Probability

An introduction

Probability theory presents a systematic way to think about such issues

Examples

- *Probability of:*
  - *A newborn will be male?*
  - *Getting HH in a coin toss?*
  - *Getting a particular face on a die?*
  - *In a bag with 1 red, 1 green and 3 blue balls. What is the probability to draw a red ball, a red or a blue ball?*

Compound Probabilities

Disjunctive probabilities linked by addition

A or B

Conjunctive probability

A and B

Disjunctive

- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that you draw a red* ***or*** *a green ball?*
-
- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that you draw a red* ***or*** *a blue ball?*

Conjunctive I

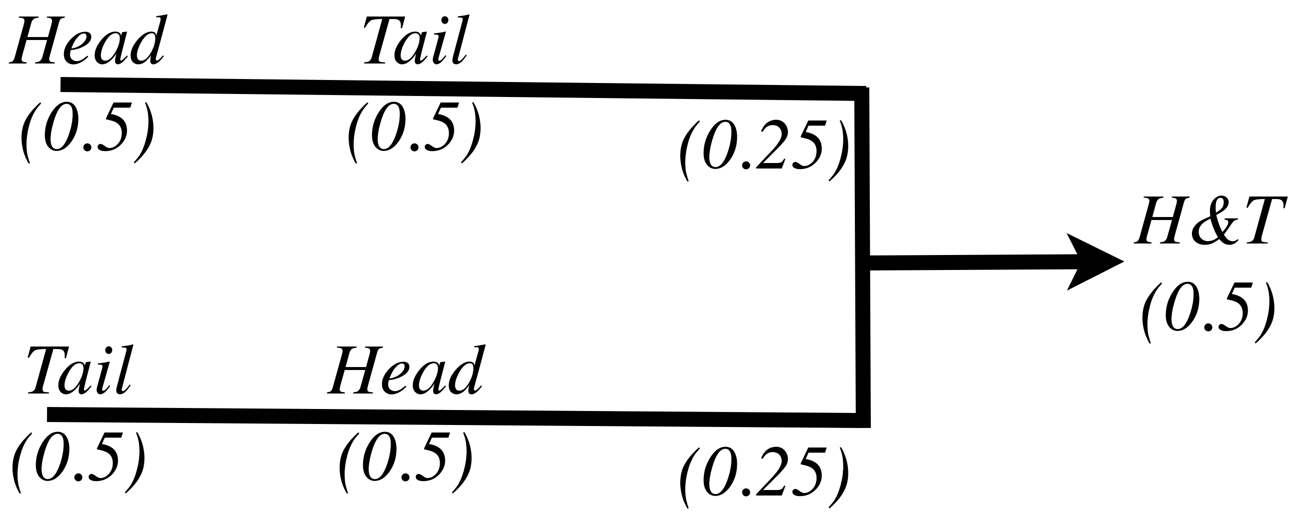
- *In 2 coin tosses, what's the likelihood of getting 2 heads?*

