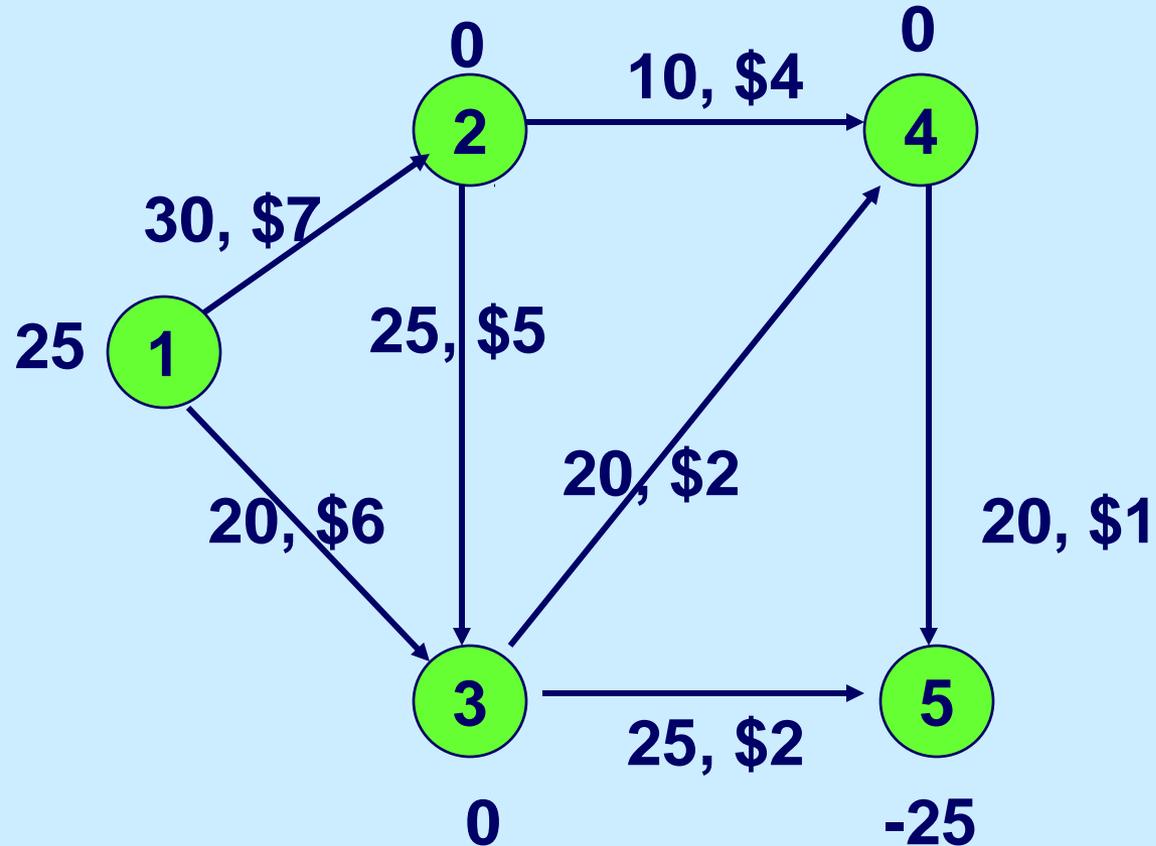


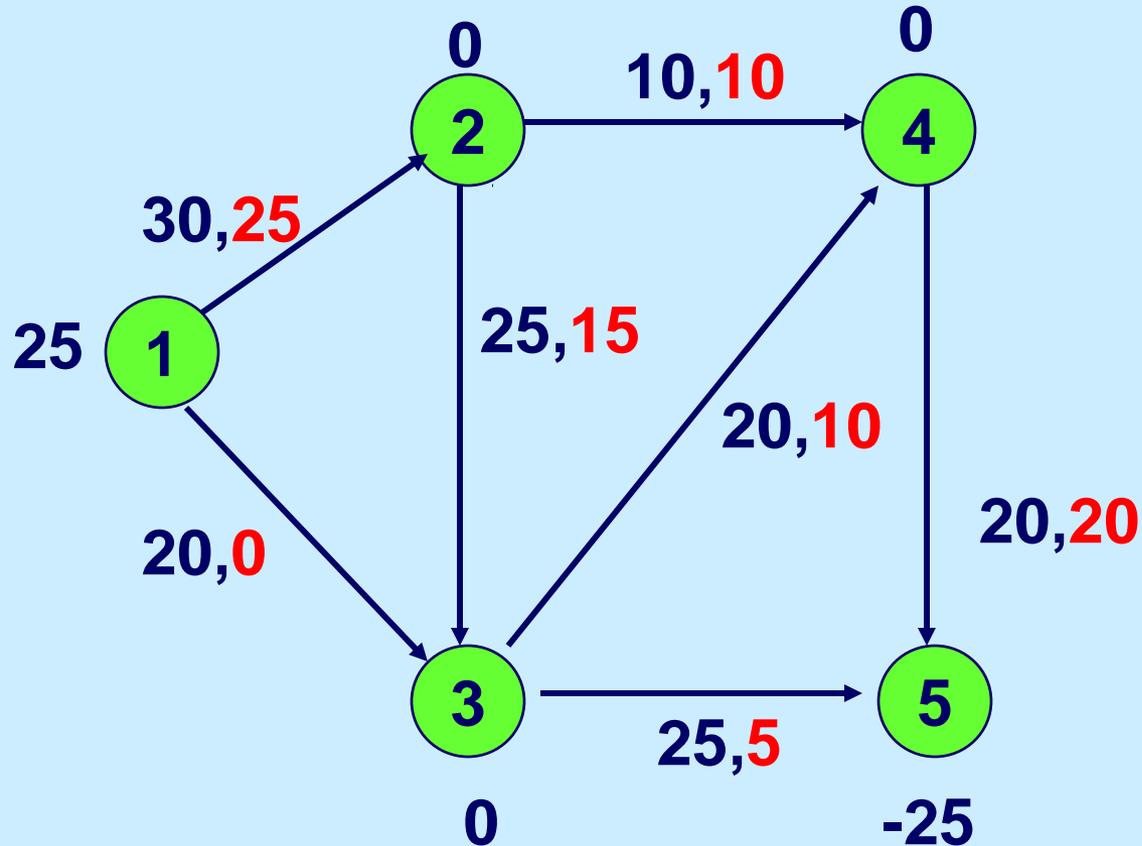
15.082J and 6.855J and ESD.78J

Cycle Canceling Algorithm

A minimum cost flow problem

