Facility Location and Distribution System Planning

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Today's Agenda

- Why study facility location?
- Issues to be modeled
- Basic models
  - Fixed charge problems
  - Core uncapacitated and capacitated facility location models
- Large-scale application (Hunt-Wesson Foods)

Logistics Industry

- U.S. logistics industry: $900 billion - almost double the size of the high-tech industry: > 10 percent of the U.S. gross domestic product
- 11 per cent of Singapore's GDP with a growth of 9 per cent in year 2000
- Singapore *Logistics Enhancement & Applications Programme (LEAP) 2001*
- Global logistics: $3.43 trillion
- 1998, U.S. trucking industry revenues just under $200 billion
- 7.7 million trucks carried over 1 trillion ton miles of freight

Singapore Retail 21 Plan

Basic Issue

- Where to locate and how to size facilities?
- How to meet customer demands from the facilities?
  - Which facility (facilities) serve each customer?
  - How much customer demand is met by each facility?

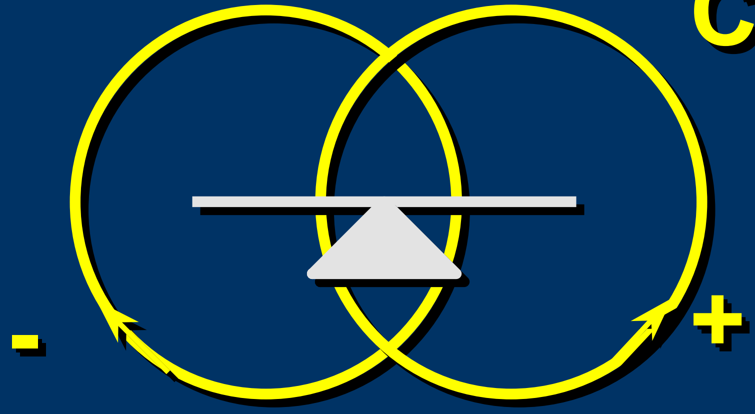
Facilities might be warehouses, retail outlets, wireless bay stations, communication concentrators

Some Elements of Cost & Service

- Transportation Costs
  - Vehicles, Drivers, Fuel
- Warehousing
  - Facility Construction/Rental, Handling Costs, Inventory
- Customer Service
  - Service Time, Single Sourcing

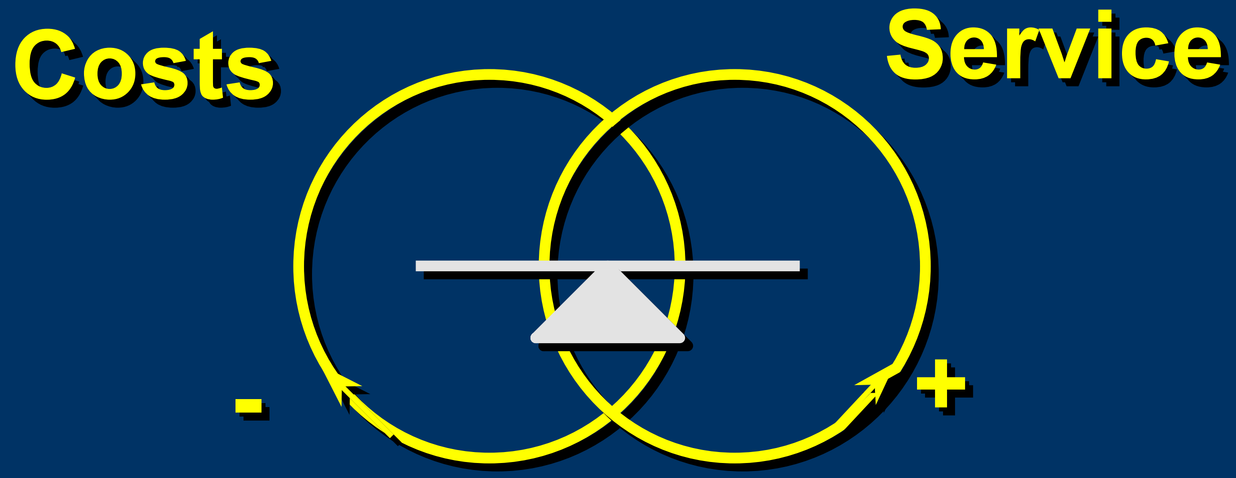
System Trade-offs

Transportation Costs | **Fixed Costs**



Effect of More Facilities

System Trade-offs



Effect of “Individualized” Service (e.g., Direct Shipments)