T	T
T	H
H	T
H	H

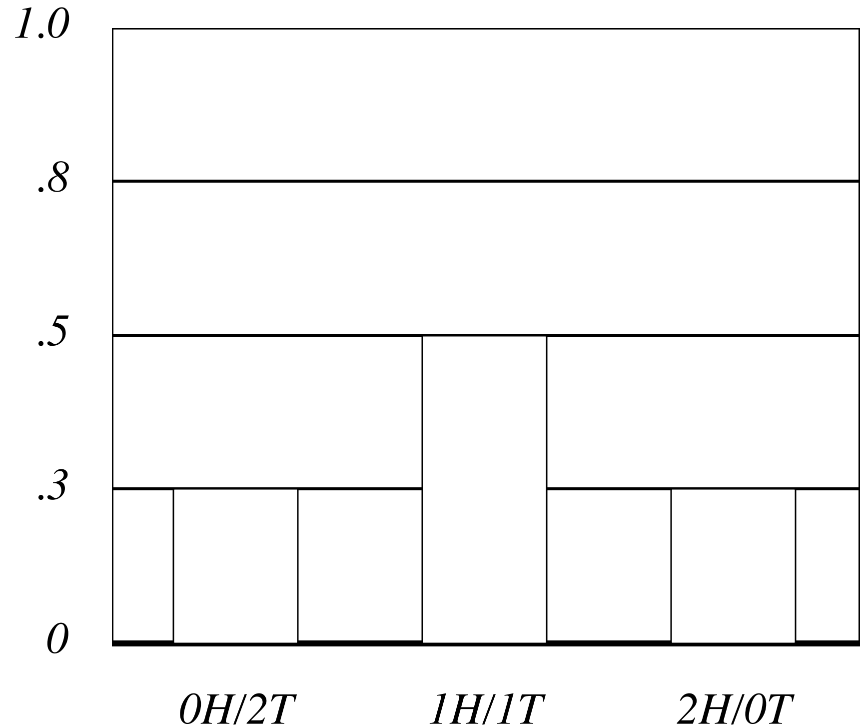
Conjunctive II

- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that in 2 drawings you get 2 red balls?*
-
- *In a bag with 1 red, 1 green and 3 blue balls. What is the likelihood that 2 drawings you get 1 red and 1 blue ball?*

