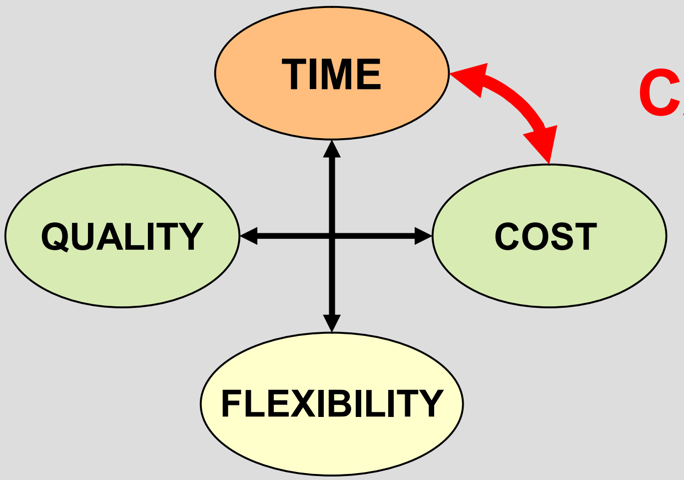
Class 3: Capacity Lecture



CAPACITY

too high:

- Capital expenses
- Labor expenses
- Resource waste
- Environmental damage
- Cash flow issues

too low:

- Customer wait / death
- Brand damage
- Customer dissatisfaction
- Lost sales
- Low employee morale
- High turnover

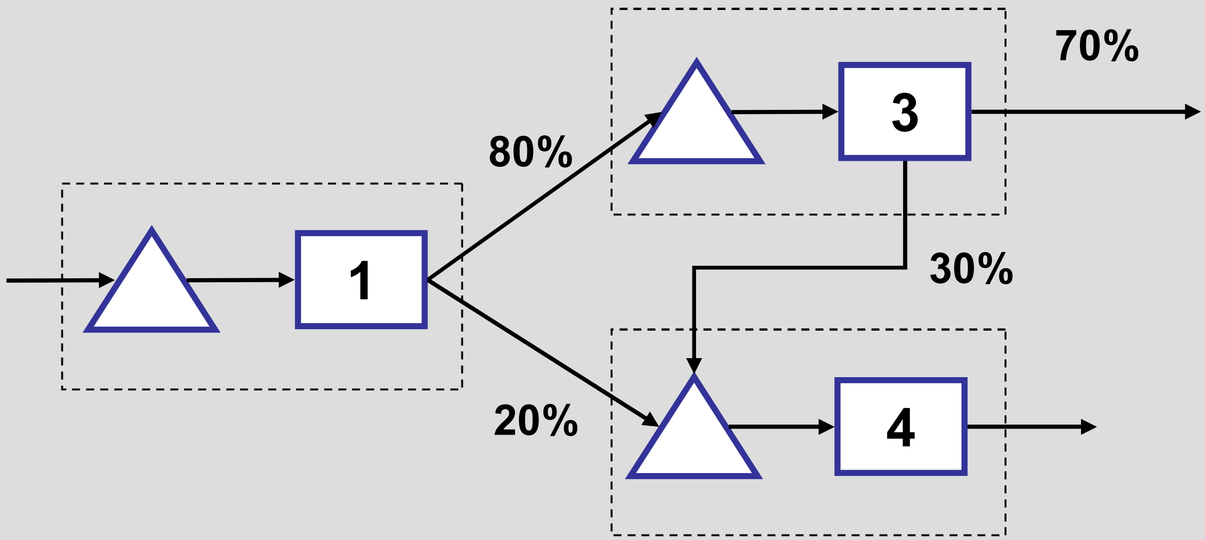
Typical Questions

- How many machines should be purchased?
- How many workers should be hired?
- Consequences of a 20% increase in demand?
- How many counters should be opened to maintain customer wait below 10 minutes?
- How many assembly stations are needed to maintain backorders below 20?
- How often will all 6 operating rooms be full?
- How will congestion at Logan change if a 5^{th} runway is built?

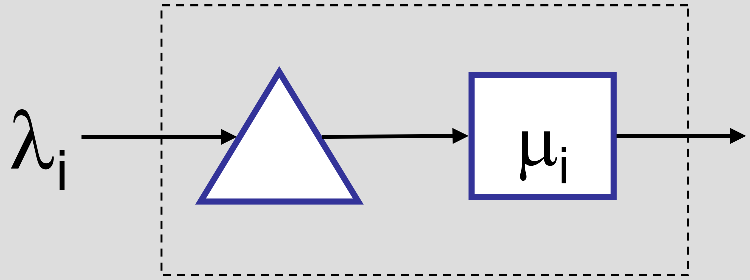
Methodology

	Step	
	Step 1:	Process Flow Diagram
This lecture	Step 2:	Demand and Capacity Analysis
This lecture	Step 3:	Congestion Analysis
	Step 4:	Financial/Decision Analysis

Step 1: Process Flow Diagram



Step 2: Demand/Capacity Analysis



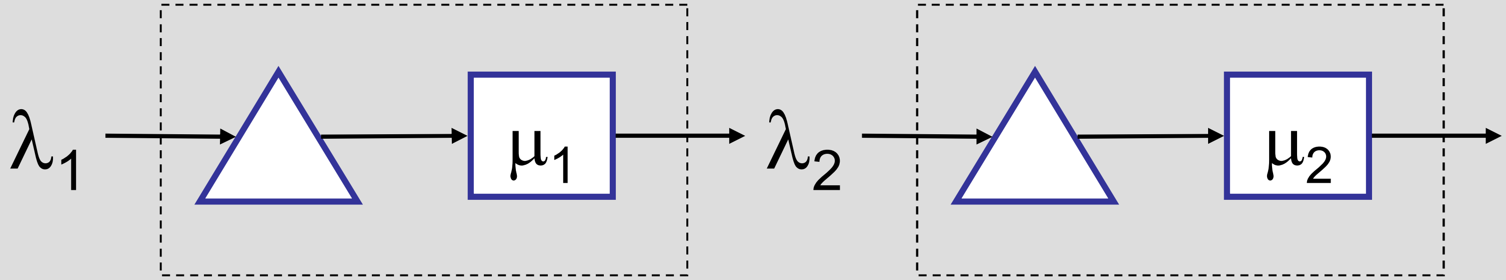
For each process step i, determine:

