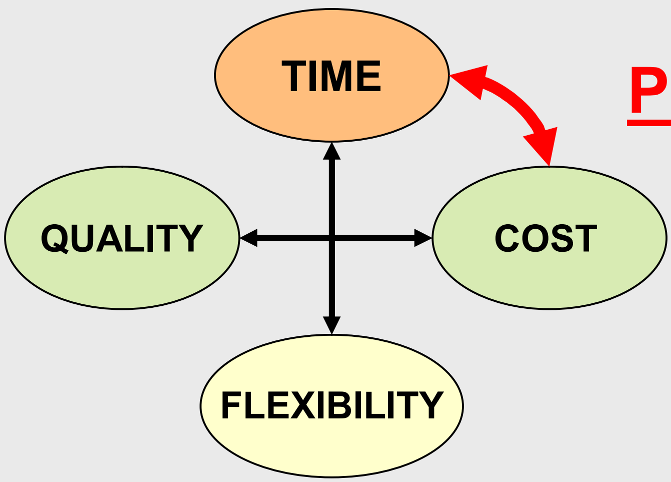
# Class 6: Inventory Lecture




**Design Decision:**
**CAPACITY (cf. Class 3)**

**Planning/Control Decision:**
**INVENTORY**

**(Oct 2001: $1.16 trillion in US!)**

**Trade-off: Inventory Cost Vs. Service Level**

# From the Trenches...

Too much:

- Liz Claiborne experiences "unexpected earnings decline as a consequence of 'higher-than-expected excess inventories'" – Agins, Teri. “Liz Claiborne Seems to Be Losing Its Invisible Armor,” *The Wall Street Journal*, July 19 1993.

- “On Tuesday, the network-equipment giant Cisco provided the grisly details behind its astonishing $2.25 billion inventory write-off in the third quarter” – Barrett, Larry. “Cisco’s $2.25 Billion Mea Culpa,” *News.com*, May 9 2001, http://cnet.news.com (accessed June 3, 2004).

Too little:

- IBM struggles with shortages in ThinkPad line due to ineffective inventory management – Hays, Laurie. “IBM to Slash Prices Up to 27% on Business PCs,” The Wall Street Journal, August 24 1994.

- “Since 1990 we have designated the Department of Defense’s management of its inventory, including spare parts, as high risk because […] its management systems and procedures were ineffective.” – US General Accounting Office. “Army Inventory: Parts Shortages Are Impacting Operations and Maintenance Effectiveness,” August 2001.

# Why Inventory Costs Money

- Cost of (stuck) capital
- Obsolescence
- Storage
- Insurance
- Security
- Theft (Shrinkage)

→ Typical per annum inventory holding cost:

# Financial Inventory Metrics

$$\text{Inventory Turns} = \frac{\text{COGS}}{\text{Inventory Value}}$$

COGS ← Earnings or P & L

Inventory Value ← Balance sheet

$$\text{Inventory Cost / Unit} = \frac{\text{Inventory Value x Holding Cost}}{\text{COGS}} = \frac{\text{Holding Cost}}{\text{Inventory Turns}}$$

Example: 10k filings, 2002 ($M)

| | *Wal Mart Stores Inc.* | *Kmart Corp.* |
|---|---|---|
| **Inventory** | $22,749 | $4,825 |
| **C.O.G.S** | $171,562 | $26,258 |

# Why Hold Inventory? How Much?

| *Type of Inventory* | *Decision Tool* |
|---|---|
| **Safety Inventory** | **Newsboy Model** |
| **Cycle Inventory** | **EOQ Model** |
| **Seasonal Inventory** | **Buildup Diagram** |
| **Speculative Inventory** | **Finance** |
| **In-Process/Pipeline Inventory** | **Little's Law** |
| **Marketing/Shelf Inventory (Retail)** | **Experience** |