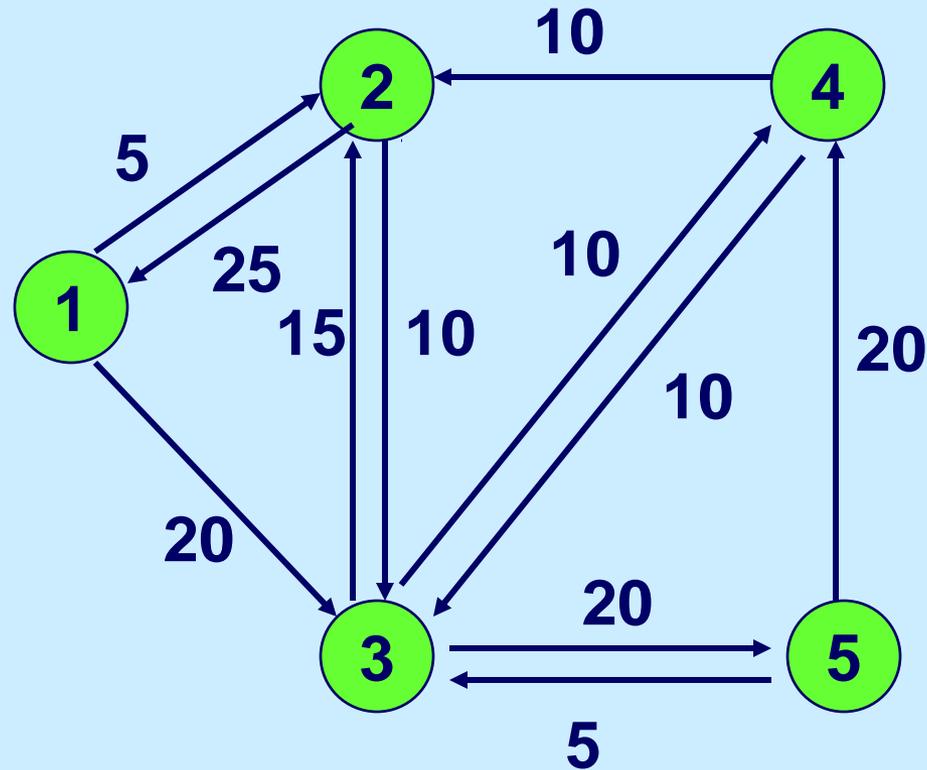


The Original Capacities and Feasible Flow

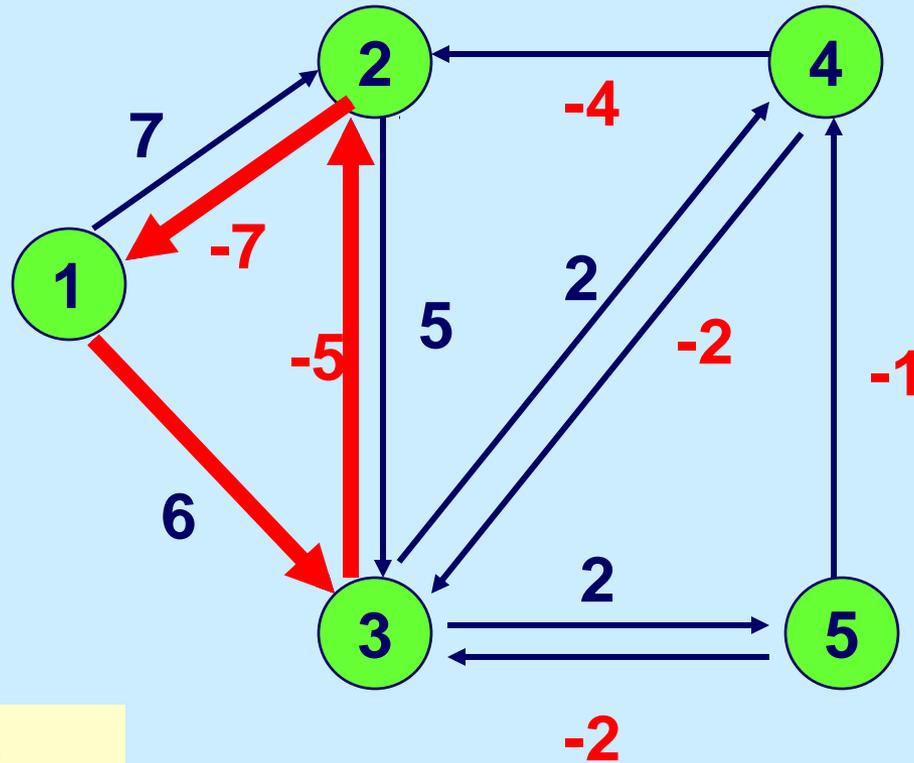


The feasible flow can be found by solving a max flow.