- λ_i : **demand or input rate (in units of work per unit of time)**
- μ_i : **realistic maximum service rate, assuming no idle time (in units of work per unit of time)**

$\rho_i = \lambda_i / \mu_i$: **capacity utilization**

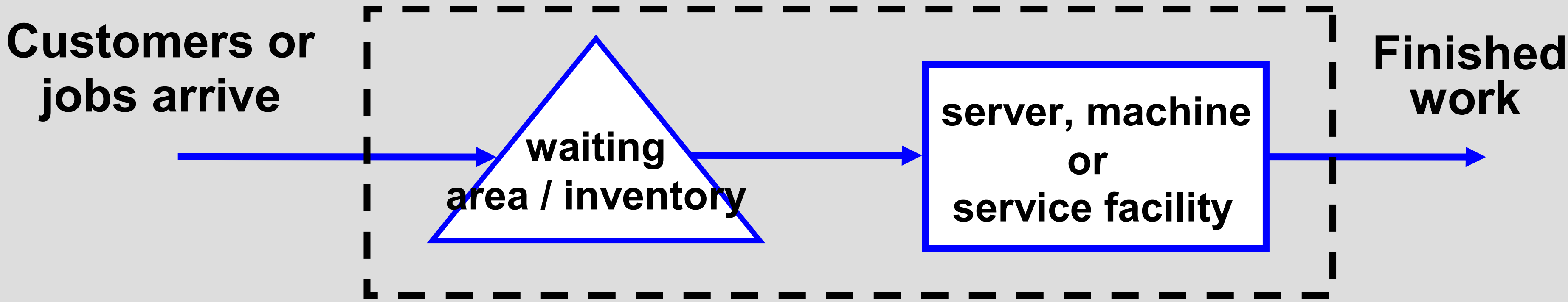
$\lambda_i - \mu_i$: **build-up rate**

Throughput



$$\lambda_2 = \min(\lambda_1, \mu_1)$$

Step 3: Congestion Analysis



System Performance = F(**System Parameters**)

- **L** Inventory level/Queue size/Line length
- **W** Waiting time
- **C** Cycle time
- P_{full} Probability queue is full

- λ Arrival rate
- μ Service rate
- **A** Inter-arrival time distribution
- **S** Service time distribution
- **N** Number of servers
- **R** Queue/Buffer capacity

Congestion Analysis Tools

Build-Up Diagrams	*Queueing Theory*
• Predictable Variability • Utilization > 1 o.k. • Short Run Analysis • Variable rates o.k.	• Unpredictable Variability • Utilization < 1 only • Long Run Analysis • Fixed rates only
• assumes workflow is continuous and deterministic	• stochastic analysis with inter-arrival and service time distributions

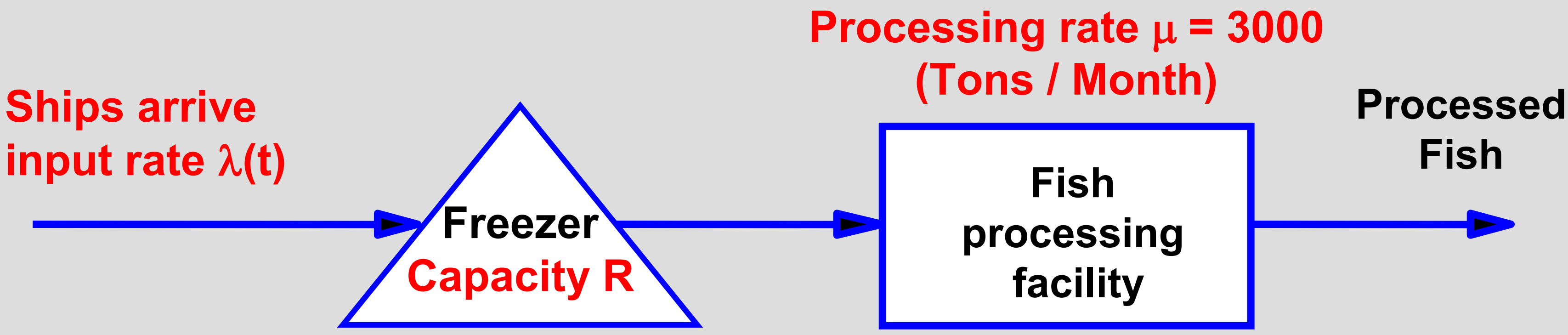
All other cases → ***Simulation / Experiments***