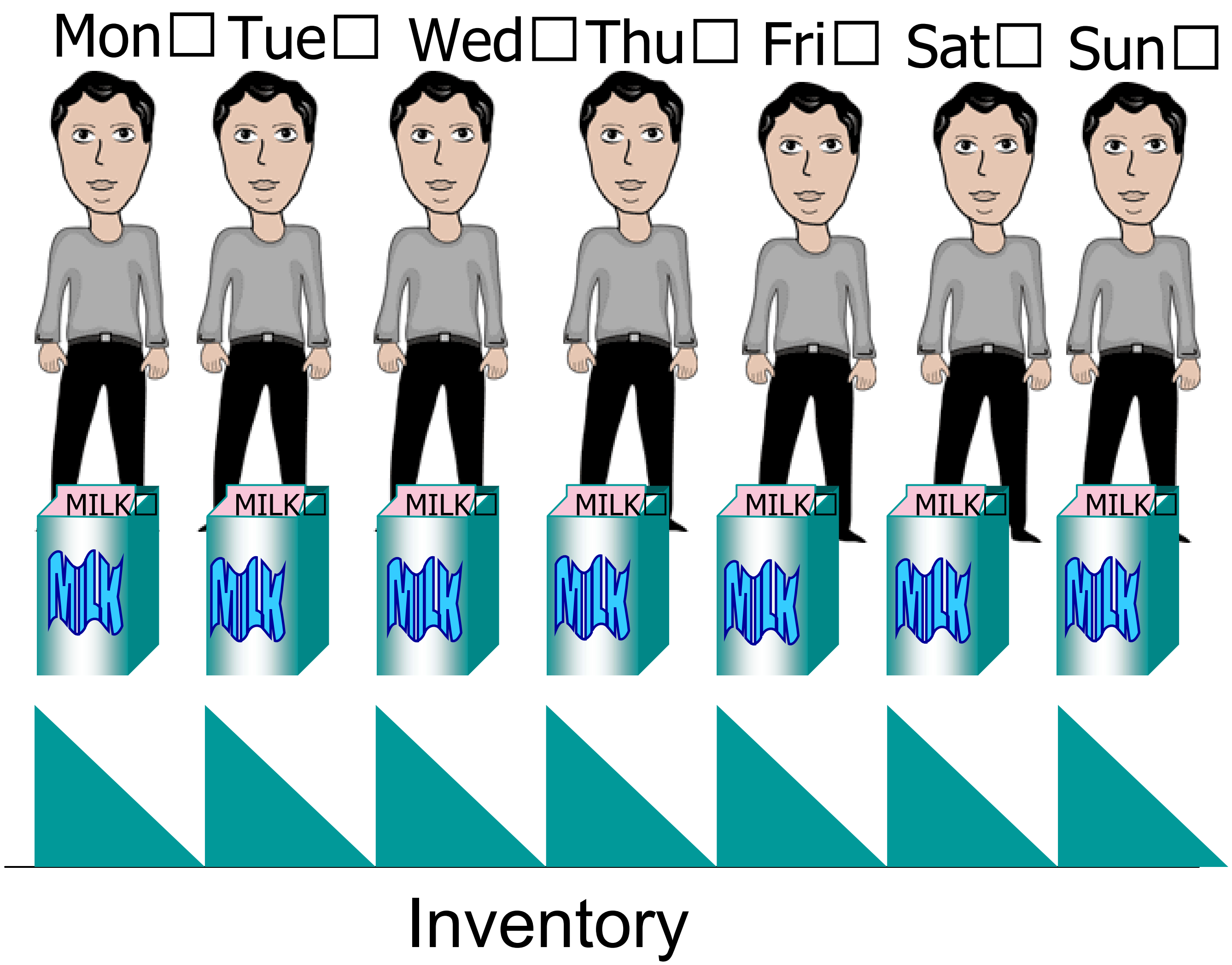
# Economic Order Quantity Model

- Set order size for repetitive ordering process with fixed order cost

- Trade-off:
  - Order size too large (too much average inventory) versus
  - Order size too small (too much ordering cost)

- Examples:
  - Ordering/Inventory replenishment policy;
  - Batch size on machine with setup time…