Capacities on the Residual Network

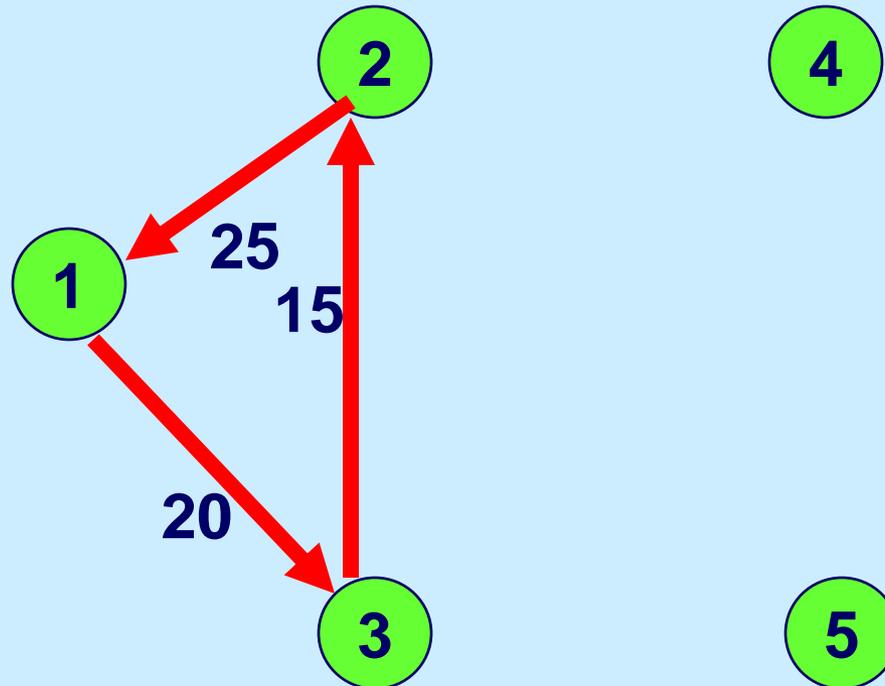


Costs on the Residual Network



Find a negative cost cycle, if there is one.

Send flow around the cycle

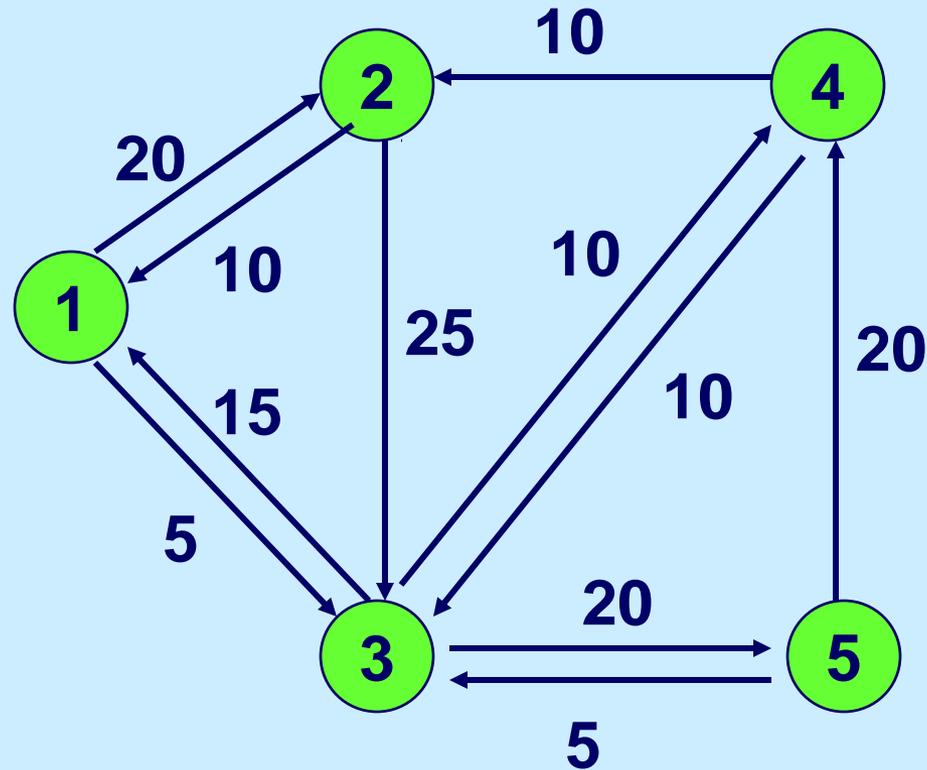


Send flow around the negative cost cycle

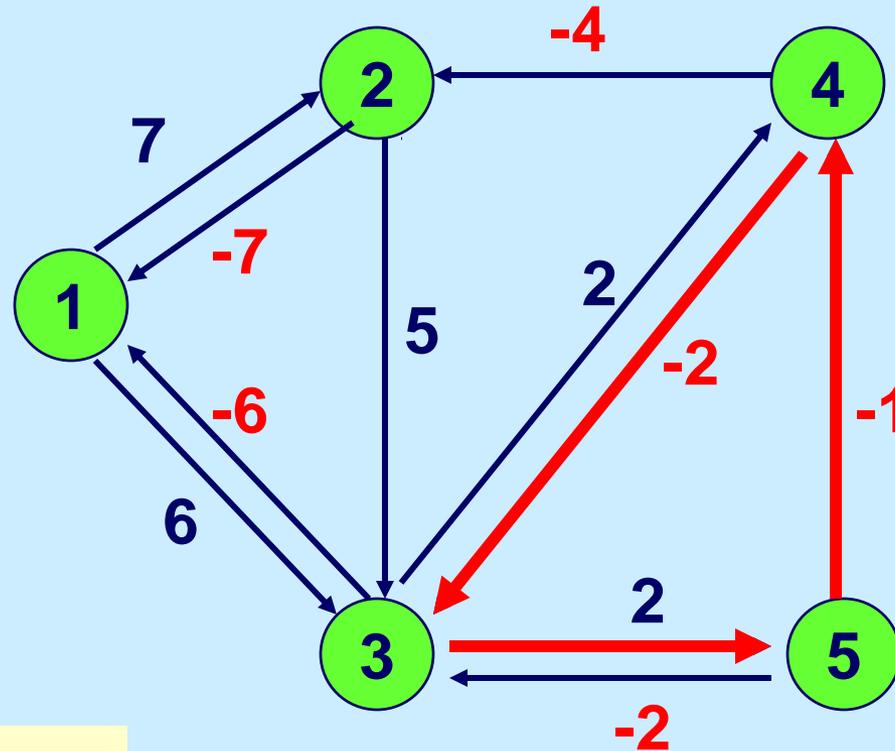
The capacity of this cycle is 15.