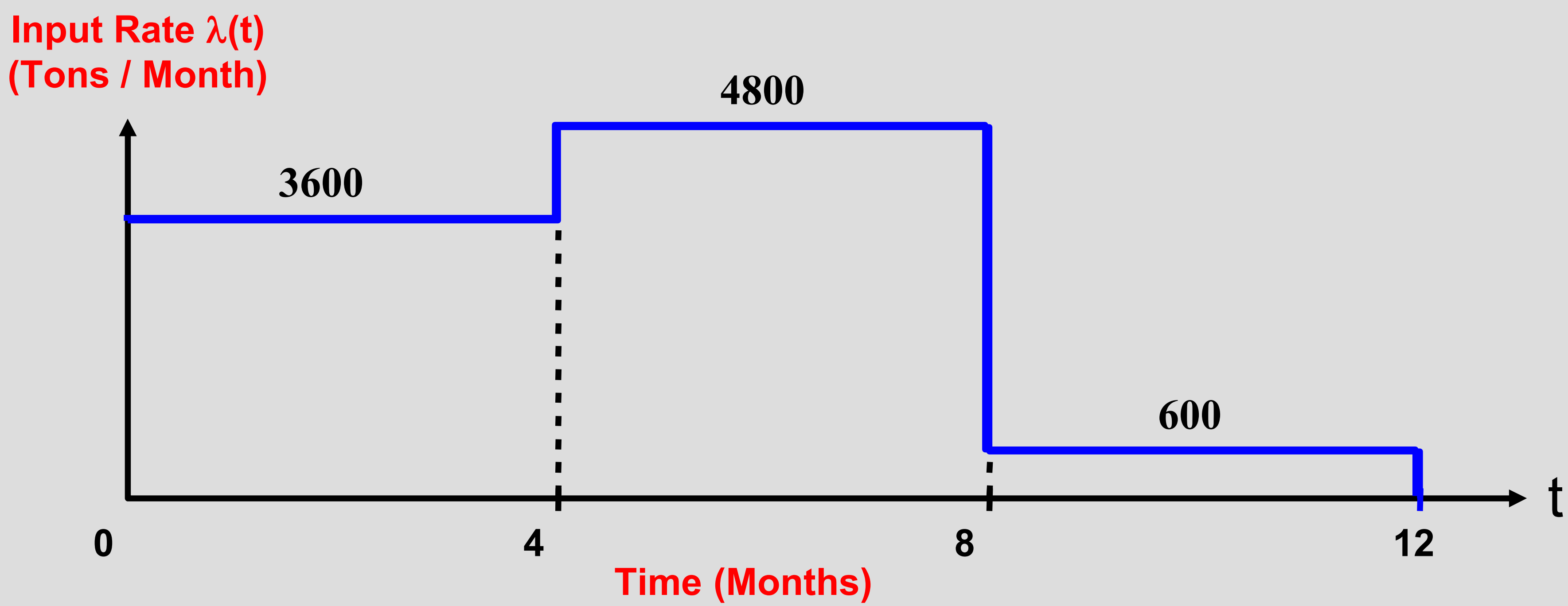
Buildup Diagrams

Think of work as being liquid

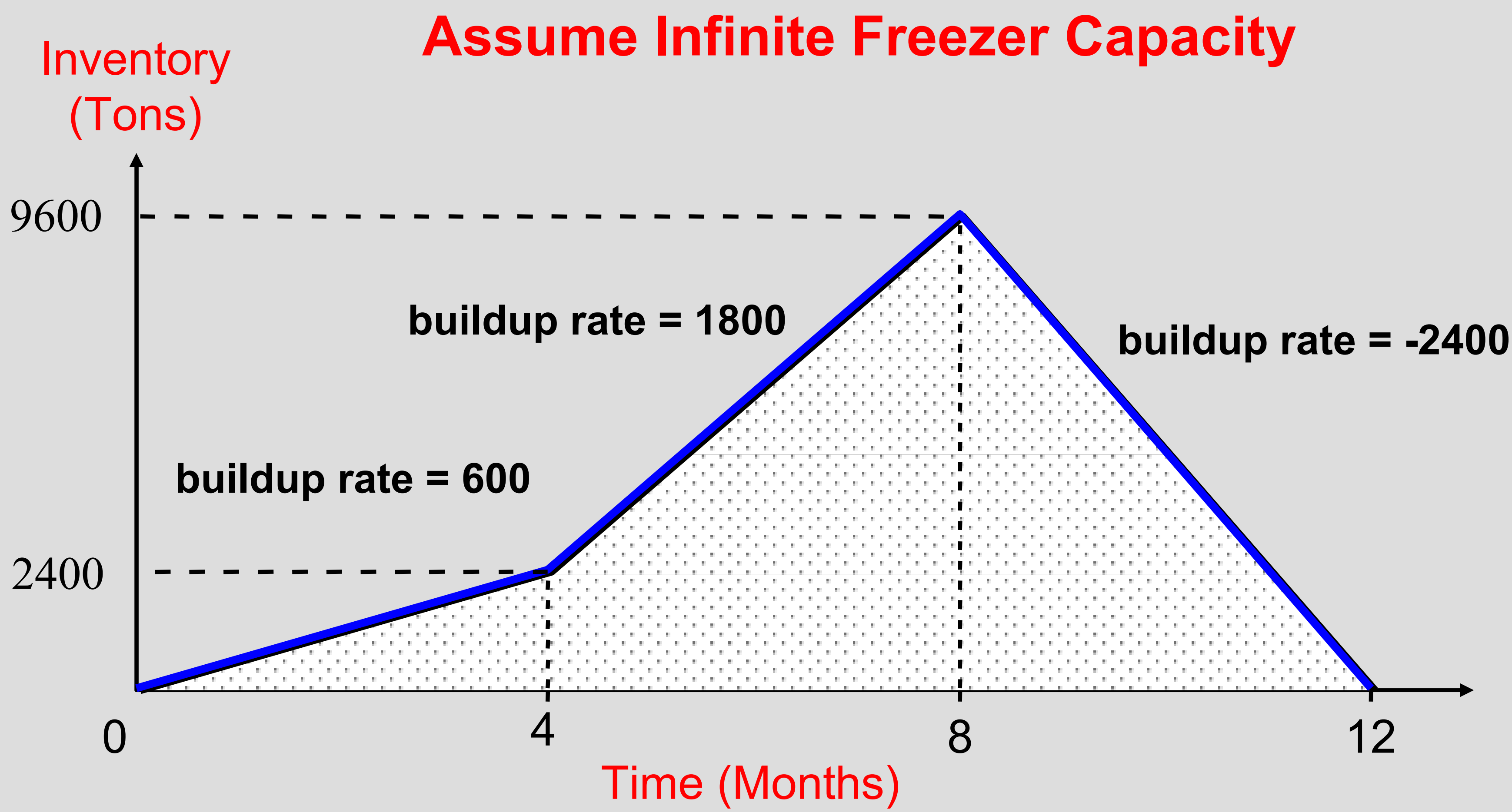
- Predictable Variability
- Utilization > 1 ok
- Short Run Analysis
- Variable rates ok
- No rocket science, but requires a little care