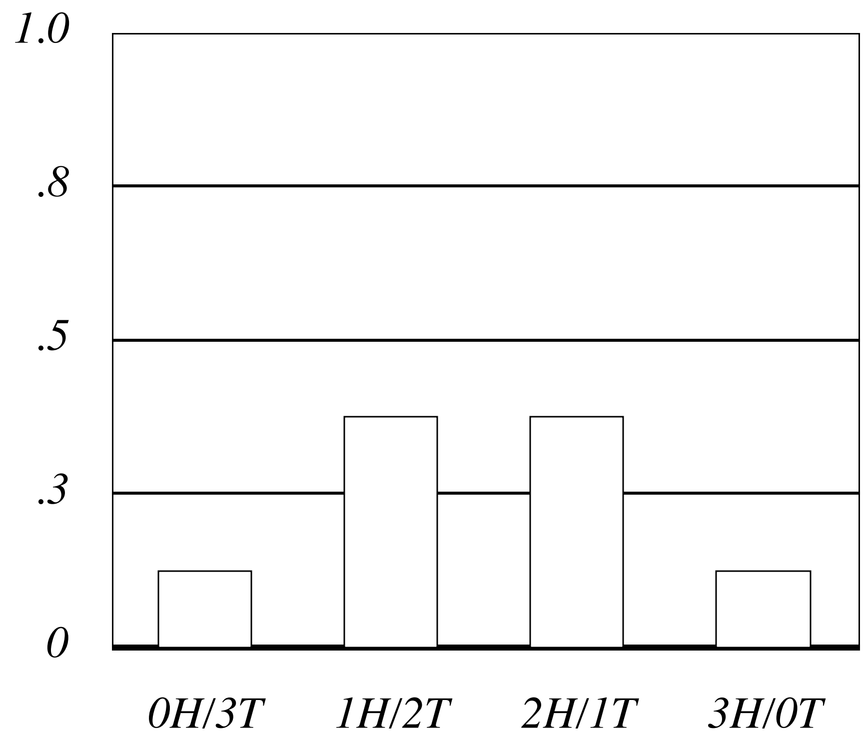
Probability pathway



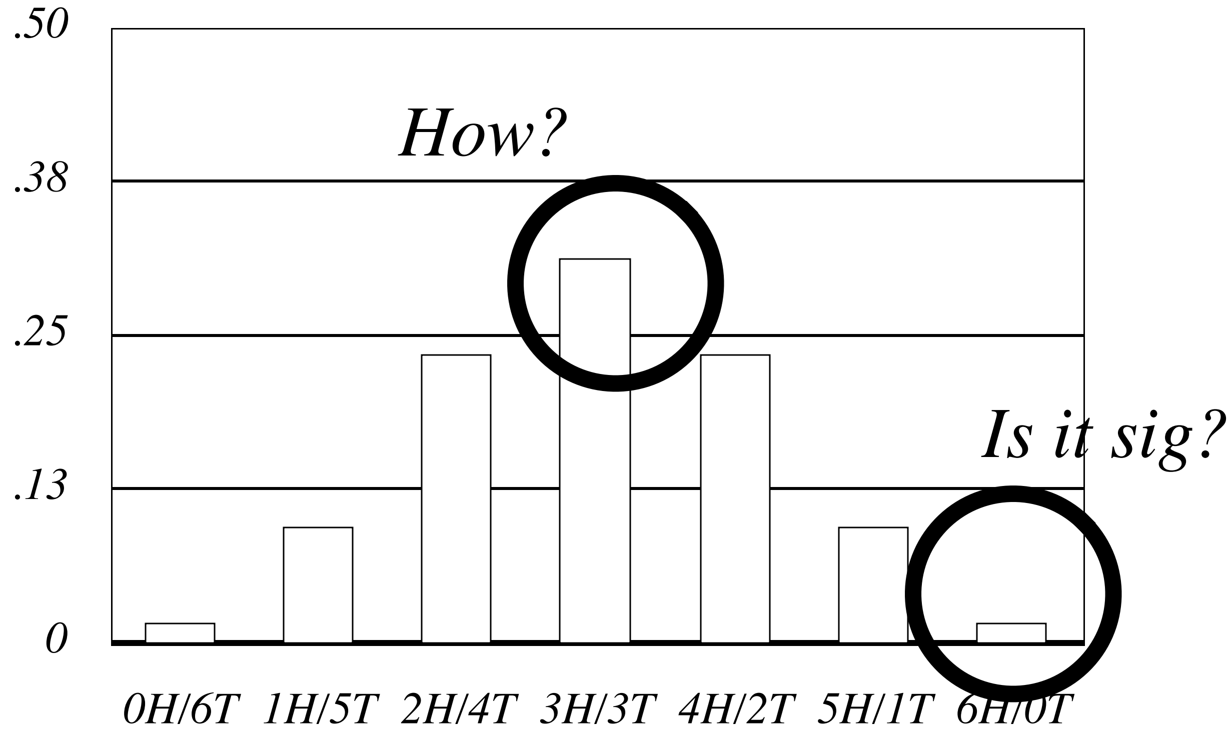
Frequency of 2 coin toss



Frequency of 3 coin toss



Frequency of 6 coin toss



Binomial Distribution I

- *A dramatic disease causes death in 60% of the cases. Only 40% people survive.*
- *What is the likelihood that 10 out of 10 people survive?*
  -
  - *Only one possible way to get 10 / 10…*
  - *.4 * .4 * .4 * .4 * .4 * .4 * .4 * .4 * .4 * .4 =.0001*

Binomial Distribution II

- *A dramatic disease causes death in 60% of the cases. Only 40% people survive.*
- *What is the likelihood that (at least) 7 out of 10 people survive?*
-
  - *A bit more complex ...*

Calculating bonomial probabilities

- *P(r out of n) = number of possible ways X Probability of each way*
- *Number of possible ways = n!/[r!(n-r)!]*
- ***P(7 out of 10) =***
  - *0.4^(r) x 0.6^(n-r) = 0.4^7 x 0.6^3 =* 0.00035389
  - *10!/7!3!=120*
  - *P(7 out of 10)* = 0.0425

7 or more out of 10?

- *7 out of 10 + 8 out of 10 + 9 out of 10 + 10 out of 10*

- *When we calculate the probability of an outcome (7), most times we calculate the probaility of the outcome or a more extreme outcome*

Binomial summary

- *By using the binomial distribution we can directly calculate the probability of en event (or a more extreme event).*

- *We can compare this probability to 0.05 and decide if it is significant or not.*

Calculating distributions:

- *6 coins tossed:*
  - $n = 6; \quad p = .5; \quad q = .5$
- *Theoretical mean (□) = np = 6 * .5 = 3*
  - *central tendency of distribution*
- *Variance (□^2) = n *p*q*
- *Standard deviation = □*
- *z = (r-□) / □*

Z scores

z = (*r*-□) / □

Probability of 6 Heads:

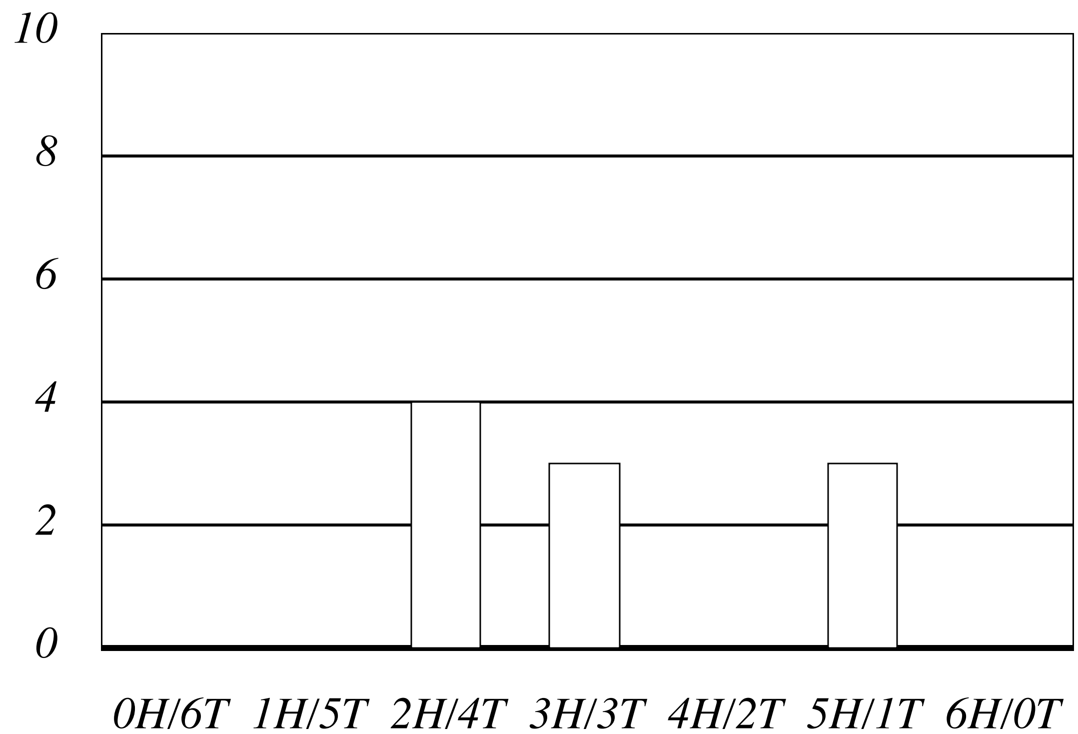
z = (6-3) / 1.225 = 2.45

Look for z = *2.45 in the Normal distribution table*

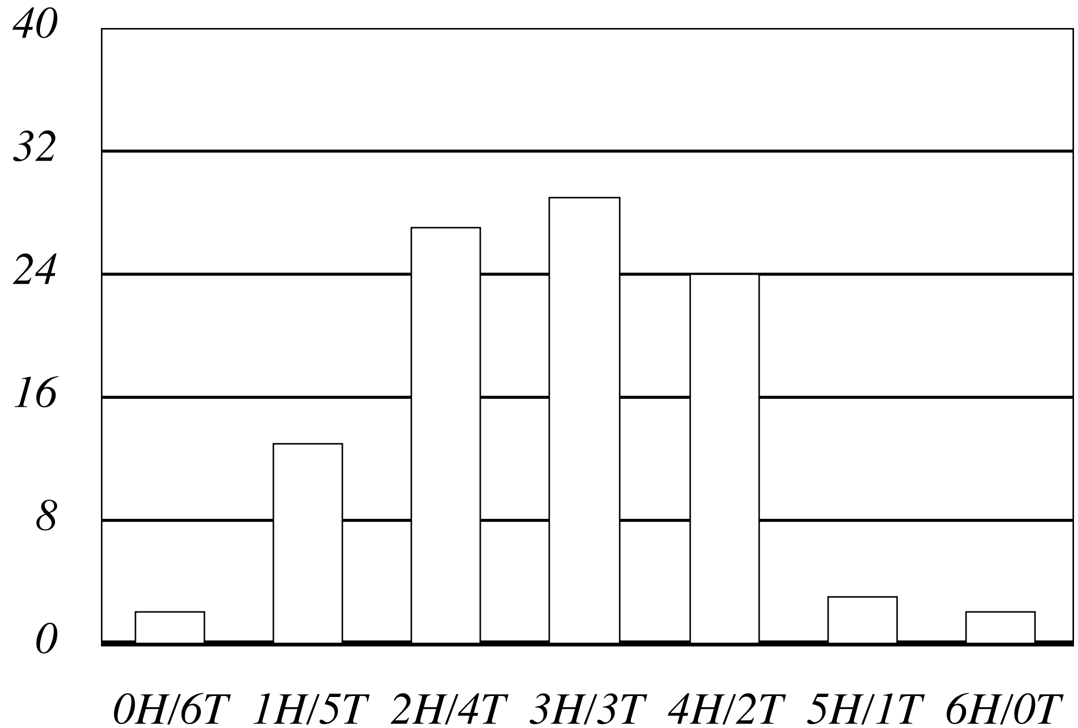
Central limit theorem

- *Approximations of normal distributions with increased number of observations*