# Running to the Store a Lot...

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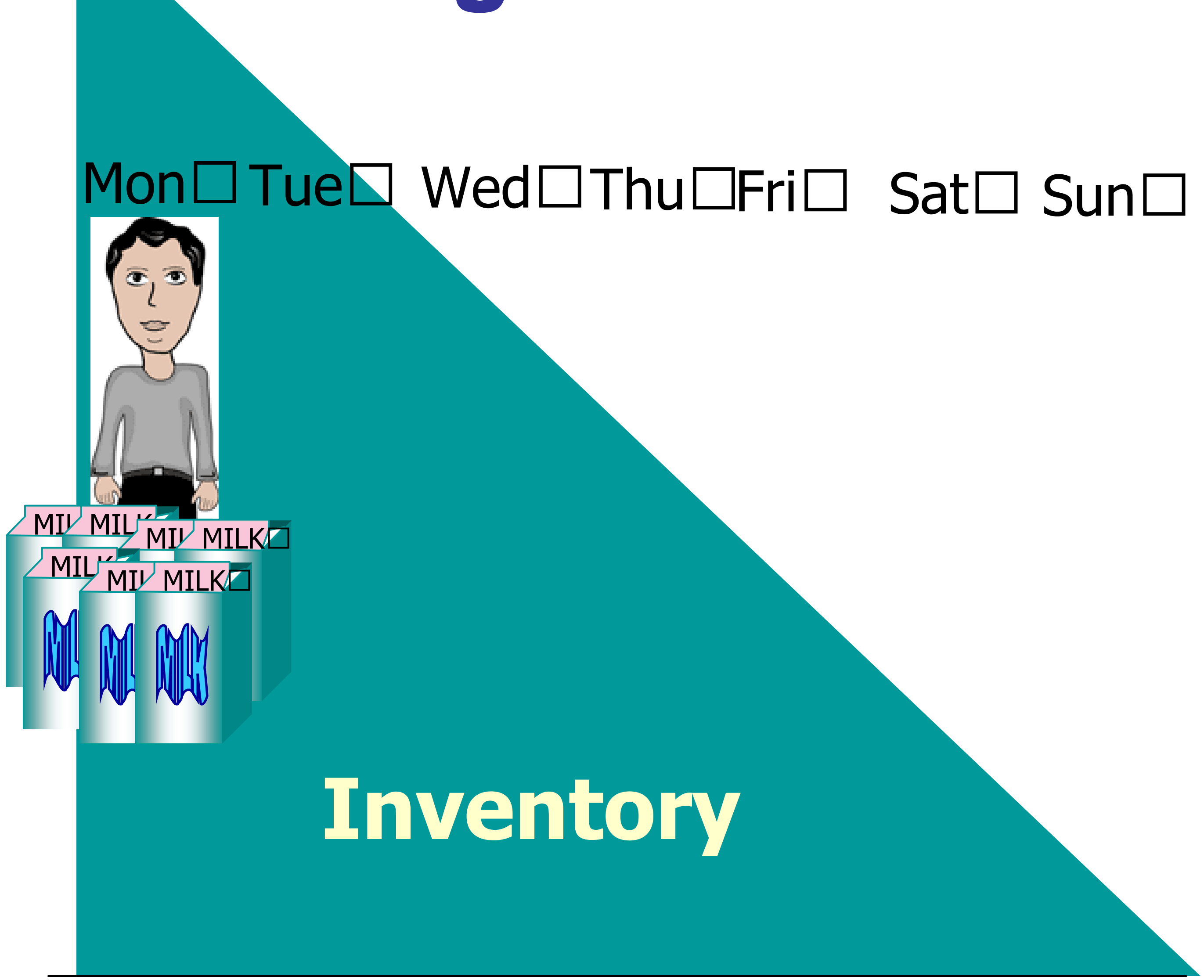
# ...Vs. Running to the Store a Little