Buildup Example: Fish Processing

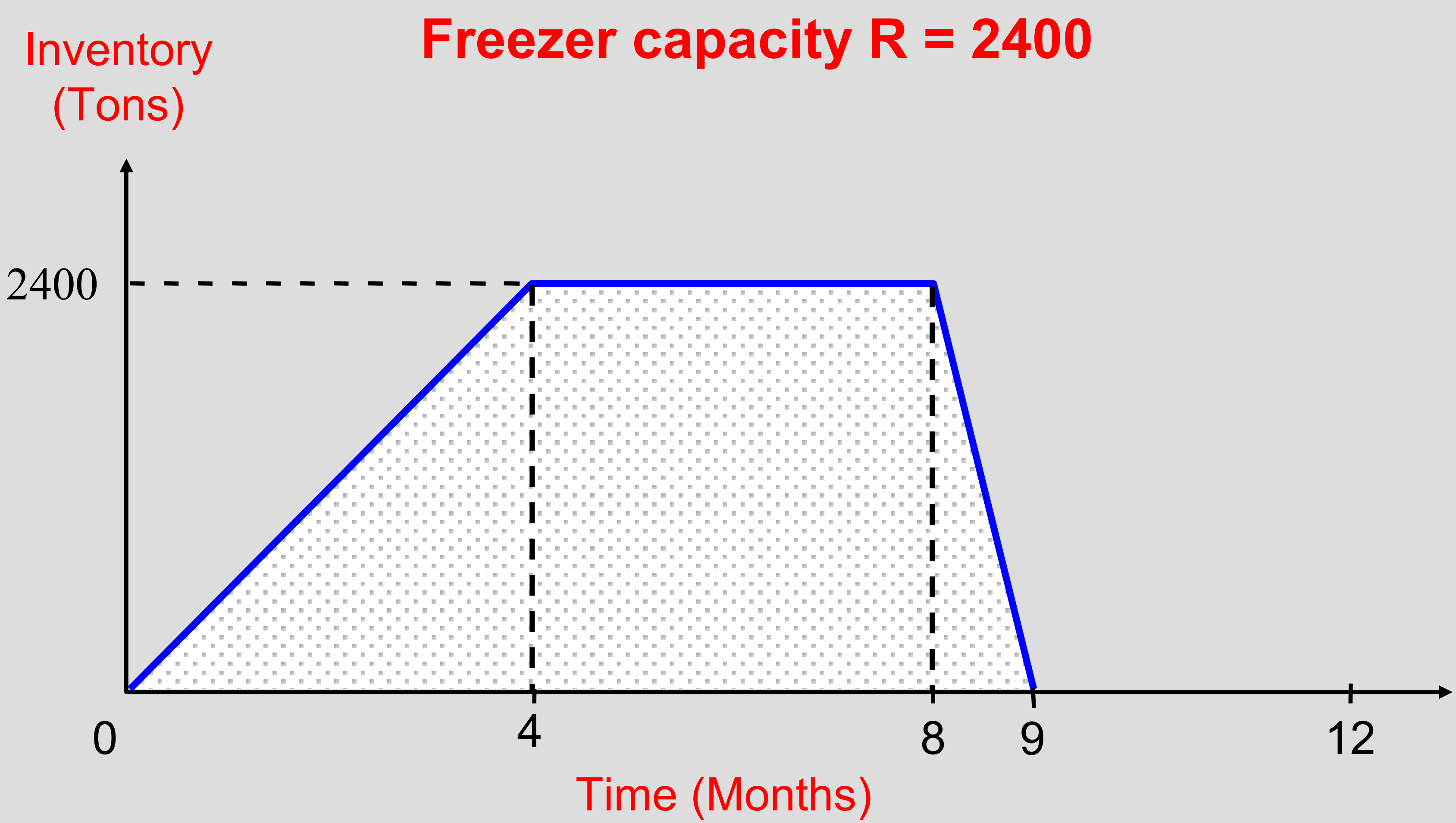




Freezer Inventory Diagram



Limited Storage Capacity



Queueing Theory

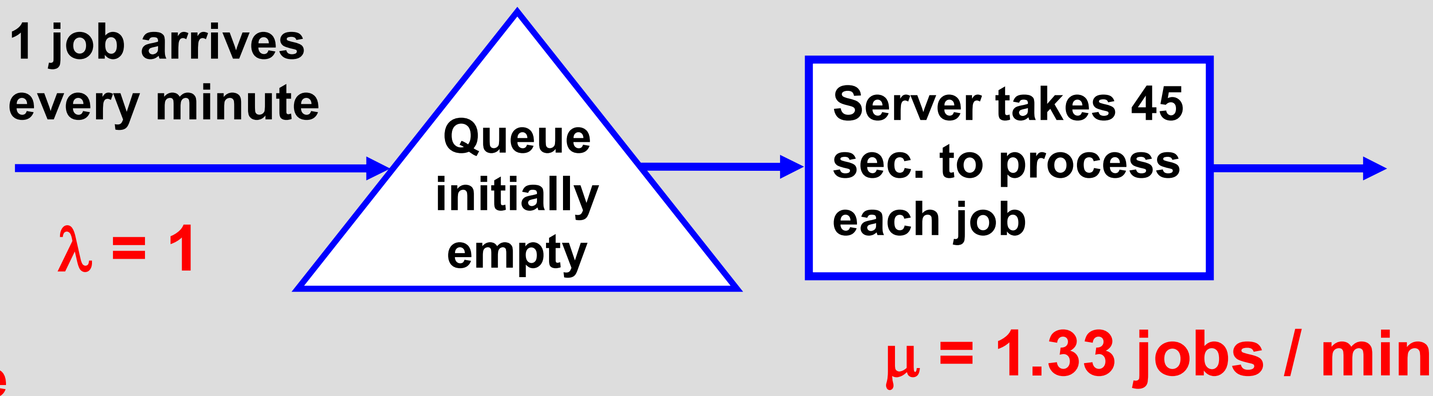
Sophisticated analysis (but easy formulas) predicting long-term impact of unpredictable variability on congestion.