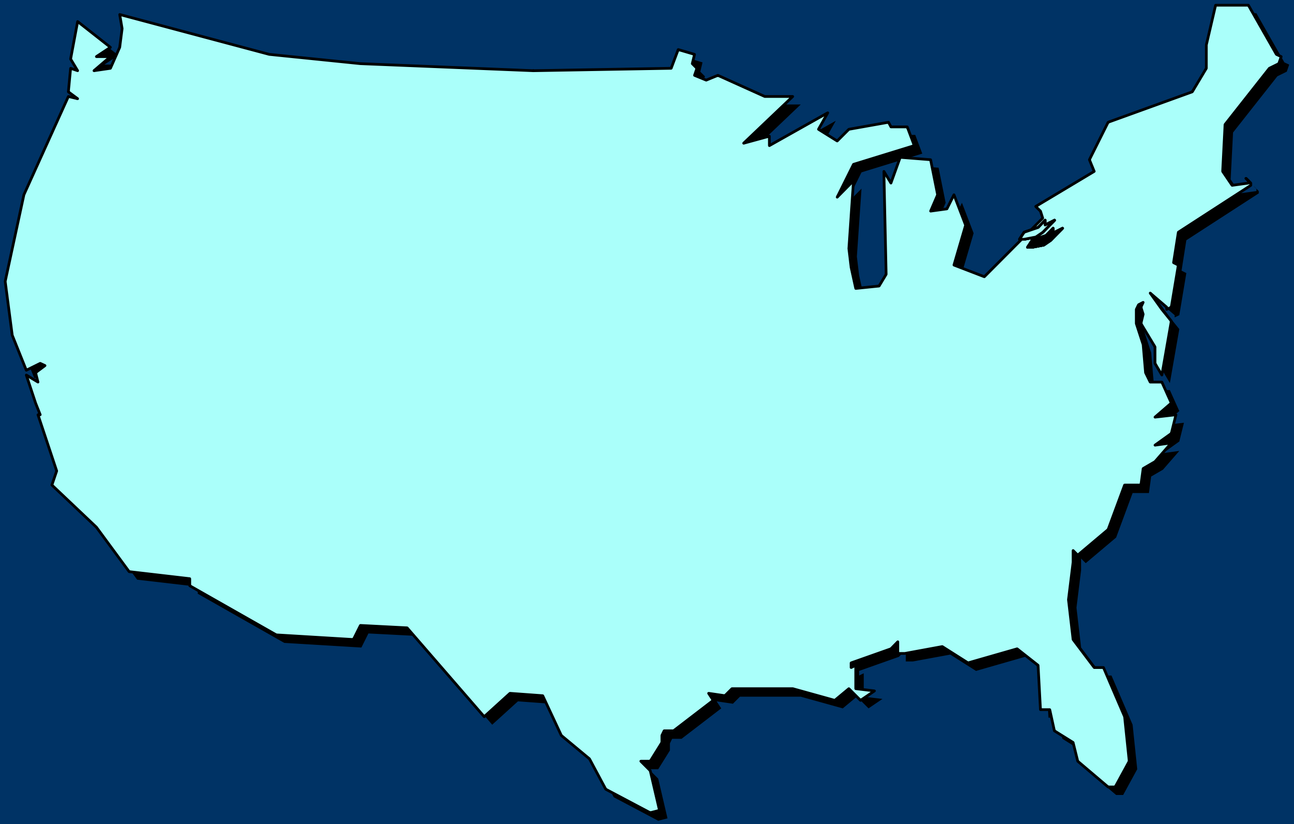
Nature of Costs

- Fixed Costs
  - Facility construction/rental
  - Vehicle purchases & rentals
  - Personnel (drivers, managers)
  - Fixed overhead
- Variable Costs
  - Inventory, handling, fuel

Optimization Applications

- Hunt-Wesson Foods saves over $1 million per year
- Restructuring North America operations, Proctor and Gamble reduces plants by 20%, saving $200 million/year
- Many, many others (e.g., supplying parts to plants)

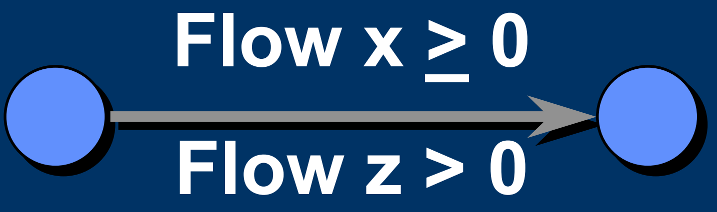
Facility Location Challenge



Modeling Issue

- How do we model "lumpiness" of the costs (e.g., fixed costs)?
- How do we model logical conditions (e.g., choice of warehouse locations)?

Modeling Fixed Costs



- Incur fixed cost F if either $x > 0$ or $z > 0$
- Suppose $x + z \le 3/2$ (demand limitation)

Model

Minimize Fy + other terms

subject to $y = 1$ if either $x > 0$ or $z > 0$

Three Models (LP Relaxations)