Form the next residual network.

Capacities on the residual network



Costs on the residual network

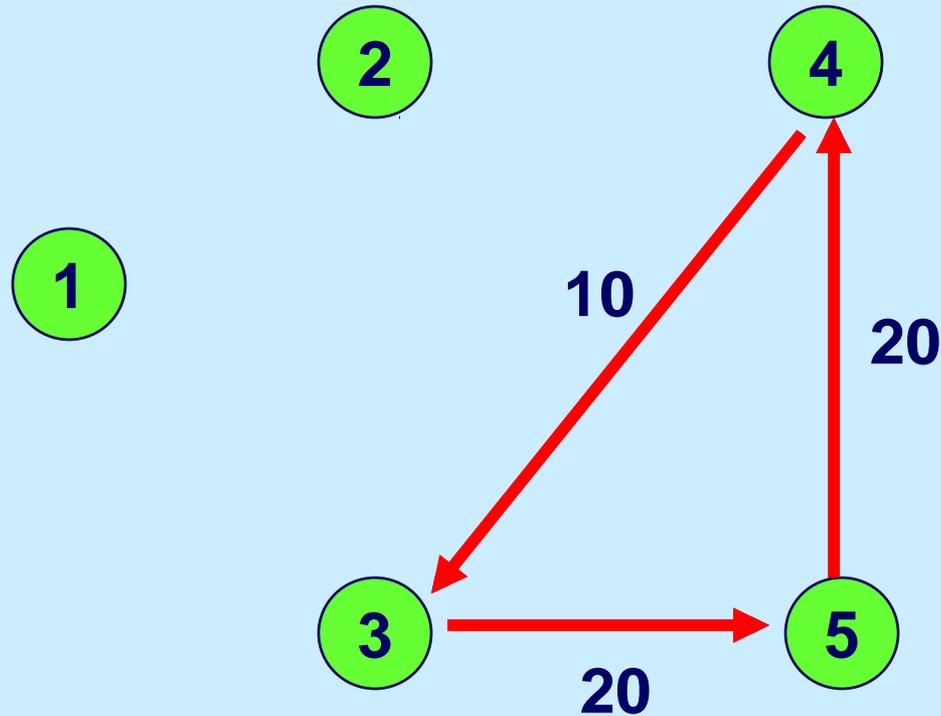


Find a negative cost cycle, if there is one.

Send flow around the cycle

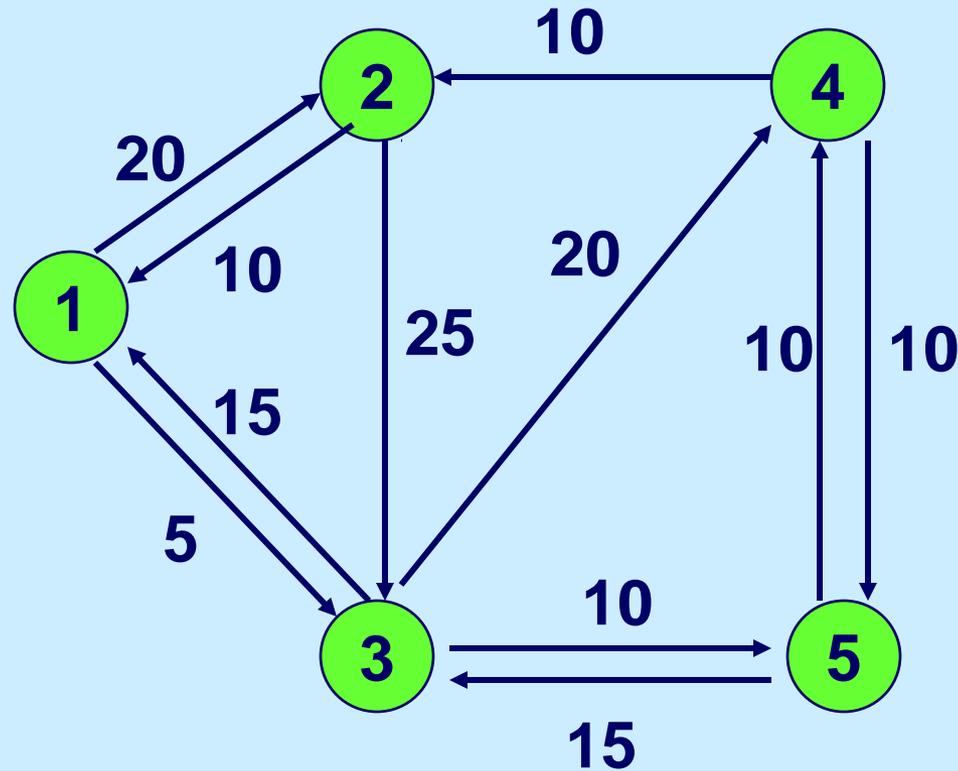
Send flow around the negative cost cycle

The capacity of this cycle is 10.