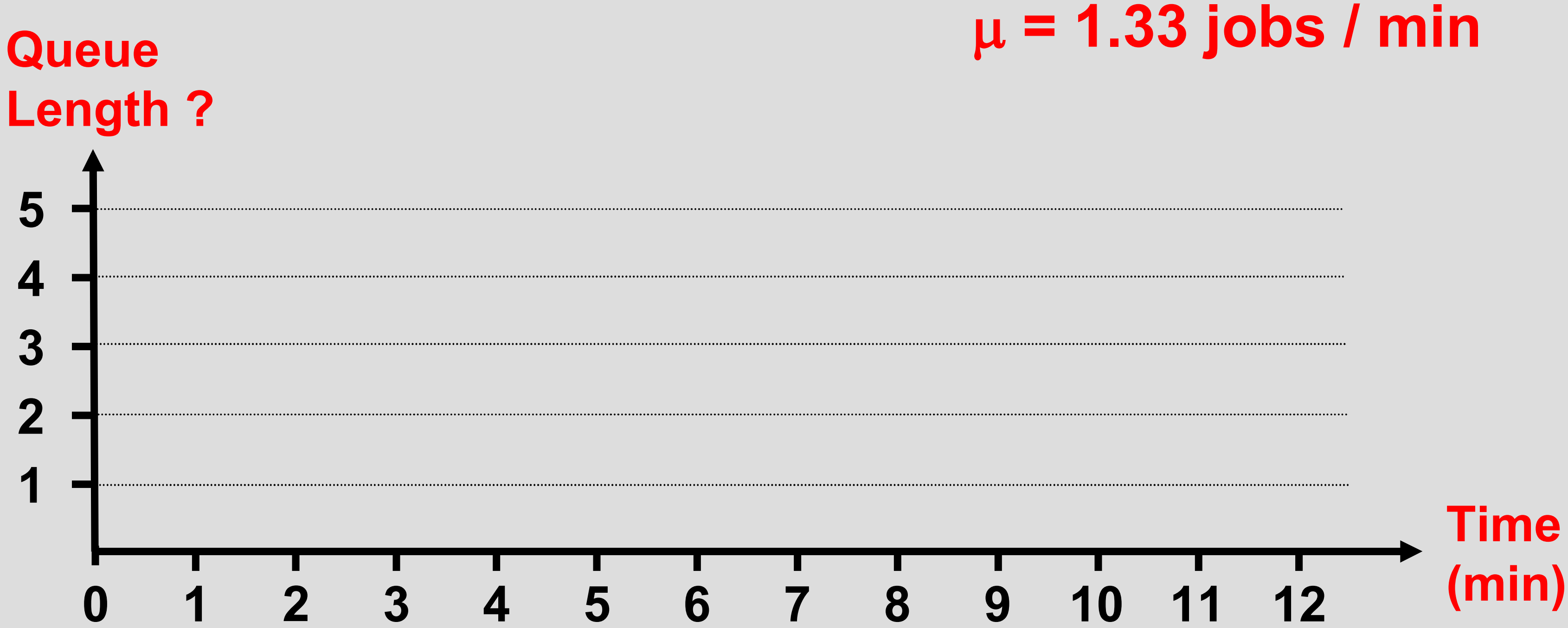
- **Unpredictable Variability**
- **Utilization < 1 only**
- **Long Run Analysis**
- **Fixed rates only**

COVERED

- **G/G/N queueing formula**
- **Little's law (flow balance)**
- **Managerial insights**