Model 1

$x + z < 3/2$

$x < 1, z < 1$

$x + z < 2y$

$x > 0, z > 0$

$0 < y < 1$

Forcing Constraints

Weak ← → Strong

Model 2

$x + z < 3/2$

$x < 1, z < 1$

$x < y, z < y$

$x > 0, z > 0$

$0 < y < 1$

Model 3

$x + z < 3y/2$

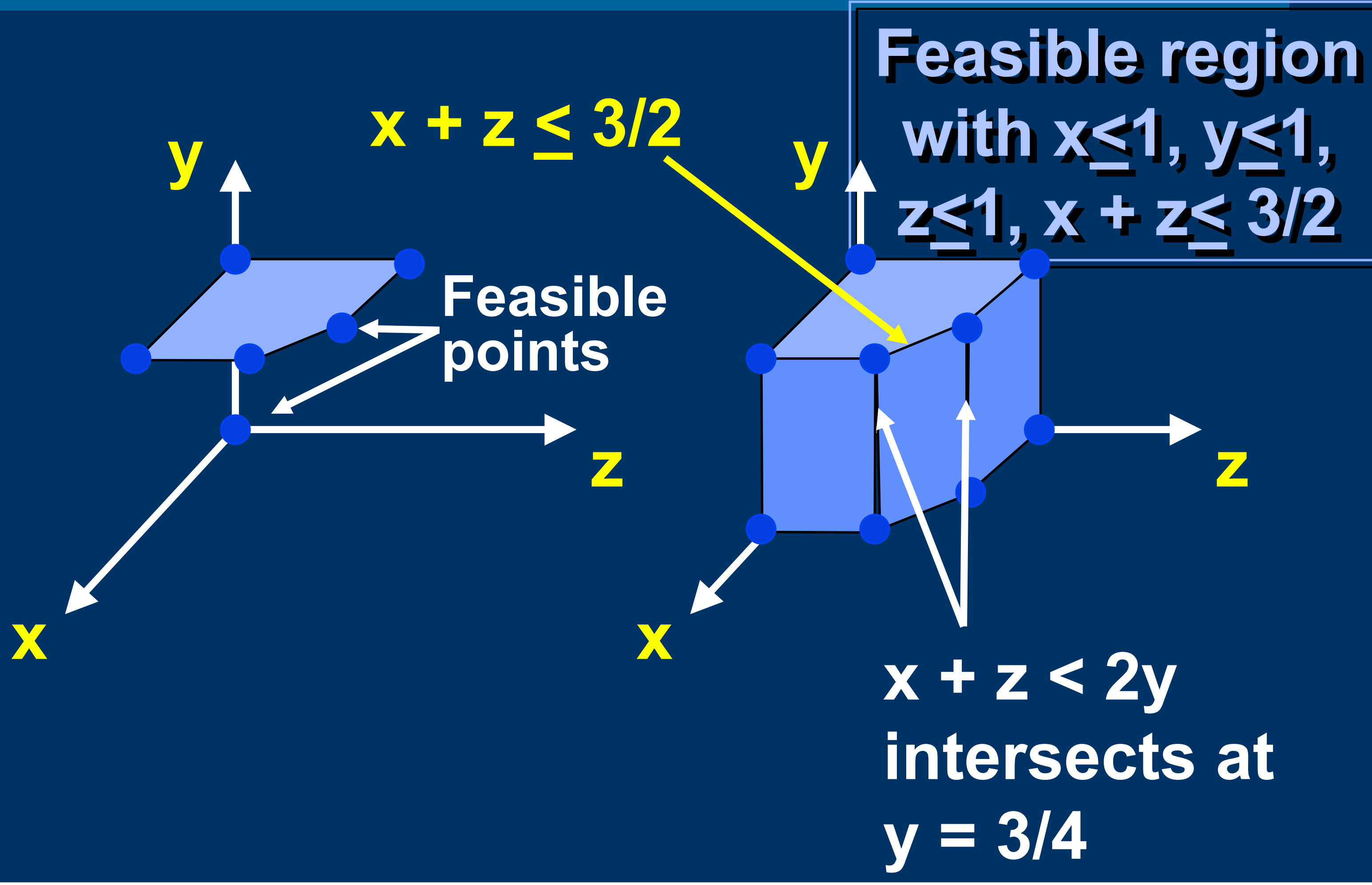