# EOQ Model Parameters

- Q = Order Quantity *decision*

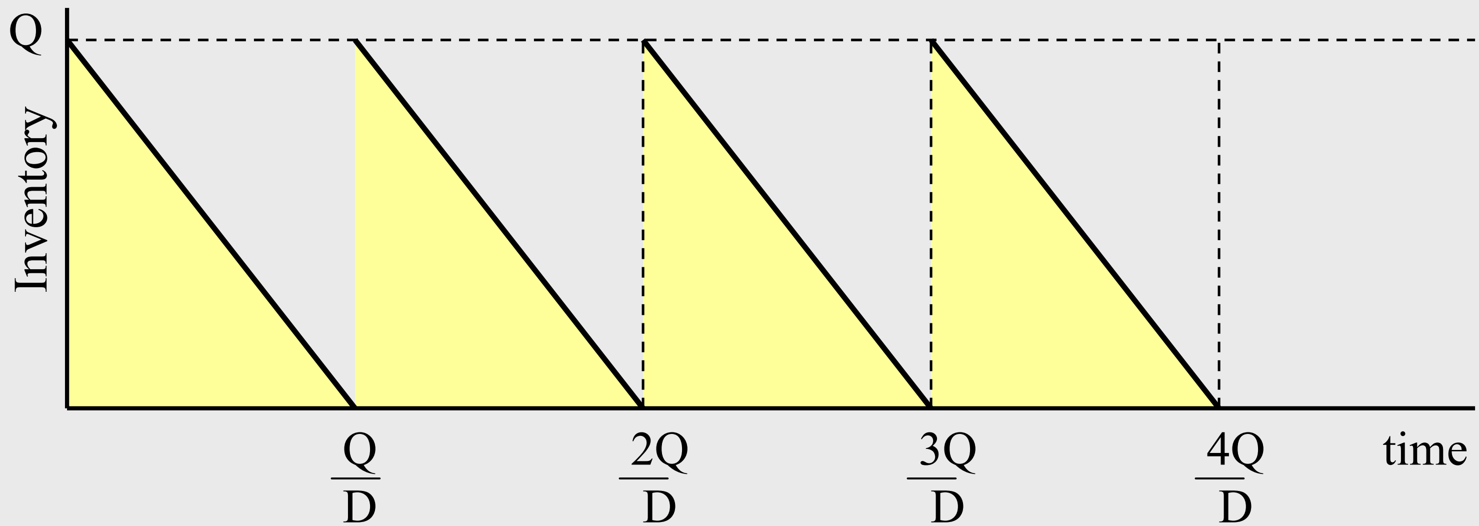
- D = Demand Rate (units/time)
- C = Purchasing Cost ($/unit) *parameters*
- F = Fixed Order Cost ($)
- H = Inventory Holding Cost (% p.a.)

**Assumptions:** - constant, deterministic demand
- instantaneous replenishment

# EOQ Model Derivation

- **Inventory Cost** $H \cdot \frac{C \cdot Q}{2}$ **; Order Cost** $F \cdot \frac{D}{Q}$ **;**

- **Total Cost** $V = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2}$

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# EOQ Formula

- **Set first derivative to 0:** $$\frac{\partial V}{\partial Q} = -\frac{DF}{Q^2} + \frac{CH}{2} = 0$$