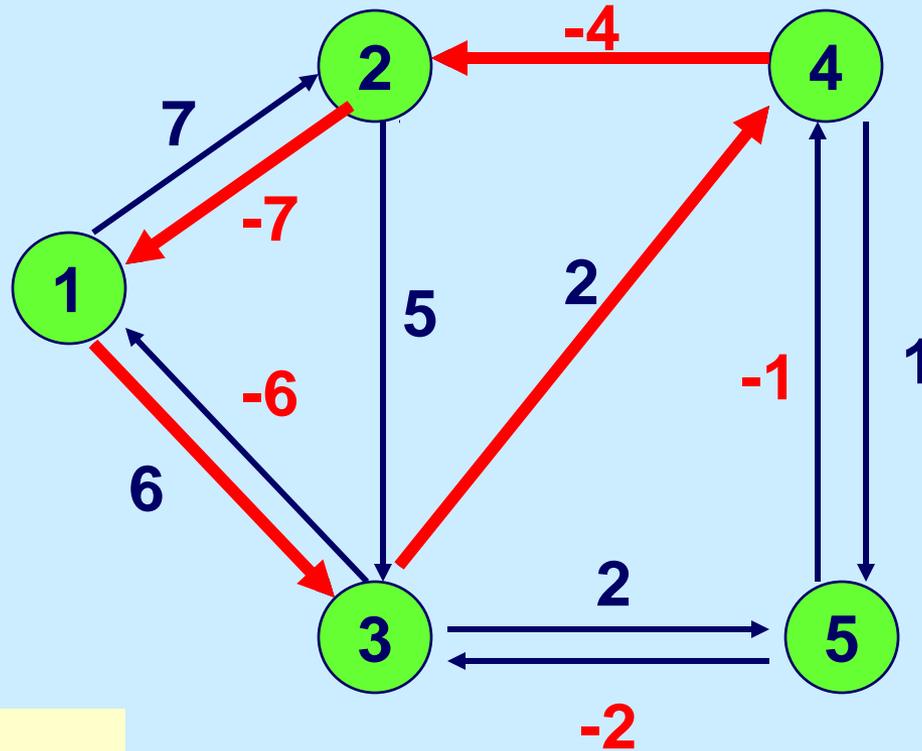


Form the next residual network.

Capacities on the residual network

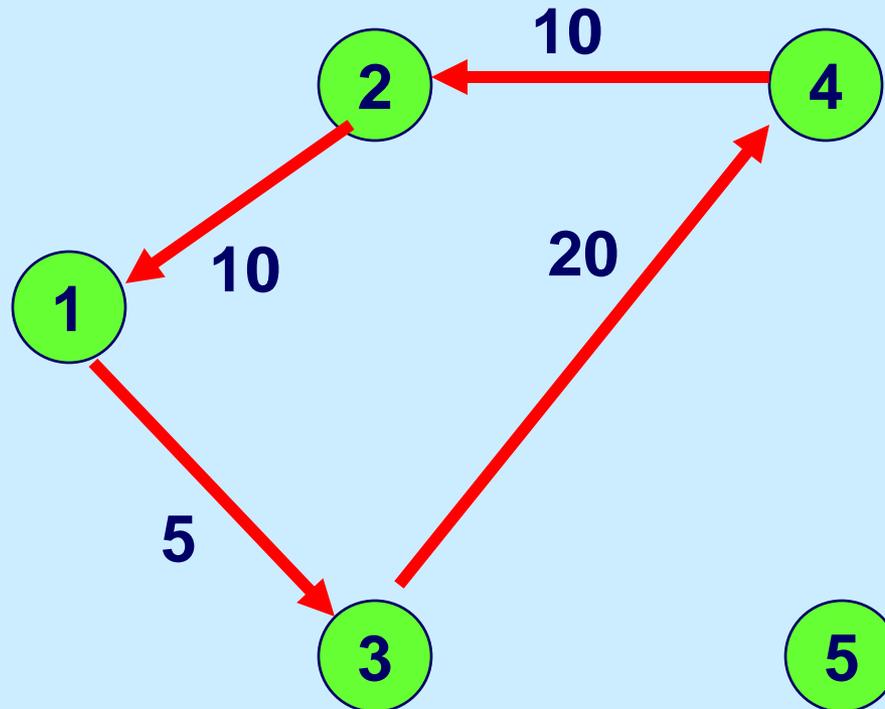


Costs in the residual network



Find a negative cost cycle, if there is one.