$x < 1, z < 1$

$x < y, z < y$

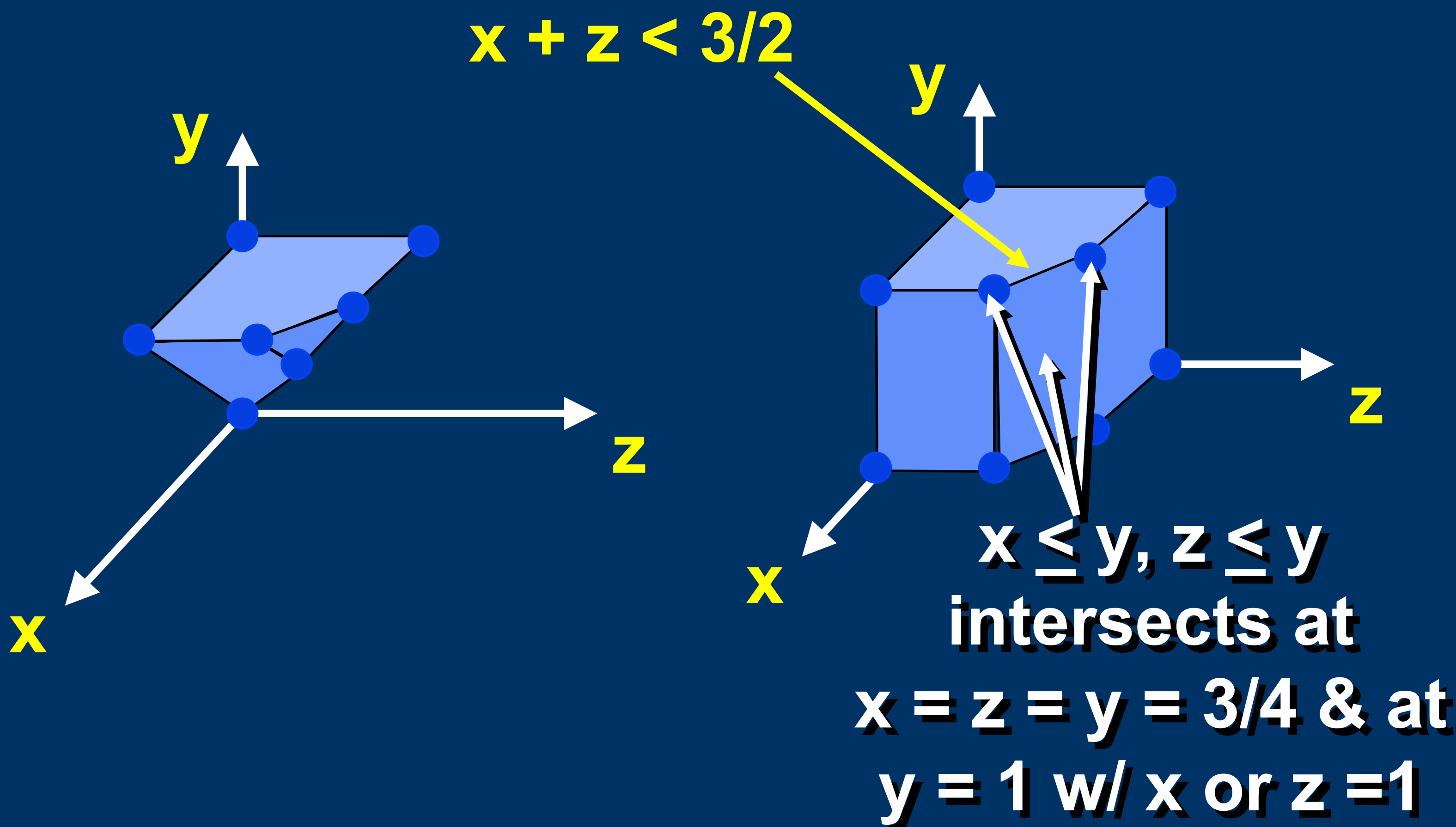
$x > 0, z > 0$

$0 < y < 1$

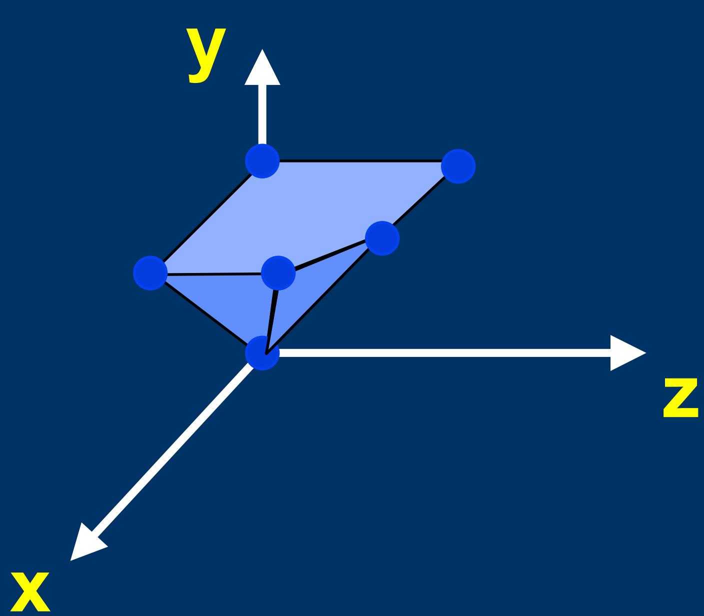
Geometry (Weak Model)



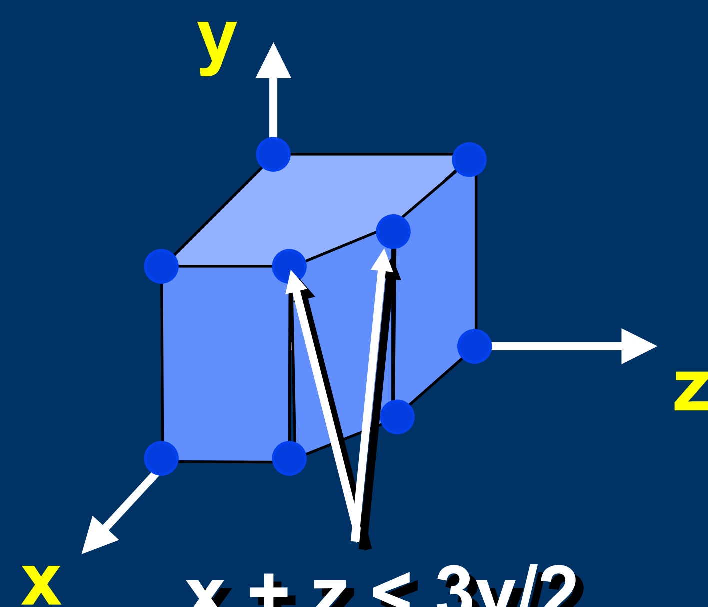
Geometry (Improved Model)