- **This yields:**

$$Q^* = \sqrt{\frac{2 \cdot DF}{CH}}$$

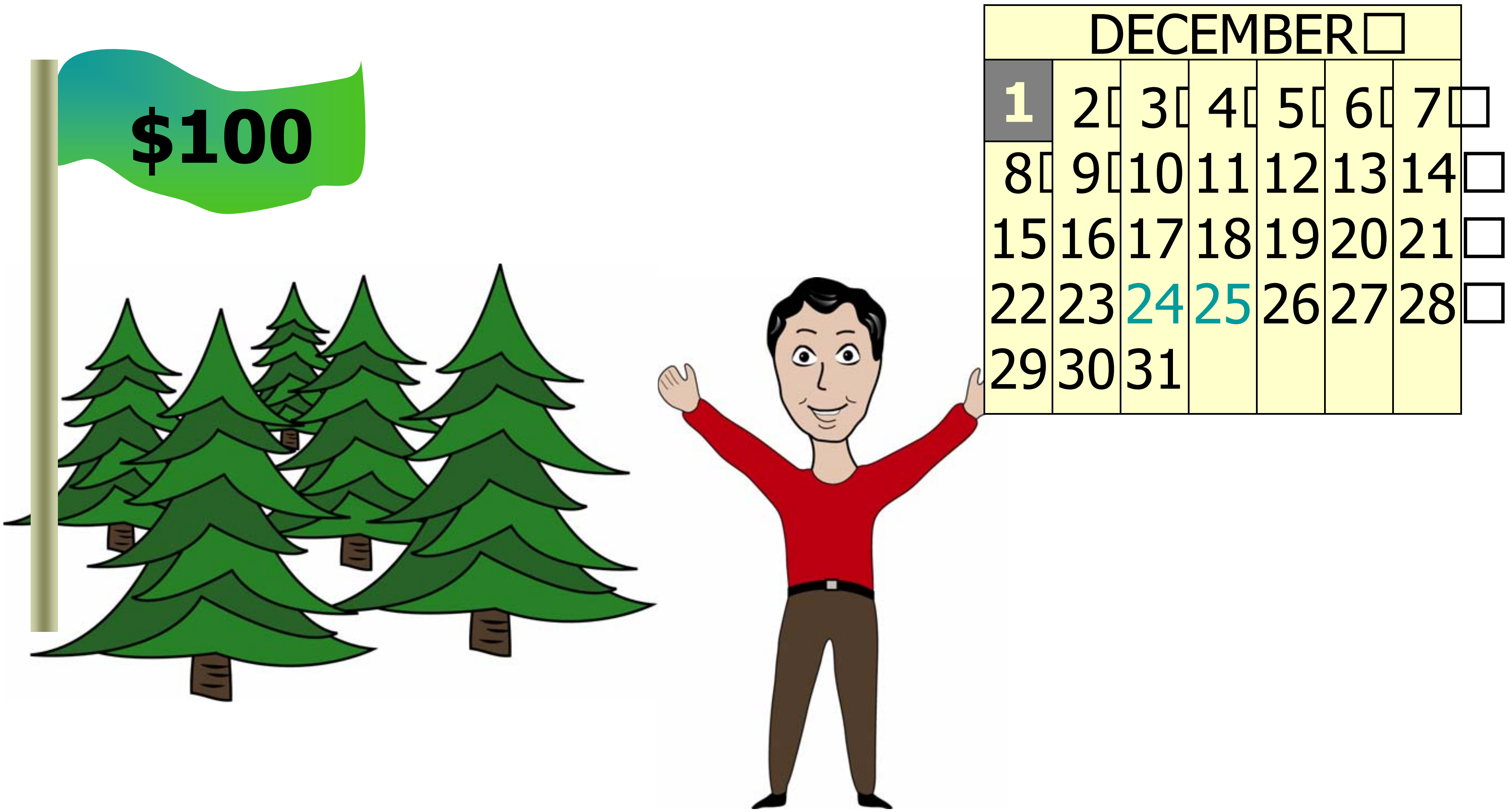
# EOQ Example

A PC assembly operation procures its 128Mb memory chips at $45 each (purchase + shipment cost) from a foreign vendor; in addition each order also costs $500 in customs fees. Assuming a constant demand of 400 chips per week and an inventory holding cost of 45%, how often would you order?

# Newsvendor Model

- One time decision under uncertainty
- Trade-off:
  - Ordering too much (waste, salvage value < cost) versus
  - Ordering too little (excess demand is lost)
- Examples:
  - Restaurant;
  - Fashion;
  - High Tech;
  - Inventory decisions…

# Christmas Tree Problem

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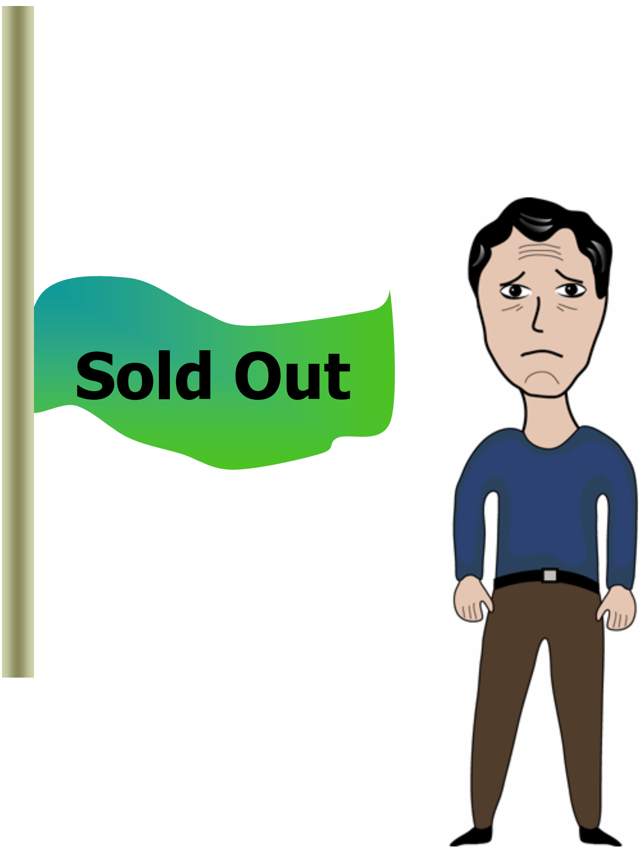
# Ordering Too Many...


| DECEMBER | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | **25** | 26 | 27 | 28 |
| 29 | 30 | 31 | | | | |