Send Flow Around the Cycle

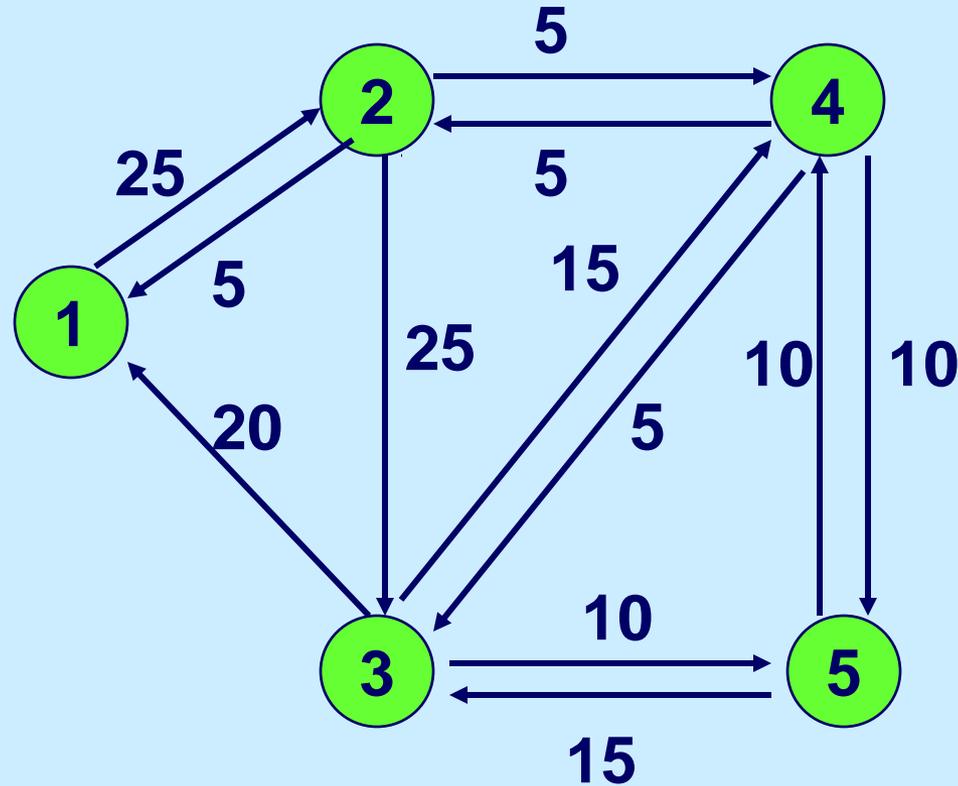


Send flow around the negative cost cycle

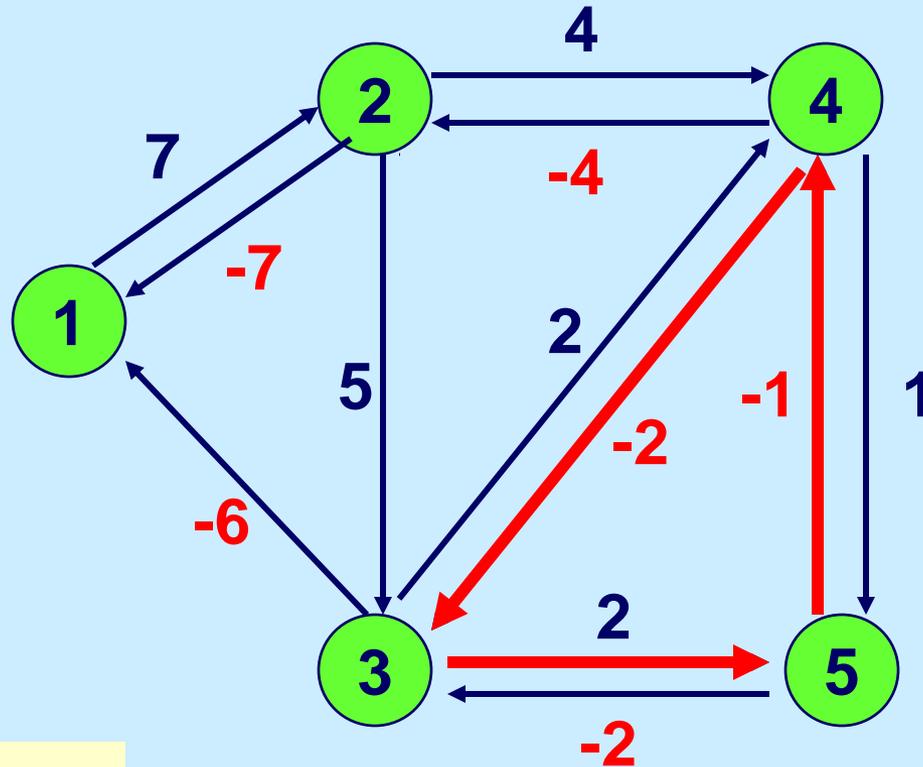
The capacity of this cycle is 5.

Form the next residual network.

Capacities on the residual network



Costs in the residual network

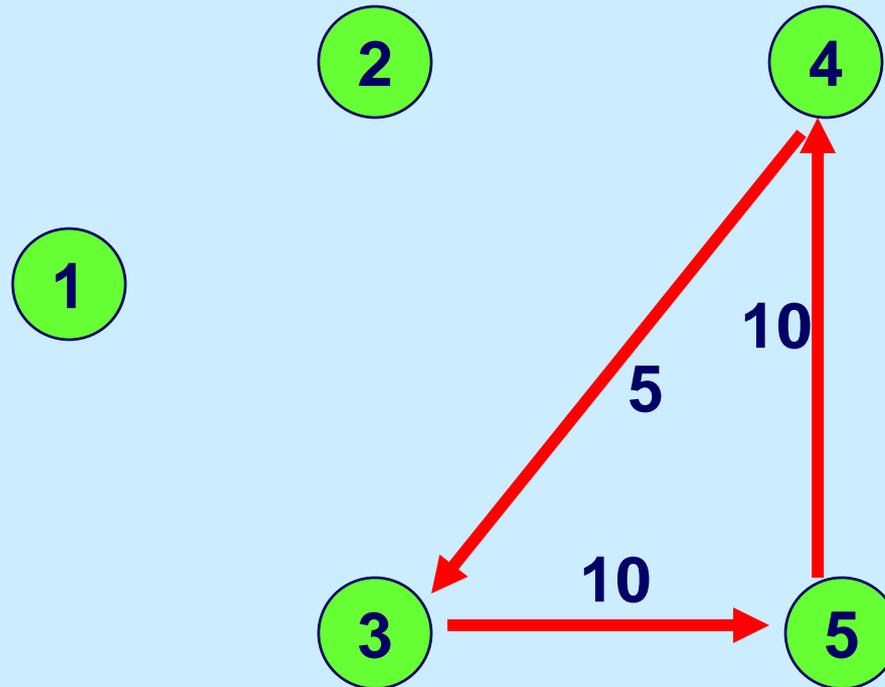


Find a negative cost cycle, if there is one.