Geometry (Strong Model)



Exact representation!



$x + z \leq 3y/2$,
$x < y$, $z < y$
intersects at $y = 1$

Core (Uncapacitated) Facility Location

Minimize Fixed + Routing Costs

Subject to

- **Meet customer demand from facilities**
- **Assign customer only to open facility**

Parameters:

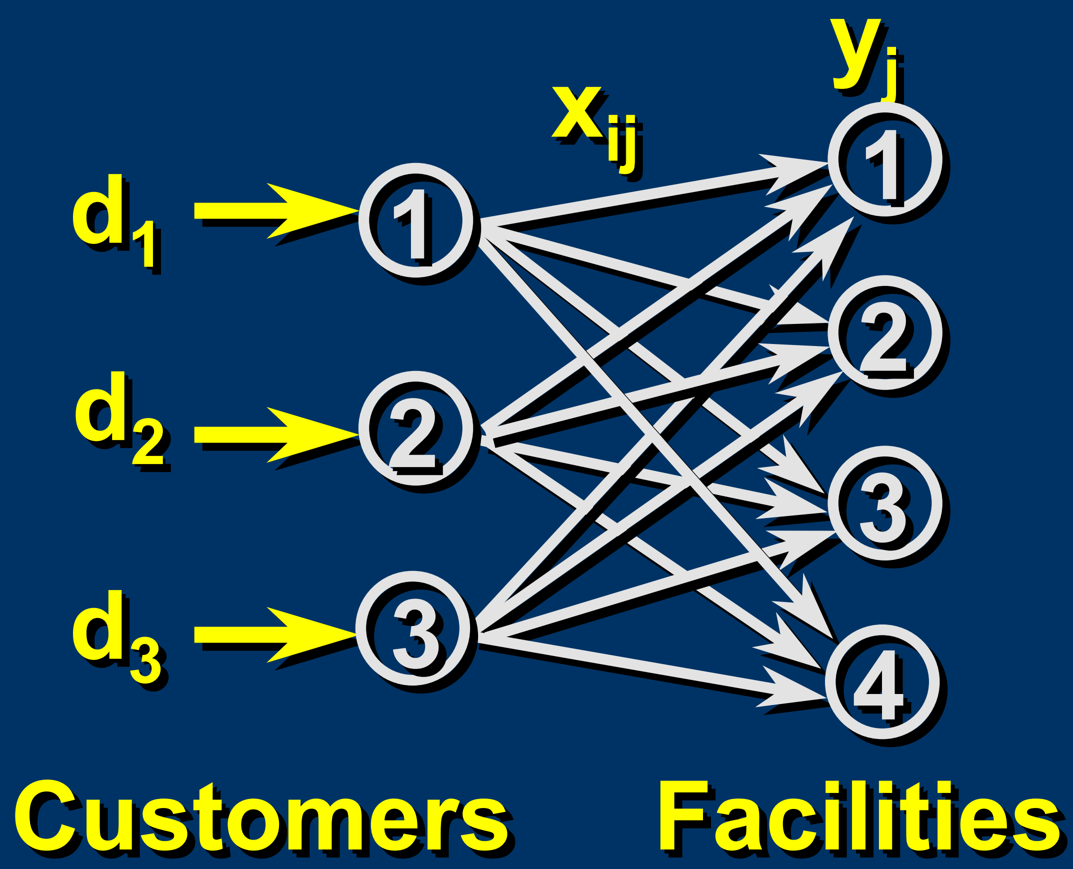
Core Facility Location Model

- Demand d_i for each customer i
- Fixed cost F_j for each facility location j
- Cost c_{ij} of routing all customer i demand to facility j = per unit cost times demand d_i

Decisions: Core Facility Location Model

- Where do we locate facilities?
  - y_j = 1 if we locate facility at location j
- Fraction of service that customer i receives from facility j (x_{ij})

Network Representation
3 Customers, 4 Facilities



Facility Location Costs

$$c_{11}x_{11} + c_{12}x_{12}$$
$$+ c_{13}x_{13} + c_{14}x_{14}$$
$$+ \; . \quad . \quad .$$
$$+ c_{31}x_{31} + c_{32}x_{32}$$
$$+ c_{33}x_{33} + c_{34}x_{34}$$