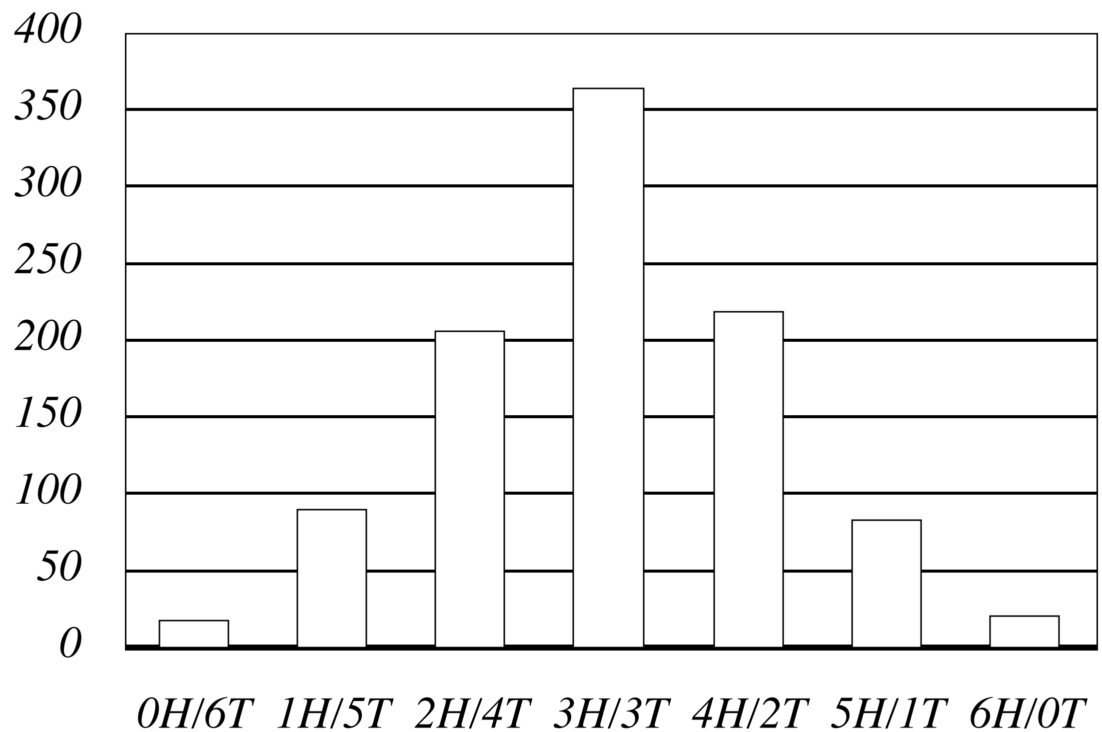
Frequency of 6 coin toss (10)



Frequency of 6 coin toss (100)



Frequency of 6 coin toss (1,000)



Summary

- *Probability*
  - *This is the base of stattistics...*
- *Binomial:*
  - *A way to calculate probabilities of events directly*
- *Central limit theorem*