Send Flow Around the Cycle

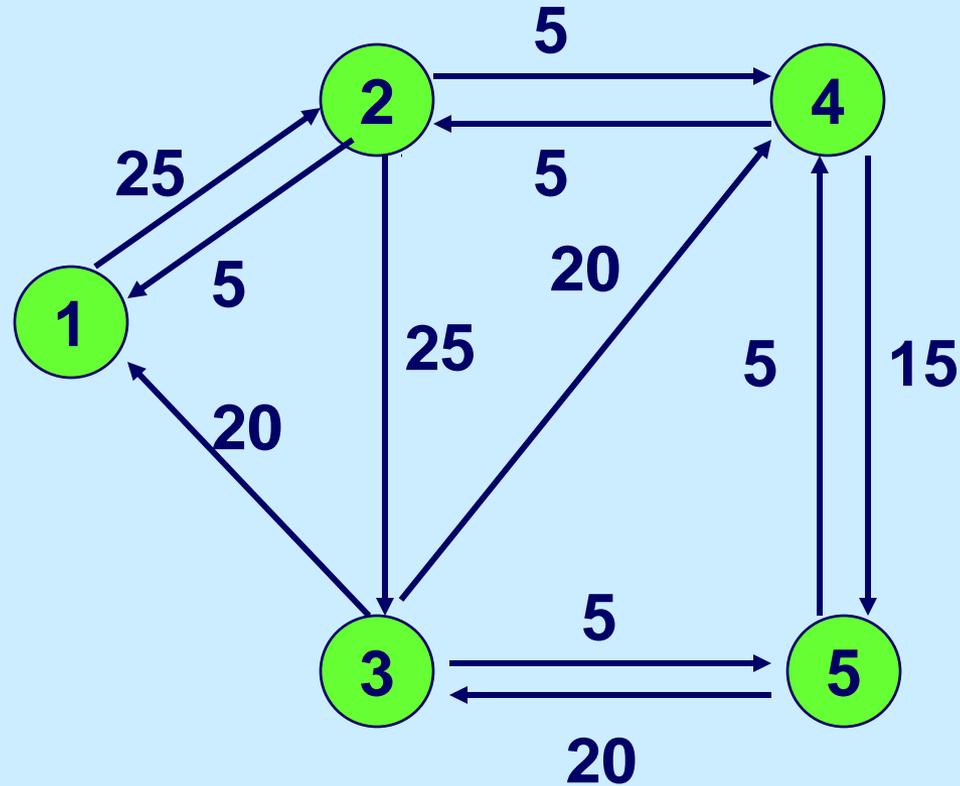
Send flow around the negative cost cycle

The capacity of this cycle is 5.

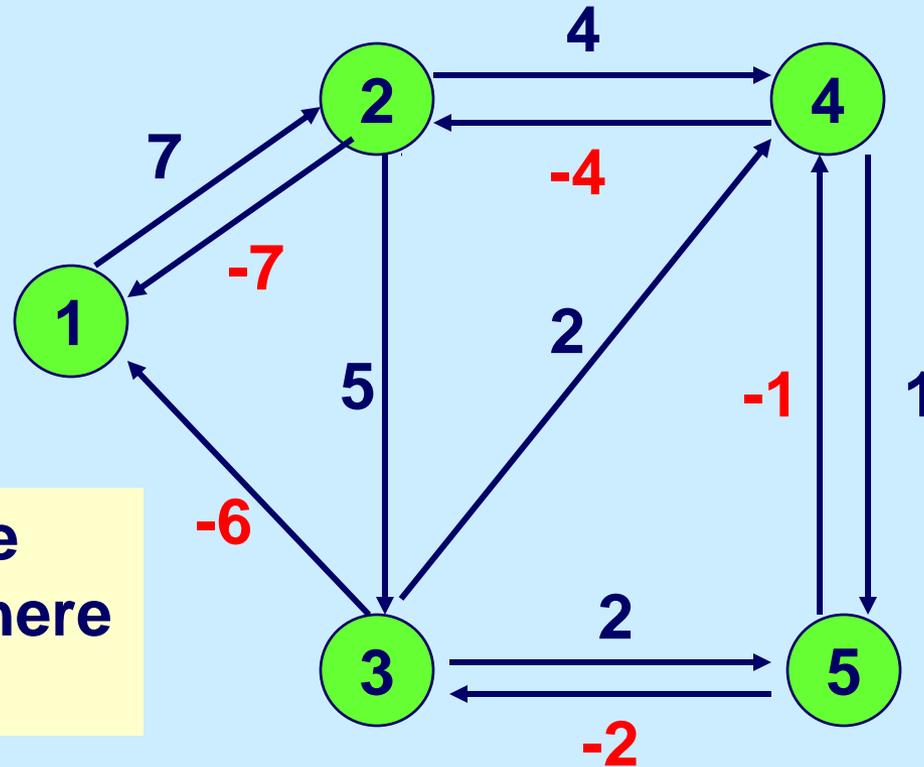


Form the next residual network.

