

2.098/6.255/15.093 Optimization Methods

Practice True/False Questions

December 11, 2009

Part I

For each one of the statements below, state whether it is true or false. Include a 1-3 line supporting sentence or drawing, enough to convince us that you are not guessing the answer, but not a comprehensive, rigorous formal justification.

- a) There exist nonempty and bounded polyhedra of the form $\{x \in \mathbb{R}^n : Ax \leq b\}$, in which every basic solution is also a vertex.
- b) Consider the problem of minimizing $c'x$ subject to $Ax \leq b$. If we increase some component of b , then the optimal cost cannot increase.
- c) A feasible linear program of minimizing $c'x$ subject to $Ax \geq b$ has finite cost if and only if c is a nonnegative combination of the rows of A .

For the next three statements, assume that we are dealing with a linear programming (minimization) problem in standard form, with the matrix A having full row rank.

- d) If a tie occurs while choosing the pivot row during the primal simplex method, then the next basic feasible solution will be degenerate.
- e) If the optimal cost is $-\infty$, then the right hand side vector b can be adjusted in some way to make the optimal cost finite.
- f) The dual of the Phase I problem will never have an infinite optimal cost.

Part II

- g) On very degenerate LP problems, the simplex method performs better than interior point methods.

- h) Given a local optimum \bar{x} for a nonlinear optimization problem it always satisfies the Kuhn-Tucker conditions when the gradients of the tight constraints and the gradients of the equality constraints at the point \bar{x} are linearly independent.
- i) In a linear optimization problem with multiple solutions, the primal-dual barrier algorithm always finds an optimal basic feasible solution.
- j) The larger the condition number of the Hessian, the slower the convergence of Newton's method.
- k) The convergence of the steepest descent method depends highly on the starting point.
- l) For the problem $\min x$ s.t. $y \leq x^3$, $y \geq 0$, the gradients of the constraints satisfy the constraint qualification condition (CQC).
- m) For a constrained optimization problem, if a point \bar{x} is feasible, satisfies the CQC, and satisfies the KKT conditions, then it is a local minimum.

Solutions

Part I

- a) True. Consider $P = \{x \in \mathbb{R} : -x \leq 0, x \leq 1\}$. Then $x = 0$ and $x = 1$ are the only basic solutions, but they are also vertices of P .
- b) True. Increasing a component of b gives a feasible set which is no smaller, so the optimal cost cannot increase. (A more complicated way to see this is to take the dual and invoke a shadow price argument using the sign of the dual variables).
- c) True. The dual problem is written as

$$\begin{array}{ll} \text{maximize} & b'p \\ \text{subject to} & A'p = c \\ & p \geq 0. \end{array}$$

If c is a nonnegative combination of the rows of A , then the dual is feasible. Since we are given that the primal is feasible, it follows that both problems have equal and bounded optimal costs. Conversely, if the primal has a bounded optimal cost, the dual must also. In particular, the dual must be feasible, which implies that c is a nonnegative combination of the rows of A .

- d) True. Let θ^* be the value of the min-ratio test. Then all basic variables x_j which have this value of $\theta_j = \frac{x_j}{u_j}$, where u_j is the j^{th} component of the basic direction become $x_j - \theta^*u_j = x_j - \theta_j u_j = 0$. Since there are more than one of these such variables, at least one stays in the basis, which implies that the resulting BFS is degenerate.
- e) False. If an LP is unbounded, its dual must be infeasible. But dual feasibility depends in no way on the vector b .
- f) True. If the dual of the phase I formulation has an infinite optimal cost, then the phase I problem must be infeasible. This cannot be true, since $(x, y) = (0, b)$ is always used as a starting BFS for the phase I problem.

Part II

- g) False. Barrier interior-point methods are unaffected by degeneracy; see BT p. 439.
- h) True. KKT conditions hold for a local minimum under the linearly independent constraint qualification condition (CQC).
- i) False. Barrier interior-point methods find an interior point of the face of optimal solutions. See BT p. 423.
- j) False. For example, Newton's method converges in one iteration for quadratic problems, regardless of condition number.

- k) True. Recall the zig-zag phenomenon shown in lecture.
- l) False. The only local minimum is $(0, 0)$. At this point, the gradient of the constraints are linearly dependent.
- m) False. Since the KKT are essentially first-order conditions, they can only “see” the linear part of the objective and constraints, and are thus unable to ensure even local optimality (unless further conditions, such as convexity, are imposed). As a simple example, consider the objective function $f(x) := -x^2$, and the feasible set defined by $g(x) := x \leq 0$. Clearly, the origin $x = 0$ is a KKT point (since $\nabla f(x) + u_1 \nabla g(x) = 0 + 0 \cdot 1 = 0$), but it is a local maximum (not a minimum).

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15.093J / 6.255J Optimization Methods
Fall 2009

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