Routing Costs

$$+ F_1y_1 + F_2y_2 + F_3y_3 + F_4y_4$$

Fixed Costs

Constraints: Tabular Representation

Facilities (Locations)

Customers

x_{11}	x_{12}	x_{13}	x_{14}	$= 1$
x_{21}	x_{22}	x_{23}	x_{24}	$= 1$
x_{31}	x_{32}	x_{33}	x_{34}	$= 1$

$x_{11} \leq y_1, x_{12} \leq y_2, x_{13} \leq y_3, x_{14} \leq y_4$

$x_{21} \leq y_1, x_{22} \leq y_2, x_{23} \leq y_3, x_{24} \leq y_4$

$x_{31} \leq y_1, x_{32} \leq y_2, x_{33} \leq y_3, x_{34} \leq y_4$

Model
(Uncapacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to

$\sum_j x_{ij} = 1$	for all customers i
$x_{ij} \leq y_j$ }	for all customers i
$x_{ij} \geq 0$ }	and facilities j
$y_j = 0$ or 1	for all facilities j

Modeling Variations

- Open at most three of facilities 1, 6 and 8-11
  - $y_1 + y_6 + y_8 + y_9 + y_{10} + y_{11} \leq 3$
- Assign each customer to a single facility
  - x_{11} integer, x_{12} integer, etc.

Modeling Variations

- Open a facility at location 3 only if we open one at location 7
  - $y_3 \leq y_7$

Note: Power of using integer variables to model logical restrictions

Modeling Enhancements

- Multiple products
- Facility capacities and operating ranges (min and max throughput if open)
- Multi-layered distribution networks
- Service restrictions
  - Single sourcing
  - Timing of deliveries
- Inventory positioning and control

Alternate Model (Uncapacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to

$\sum_j x_{ij} = 1$	for all customers i
$\sum_i x_{ij} \leq n y_j$	for all facilities j
$x_{ij} \geq 0$	for all pairs i,j
$y_j = 0$ or 1	for all facilities j

Alternate Model (Uncapacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to

$\sum_j x_{ij} = d_i$ for all customers i

$\sum_i x_{ij} \leq (\sum_i d_i) y_j$ for all facilities j

$x_{ij} \geq 0$ for all pairs i,j

$y_j = 0 \text{ or } 1$ for all facilities j

Model (Capacitated Facilities)

Minimize $\sum_i \sum_j c_{ij} x_{ij} + \sum_j F_j y_j$

Subject to K_j

$\sum_j x_{ij} = d_i$	for all customers i
$x_{ij} \le d_i y_j$	for all i, j pairs
$\sum_i x_{ij} \le CAP_j\, y_j$	for all facilities j
$x_{ij} \ge 0$	for all i, j pairs
$y_j = 0 \text{ or } 1$	for all facilities j