Capacities on the residual network



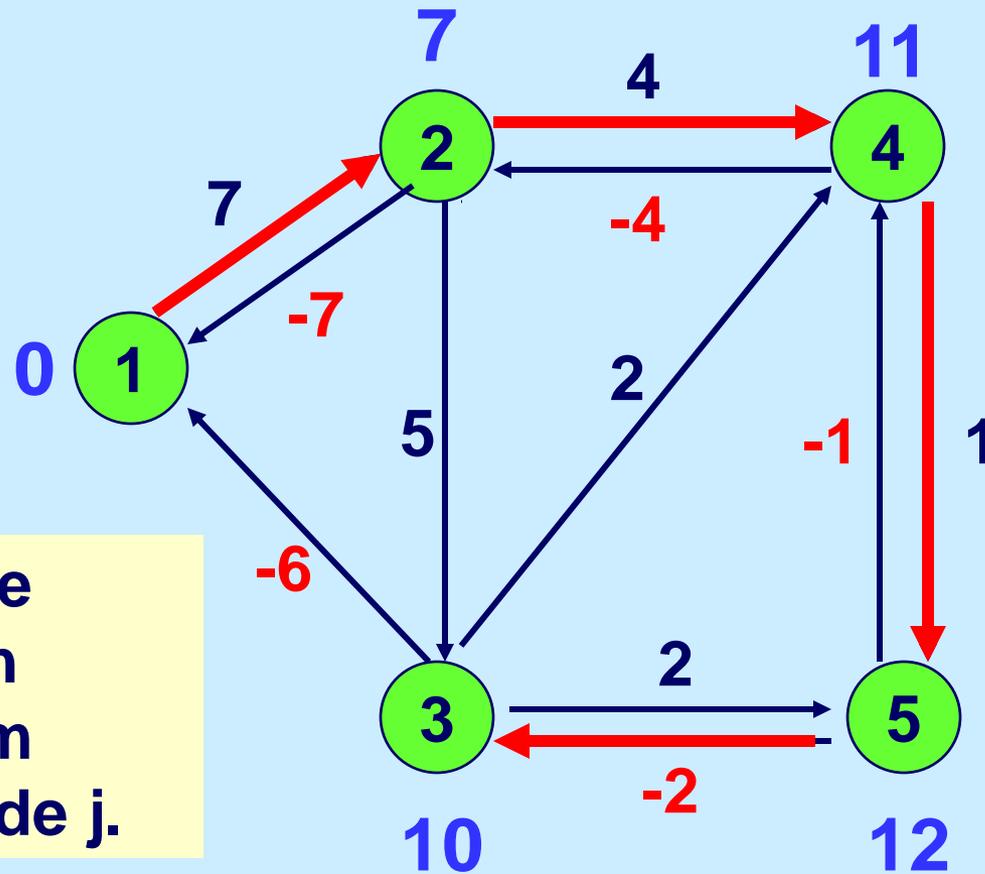
Costs in the residual network



Find a negative cost cycle, if there is one.

There is no negative cost cycle. But what is the proof?

Compute shortest distances in the residual network

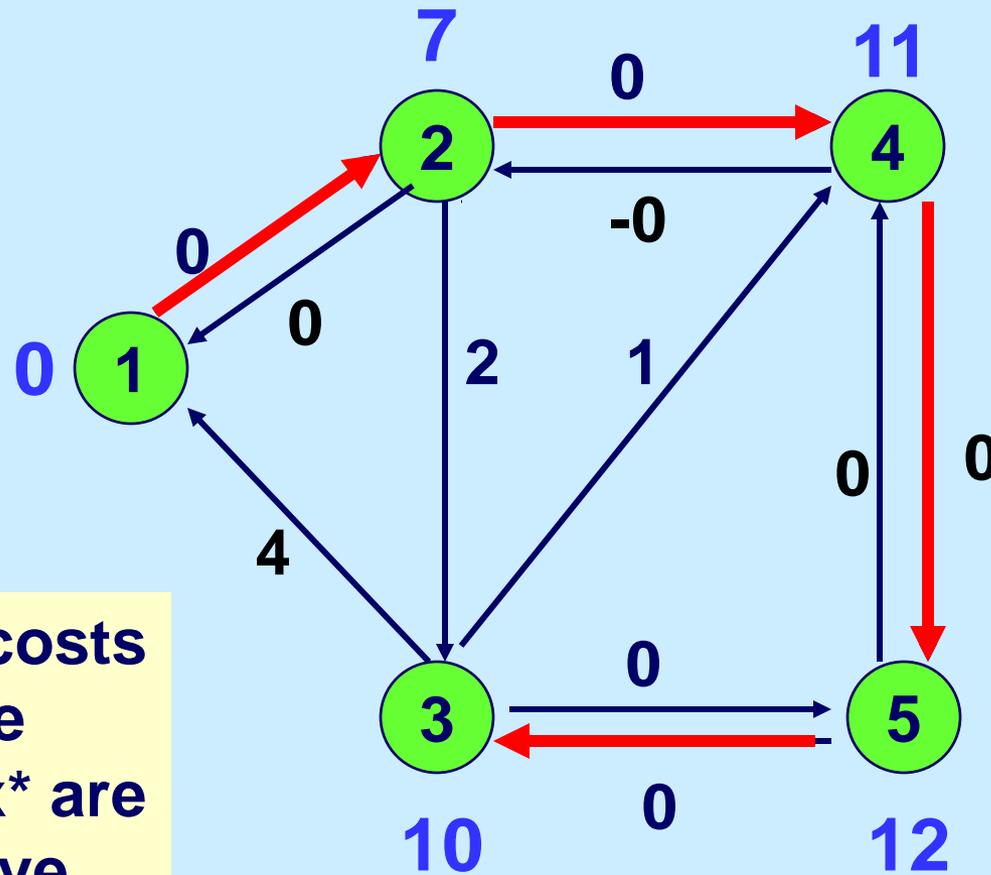


Let $d(j)$ be the shortest path distance from node 1 to node j .

Next let $\pi(j) = -d(j)$

And compute c^π

Reduced costs in the residual network



The reduced costs in $G(x^*)$ for the optimal flow x^* are all non-negative.

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15.082J / 6.855J / ESD.78J Network Optimization
Fall 2010

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