A Deterministic Queue

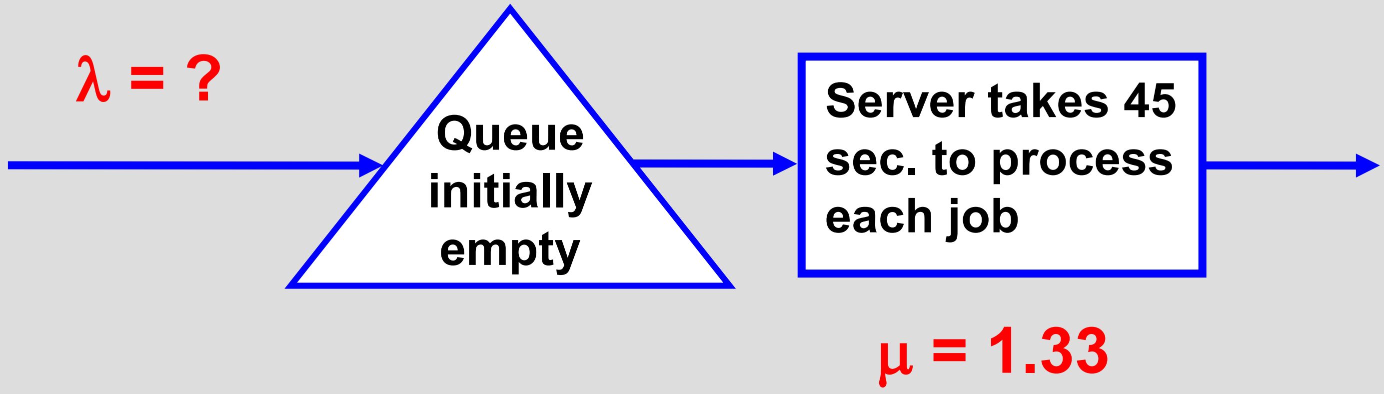




A Queue with Bursty Arrivals

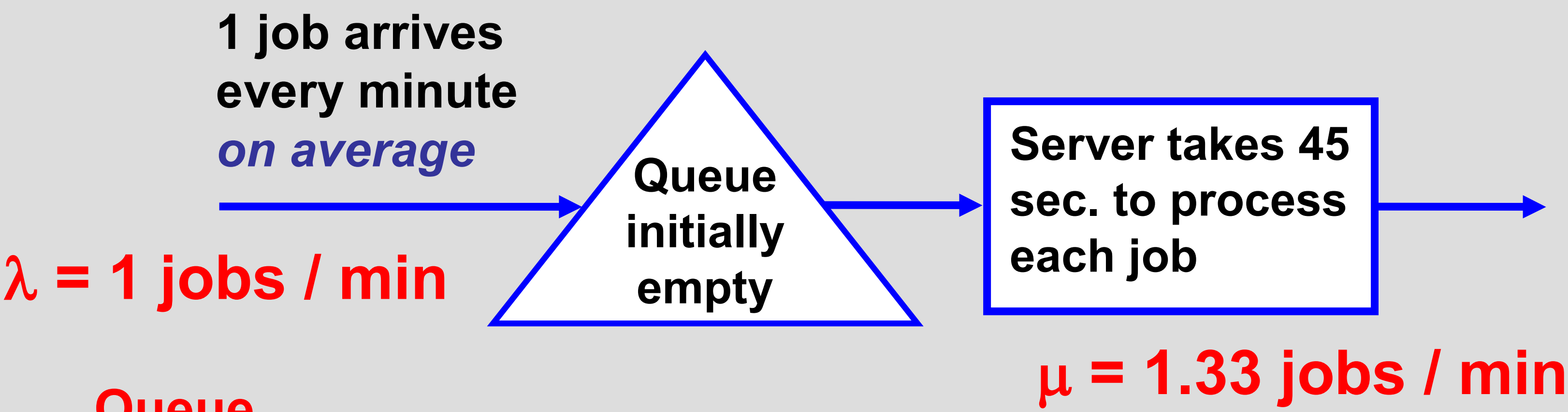
Next job arrives:

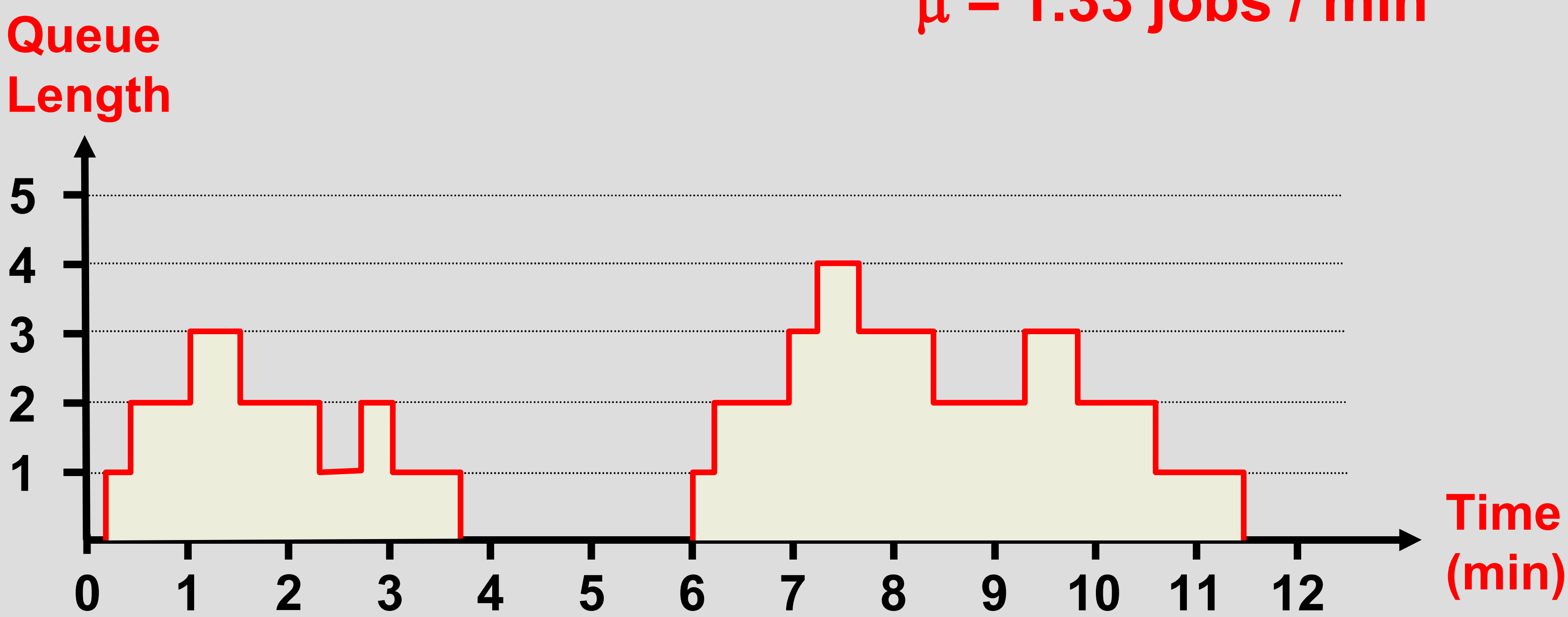
- after 15 sec. with probability 1/2
- after 1 min 45 sec. with probability 1/2



