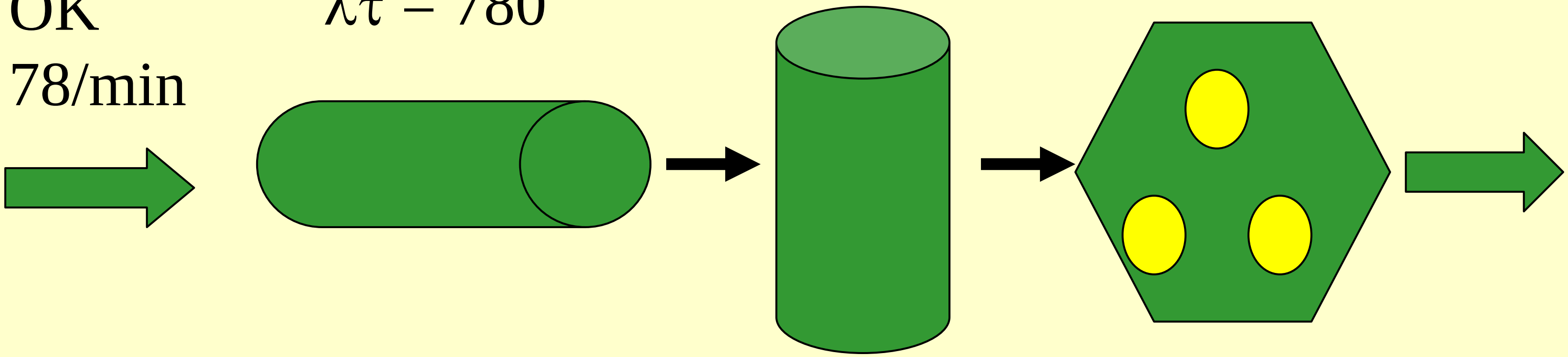
Recovery
τ = 20 min

Services
Rest rooms (5 min)
Phones (2 min)
Clothes (1 min)
Food (2 min)

OK
78/min

Cool off
τ= 10 min
$\lambda\tau$ = 780

Recovery
τ= 20 min
$\lambda\tau$ = 1560



Services
Rest rooms: $\lambda\tau$ = 390
Phones: $\lambda\tau$ = 156
Clothes: $\lambda\tau$ = 78
Food: $\lambda\tau$ = 156

$\lambda\tau$ = workload per minute