# ...Versus Ordering Too Few




| DECEMBER | | | | | | |
|---|---|---|---|---|---|---|
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| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | | | | |

# Newsvendor Model Parameters

- q = Order Quantity *decision*

- c = Unit Cost
- r = Unit Revenue
- b = Unit Salvage Value

} *parameters* (r > c > b)

- d = Demand (unknown) *random variable*

# Newsboy Objective

**<u>IF d > q</u>**
**(demand > quantity ordered)**

**Opportunity cost:**
**$(r - c) \times (d - q)$**

**<u>IF q > d</u>**
**(quantity ordered > demand)**

**Disposal cost:**
**$(c - b) \times (q - d)$**

**Objective:**
**minimize expected opportunity + disposal cost**

# Model Derivation

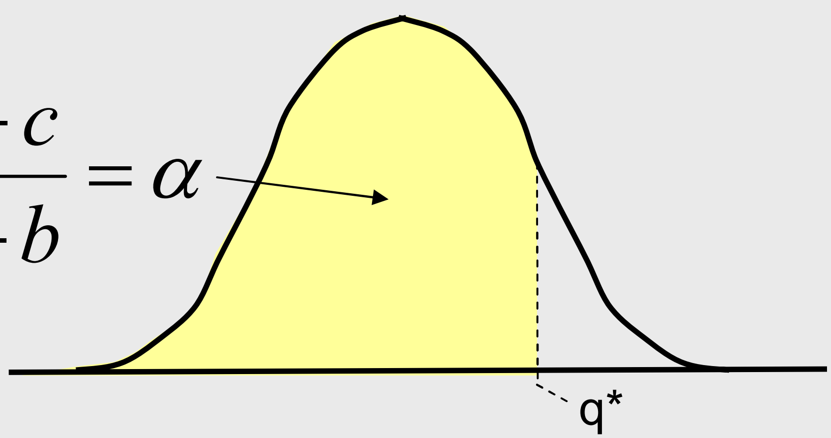
| | • IF d > q (demand > order qty) | • IF d < q (demand < order qty) |
|---|---|---|
| Profit: | $q \cdot (r - c)$ | $d \cdot (r-c) + (q-d) \cdot (b-c)$ |
| Incremental Analysis: $q \to q + 1$: | | |
| $\Delta$ Profit: | *r - c* | *b - c* |
| EAP: | $P(d > q) \cdot (r\text{-}c) + P(d \leq q) \cdot (b\text{-}c)$ | |

**As long as the *Expected Additional Profit* [EAP] is positive, it is lucrative to increase *q* to *q + 1 !!!***

# Newsvendor Formula

$$P(d < q^*) = \frac{r-c}{r-b} = \frac{r-c}{\underbrace{(r-c)}_{\text{cost of under-stocking}} + \underbrace{(c-b)}_{\text{cost of over-stocking}}} = \frac{u}{u+o}$$