- This model captures unpredictable variability

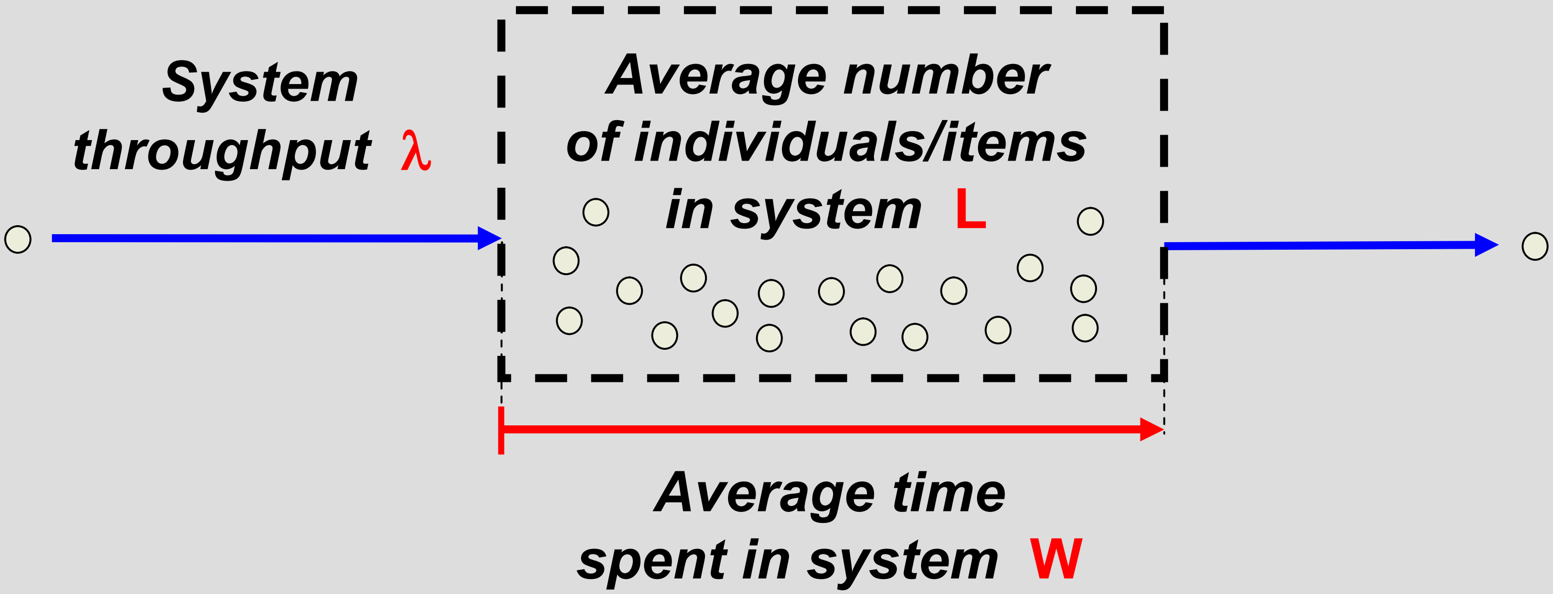
A Queue with Bursty Arrivals





Little's Law

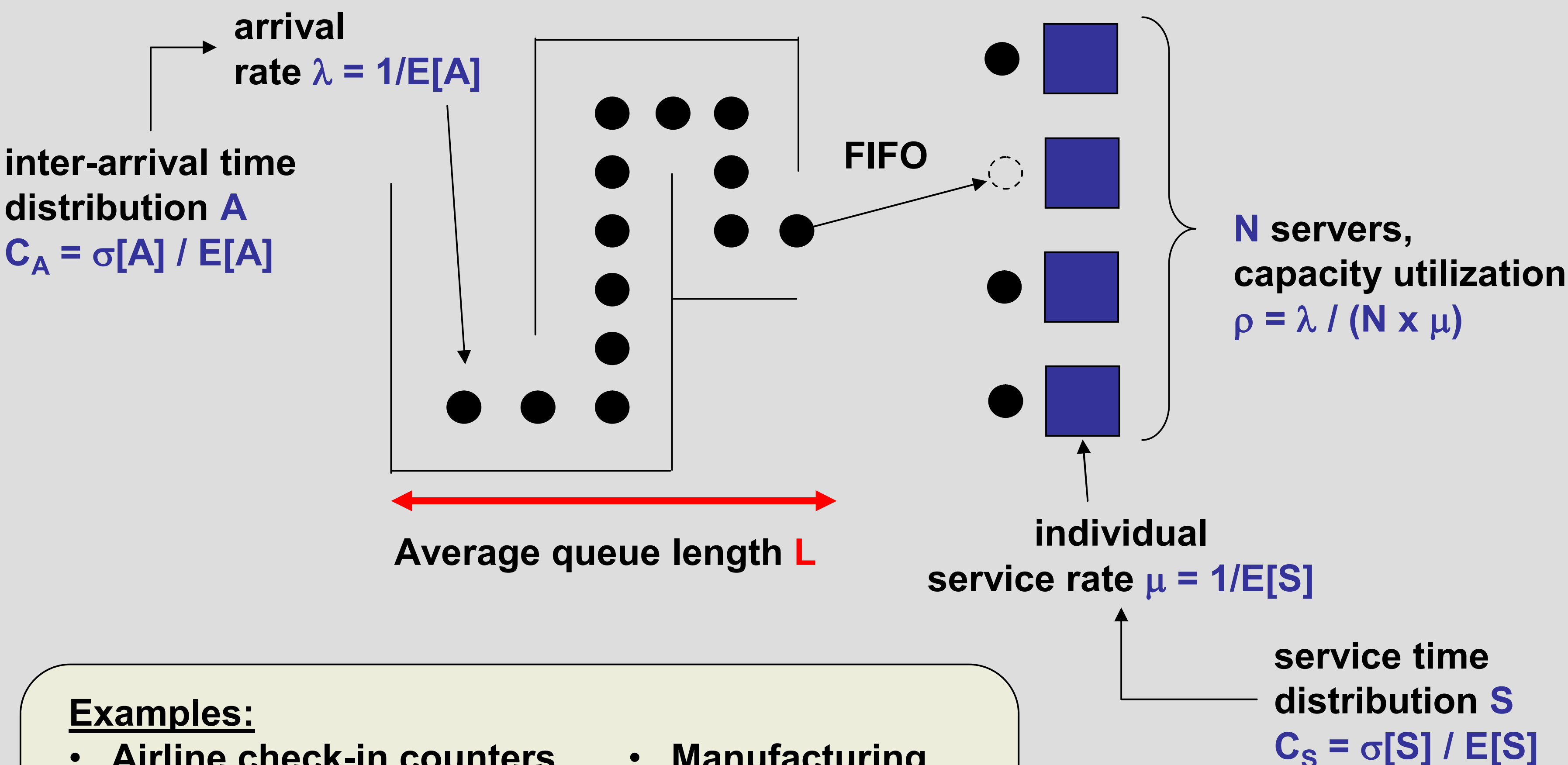
- 300 new MBA's/Year x 2 Years MBA = 600 students in Sloan