Tabular Representation

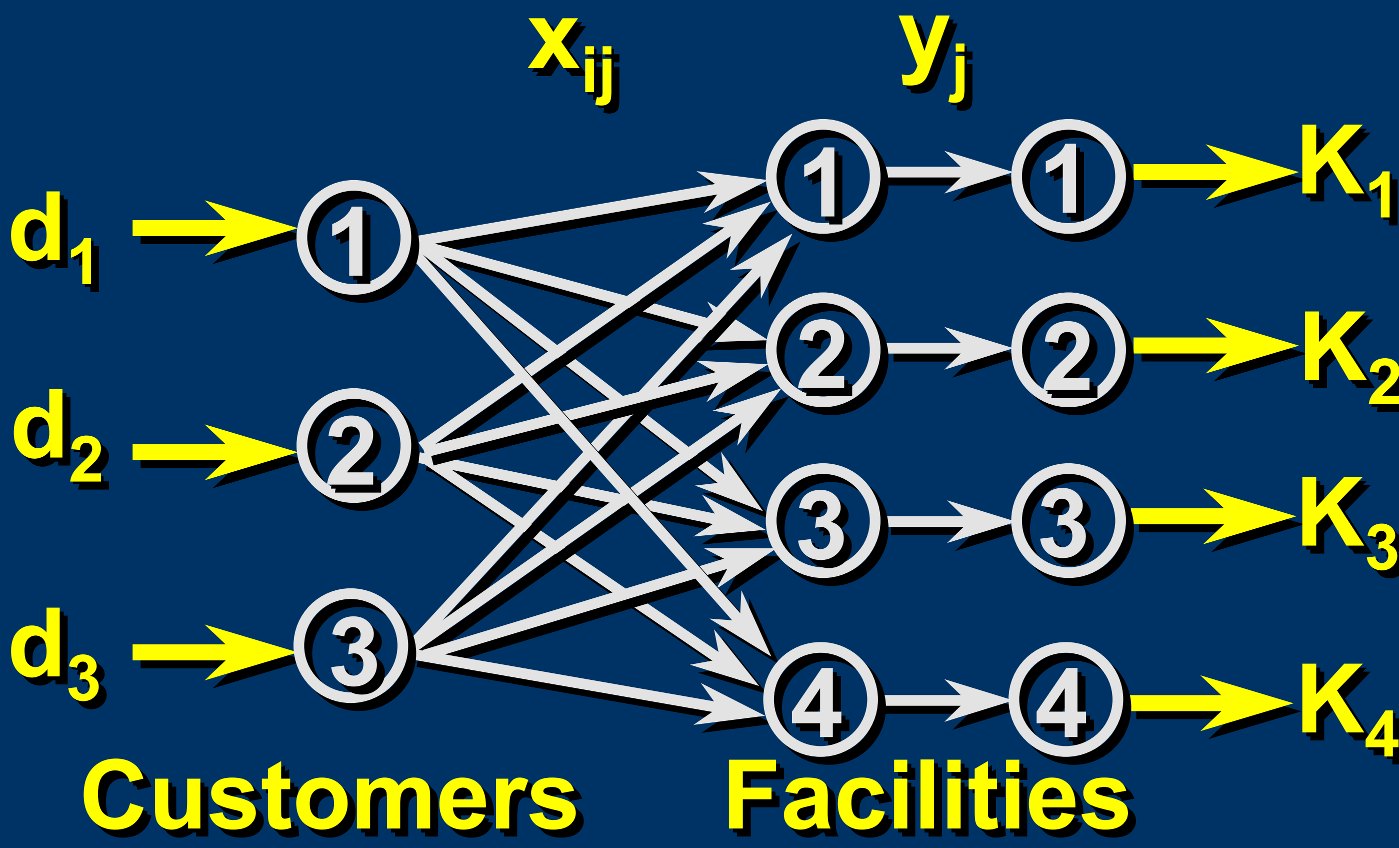
Locations

Customers

x_{11}	x_{12}	x_{13}	x_{14}	$= d_1$
x_{21}	x_{22}	x_{23}	x_{24}	$= d_2$
x_{31}	x_{32}	x_{33}	x_{34}	$= d_3$
$\leq$	$\leq$	$\leq$	$\leq$	
$K_1 y_1$	$K_2 y_2$	$K_3 y_3$	$K_4 y_4$	

plus cell constraints $x_{ij} \leq d_i y_j$

Network Representation



Solution Approaches

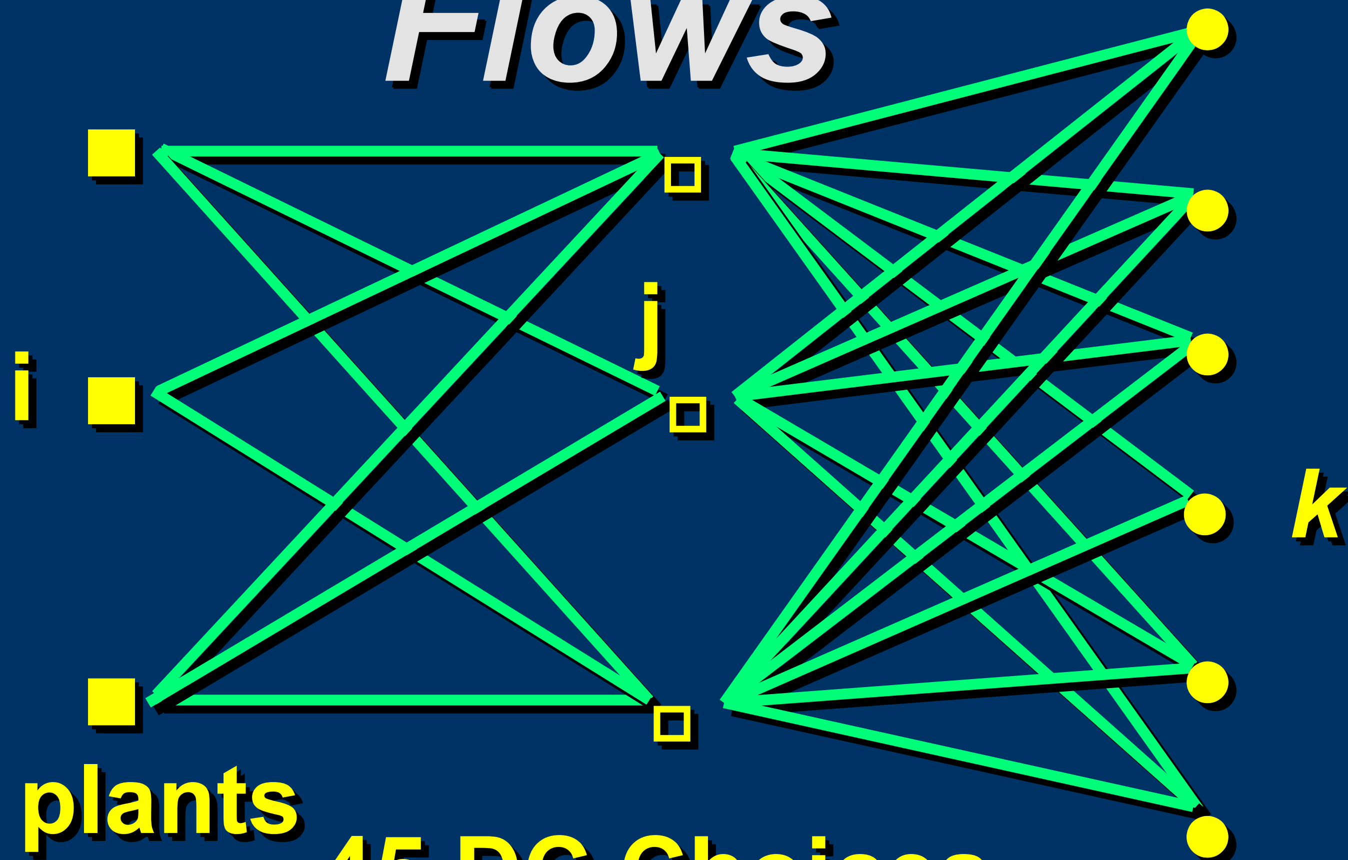
- Heuristics
  - Add, drop, and/or exchange
  - Linear programming relaxation
  - Bounding (Lagrangian relaxation)
- Optimization methods
  - Large-scale mixed integer programming
  - Benders decomposition
  - Lagrangian relaxation (e.g., dualize capacity constraints to give uncapacitated facility location subproblem