$$\frac{r-c}{r-b} = \alpha$$




Demand Distribution

Remark: If d is Normal($\mu$,$\sigma$),

$q^* = \mu + k.\sigma$ with

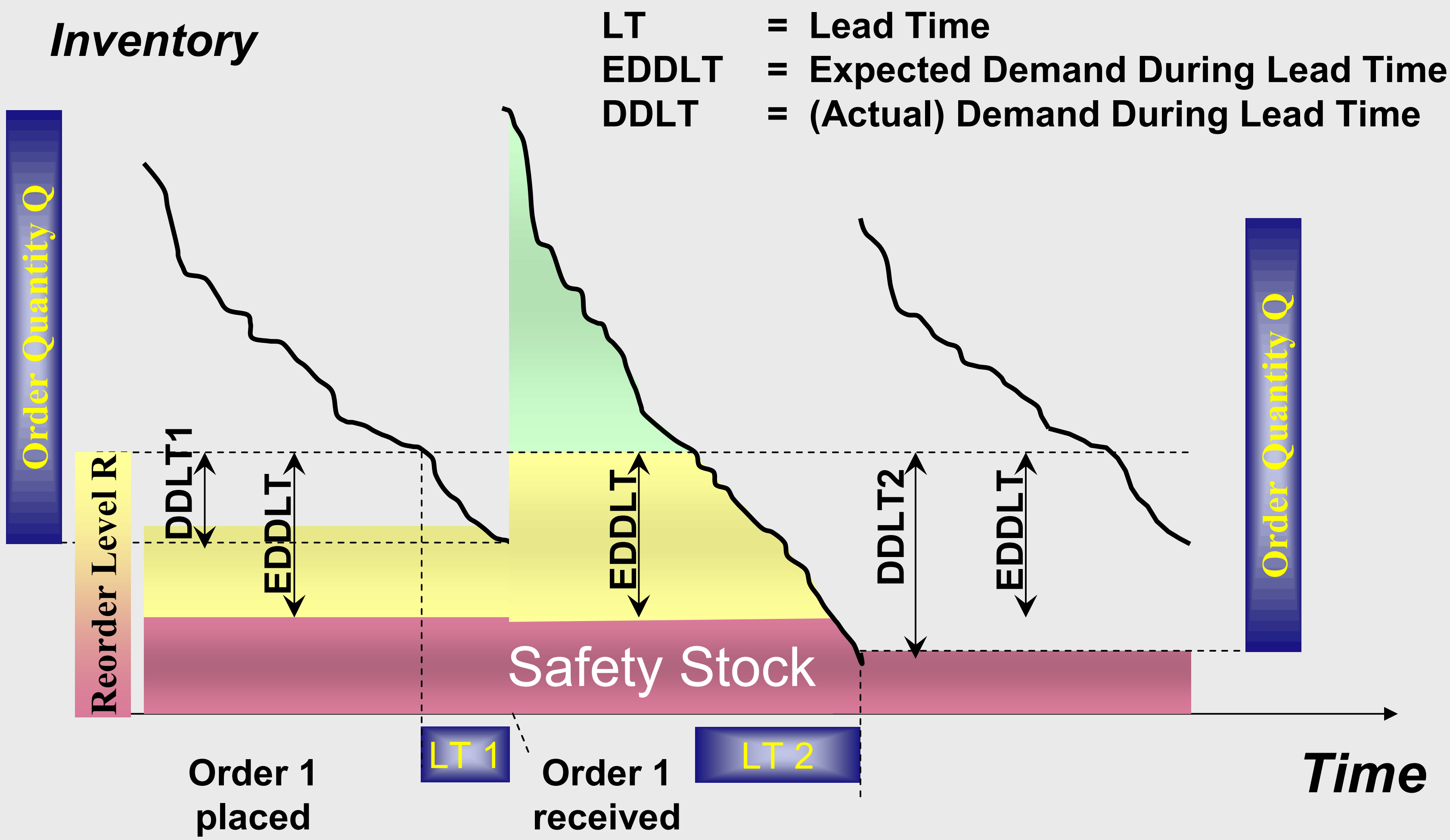
$\alpha$ = 95% → k = 1.64
$\alpha$ = 99% → k = 2.32
$\alpha$ = 99.9% → k = 3.09

# Newsvendor Example

Based on forecasts and marketing studies you are expecting a total lifecycle demand N(60,000;20,000) for a new product due to launch in the future. The product has a gross margin of $750 and a liquidation/disposal cost (for unsold inventory) of $250. Because of long lead-times you must commit orders to supplier for the entire product life-cycle now. How much should you order?

# Continuous Review System

"order Q whenever inventory reaches R"

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# (R,Q) Parameters

"order Q whenever inventory reaches R"

- Set Q as the EOQ solution
- Set R as the newsboy solution:

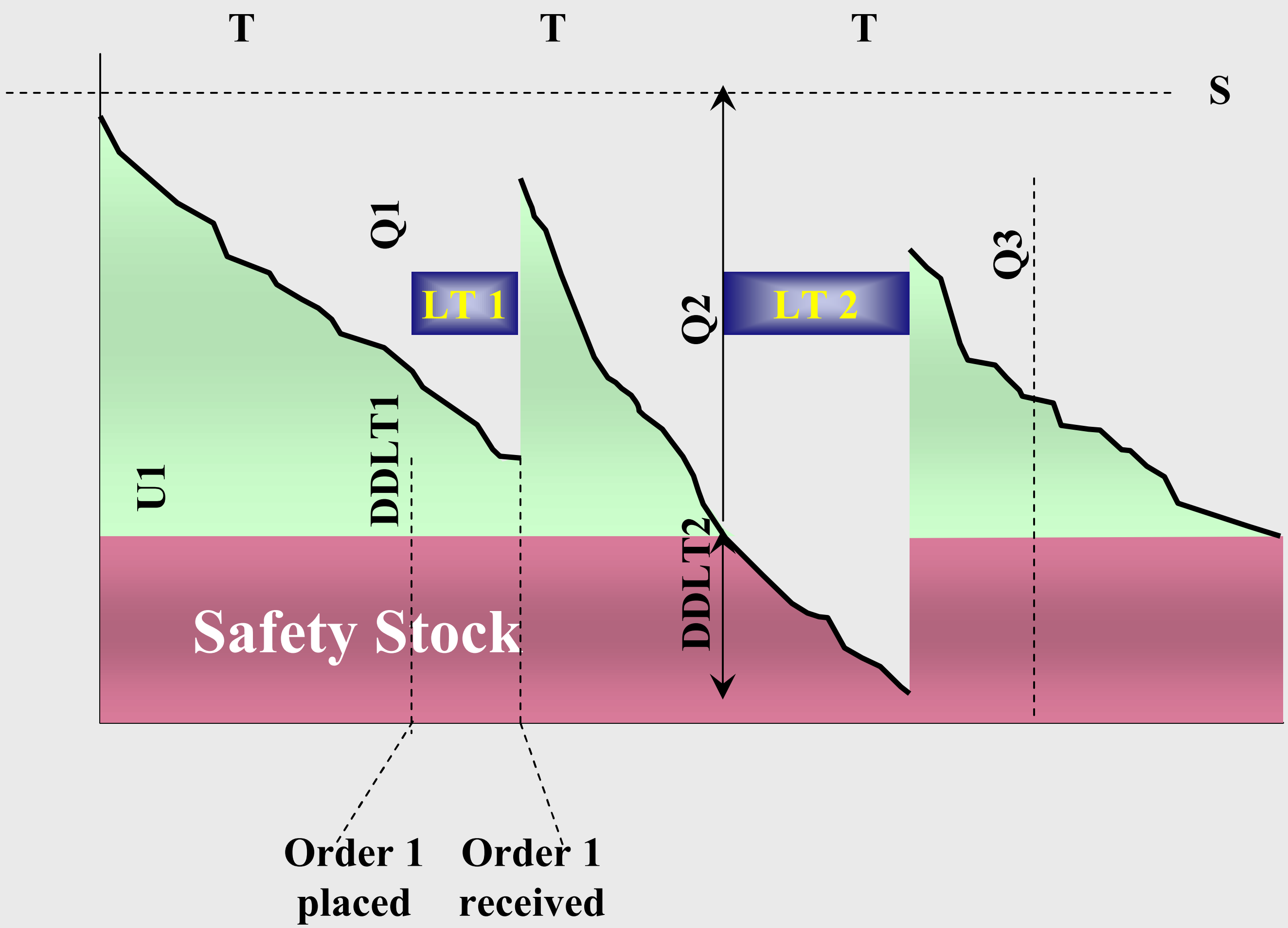
$$P(DDLT < R) = \alpha$$

where $\alpha$ is a desired service level (e.g. 95%)

<u>Example (cont'd):</u> if weekly demand for 128Mb chips is in fact N(400,80) and delivery time is 2 weeks, for a 95% service level:

# Periodic Review System

"order back to S every T time units"




LT = Lead Time
T = Cycle Time or Review Period
DDLT = Actual Demand During Lead Time
Qi = Order Size
S = Order Up To Level

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# (S,T) Parameters

"order back to S every T time units"

- Set T as the EOQ solution divided by the demand rate
- Set S as the newsboy solution:

$$P(DDLTRP < S) = \alpha$$

where: - $\alpha$ is the desired service level (e.g. 95%)

- DDLTRP = Demand During Lead-Time and Review Period

# Safety Stock Formula

- Under periodic and review systems, safety stock SS (under normally distributed demand) is given by:

$$SS = k\,\sigma$$

fractile depending on service level, e.g.
95% → k = 1.64
99% → k = 2.32
99.9% → k = 3.09

standard deviation of DDLT or DDLTRP

# Class 6 Wrap-Up

1. Financial inventory metrics: inventory turns, per unit inventory cost

2. Functions of inventory: seasonal, cyclical, safety, speculative, pipeline, shelf

3. EOQ & newsboy models

4. Continuous and discrete replenishment policies, safety stock formula