- Conservation of Flow (equilibrium):

$$L = \lambda \times W$$

G/G/N Queueing Model



Examples:

- Airline check-in counters
- Bank ATMs
- Retail cashiers
- Computer processing
- Manufacturing
- Call centers
- 911 response
- …

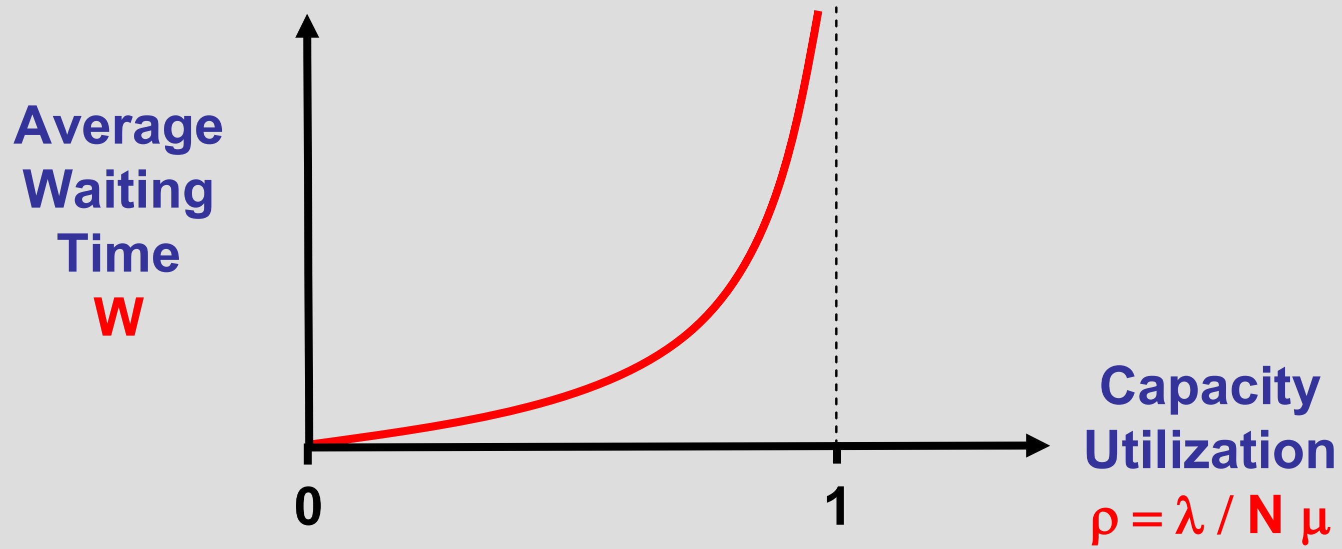
G/G/N Queueing Formula

Approximation with an infinite buffer size:

$$L = \frac{\rho^{\sqrt{2(N+1)}}}{1-\rho} \times \frac{C_A^2 + C_S^2}{2}$$

L	**average number waiting**
ρ	**capacity utilization (= λ / $N\mu$)**
C_A	**coefficient of variation: inter-arrival times**
C_S	**coefficient of variation: service times**
N	**number of servers**

Main Queueing Insight



- **The relationship between waiting time and capacity utilization is strongly non-linear!**

Managing the Psychology of Queueing

1. Unoccupied time feels longer than occupied time
2. Process waits feel longer than in process waits
3. Anxiety makes waits seem longer
4. Uncertain waits seem longer than known, finite waits
5. Unexplained waits are longer than explained
6. Unfair waits are longer than equitable waits
7. The more valuable the service, the longer the customer will wait
8. Solo waits feel longer than group waits

Class 3 Wrap-Up

1. **Inventory buildup diagrams and predictable variability**

2. **Little's law (systems in equilibrium) $L = \lambda \times W$**

3. **Queueing theory and unpredictable variability**

4. **Non-linear relationship between W or L and ρ**

5. **Queue Psychology Management**