Hunt-Wesson Foods

Ingredients

- Multiple products
- Multiple plants
- Many DCs, many customers
- Site selection and sizing
- Customer service levels
- Complex costs

Flows



14 plants

45 DC Choices

121 Customer Zones

17 Product Groups *p*

Data Preprocessing

- 49 product-plant combinations
  - (from 14x17 = 238)
- 682 DC-customer zone combinations
  - (from 45x121 = 5,445 possibilities)

Data Preprocessing

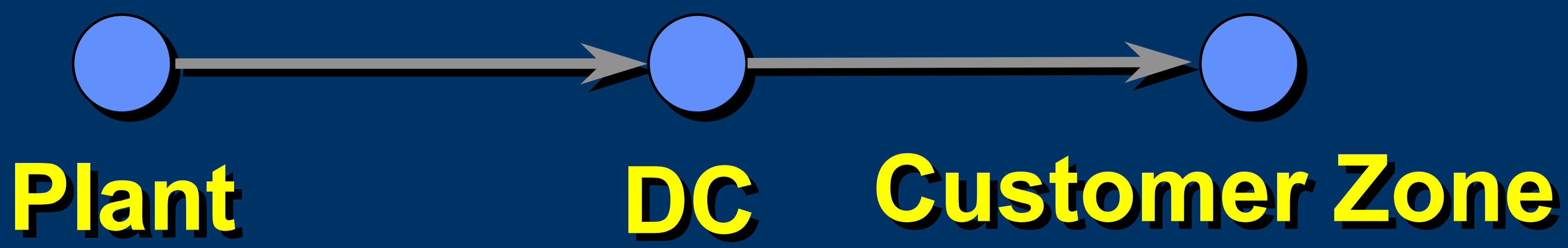
- 23,513 product-plant-DC-customer combinations
  - (from 49x682 = 33,418 possibilities)

System Requirements

- Data easy to acquire
- Inexpensive/quick to run
- Easily updated
- User-oriented
- Flexible (what if capabilities)
- Measurable benefits

Indices

- p = products
- i = plants
- j = distribution centers
- k = customer zones



Decision Variables

- x_{pijk} = amount of product p shipped from plant i to customer zone k through DC j
- z_j = 1 if DC j open
- y_{jk} = 1 if DC is sole source of customer zone k

Constraints

$$\Sigma_{jk}\, x_{pijk} \leq S_{pi}$$

$$\Sigma_{i}\, x_{pijk} = D_{pk} y_{jk}$$

$$\Sigma_{j}\, y_{jk} = 1$$

$$\underline{V}_j z_j \leq \Sigma_{pk} D_{pk} y_{jk} \leq \overline{V}_j z_j$$

$$x_{pijk} \geq 0$$

$$z_j, y_{jk} = 0 \text{ or } 1$$

+ Configuration Constraints on y,z

Objective

$\Sigma_{pijk}\ c_{pijk} x_{pijk}$ Transportation Cost

$+\ \Sigma_j\ f_j z_j$ Fixed DC Cost

$+\ \Sigma_j\ v_j \Sigma_{pk}\ D_{pk} y_{jk}$ Variable DC Cost

Model Development

- Aggregation of Data
- Preselection of Certain Decisions in Large Applications

Choice of Models

Why integer instead of linear programming?

Power of Integer Programming

- Fixed costs
- Bounding # of facilities
- Precedence constraints
- Mandatory service constraints
  - Sole sourcing
- Service timing

Stages in Model Development

- Probationary analysis
- Analyzing results
  - Sensitivity analysis
  - What if analysis
  - Priority analysis

Today's Lessons

- Facility location and distribution important in practice
- Geometry of fixed cost modeling
- Model choice is important in problem solving
  - Strong vs. weak forcing constraints
- Optimization models are able to solve large-scale practical problems