

## MEMORANDUM

**To:** Professor Graves

**From:** Team 3 - Tammy Greenlaw, Chris Caballero, Aaron Raphel,  
Minja Penttila, Cliff Smith

**Date:** August, 2003

**Re:** 15.066 - System Optimization  
Pump System Design: Optimizing Total Cost over System Life Cycle

**Executive Summary**

Traditionally, pump and pipe systems are designed by beginning with a given pipe design (diameter and physical layout). The pump is then selected for the pipe layout by considering the operating costs for the pump and the capital costs of the pipe. Additionally, the pumps are often designed by engineering consultants who oversize the pumps for the system to guarantee that pump is not undersized. Unfortunately, this causes poor efficiencies and, consequently, higher operating costs. We believe that an improved methodology includes:

1. Selecting the pipe design concurrently with the pump design
2. Including the capital costs of the pumps in the life cycle cost analysis during the system design

The utility of this methodology was demonstrated by using a linear, integer optimization model to select optimal combinations of pipe systems and pump systems.

The optimization model was then extended to assess the impact of various energy rate structures and potential (pending) tax implementations on carbon emissions. These analyses did show how certain “price-break” structures give incentives to build less-efficient pump and pipe systems.

**Background**

The total energy in a pumping system moving water from Point A to Point B in a full pipe at a constant flowrate (Q) can be calculated at any point in the pipe using the Bernoulli equation.

$$P_A + V_A^2/2g + Z_A = P_B + V_B^2/2g + Z_B + H_f$$

where:

- $P_X$  = Pressure
- $V_X$  = Velocity
- $g$  = Gravitational constant
- $Z_X$  = Elevation
- $H_f$  = Energy lost as heat due to friction

If you consider only conditions at Points A and B, one can assume that the velocity at Point A (pump) and the velocity at Point B (exit) is zero, and assume that pressure at Point B is zero (atmospheric pressure), the equation simplifies to:

$$P_A = H_f + \Delta Z$$

$P_A$  is the pressure that the pump must add to move water from Point A to Point B at flow Q.  $P_A$  is usually expressed in feet (similar to inches of mercury) and referred to as System Head (H) or Total

Dynamic Head (TDH).  $\Delta Z$  is the change in elevation from Point A to Point B. Where  $\Delta Z$  is positive, water is pumped to a higher elevation and energy is stored as potential energy. Where  $\Delta Z$  is negative,  $P_A$  is reduced or water flows by gravity.  $H_f$  is also expressed in feet and often referred to as Frictional Loss or Headloss due to friction.  $H_f$  is the amount of energy lost as heat due to the friction of water moving along pipe walls or fittings. In industrial pumping, most pumping energy is actually spent overcoming frictional losses (Hawken, et. al, p. 115). Appendix X shows the frictional loss and elevation change calculations used to generate System Head values for each pipe diameter at flow  $Q$ . Frictional losses are calculated along straight pipe and through fittings as follows:

$$\begin{array}{ll} \text{Fittings:} & H_f = kV^2/2g \\ \text{Pipe:} & H_f = fLV^2 / 2gD \end{array} \quad (\text{See Appendix B1 for further details})$$

Fluid flow in full pipes can be expressed as:

$$Q = VA$$

where:  $Q$  = Flowrate  
 $V$  = Velocity  
 $A$  = Cross-sectional area of pipe

Therefore, at a constant flowrate ( $Q$ ), velocity ( $V$ ) can be reduced by increasing the pipe diameter. Since frictional energy losses along straight pipe and through fittings are directly proportional to  $V^2$ , increasing pipe diameter significantly reduces frictional losses.

### **Project Summary**

We have created a linear, integer model to aid in the concurrent selection of a pump size and a pipe diameter. The model optimizes the pumping system design by selecting two components, pump size and pipe diameter, based on their impact on the system life cycle costs. Binary decision variables include four pump options and four pipe diameter options; the program is structured to pick one pump and one pipe option to minimize the life cycle costs ( $Z$ ) as follows:

$$\text{Minimize } Z = C_j + C_i + n \times C_{om}$$

where:  
 $C_j$  = Capital costs to purchase and install pump,  
 $C_i$  = Capital costs to purchase and install pipe, fittings, and valves),  
 $C_{om}$  = Operating costs due to pump energy consumed over assumed life cycle,  
 $n$  = Life cycle in years

Preliminary engineering calculations provided performance and capital cost parameters for each piping configuration option and pump size option. Performance parameters including pump efficiency ( $\eta_p$ ), motor efficiency ( $\eta_m$ ), and system head are used to calculate the annual energy consumption for any combinations of pump and pipe diameter. Capital cost estimates for each pump option and each pipe option are utilized directly as part of the objective function. The main engineering assumptions made to calculate energy consumption and capital costs for each combination of pump and pipe options are:

- Flow ( $Q$ ) is constant at 750 gallons per minute (gpm)
- The pump is centrifugal with On/Off controls
- Total pipe length is 400 linear feet + fittings and valves
- Pipe is Schedule 40 steel with grooved connections
- Point B is 40 feet higher (elevation) than Point A

- The pump is running continuously (95% service factor)
- Life Cycle = 20 years

Four energy cost structure options are modeled to convert energy consumption (kwh) to annual operating costs. Table 1 shows the Option 1 rate structure that decreases in a step-wise fashion with increasing energy use. This option represents a simplified public utility rate structure that provides volume discounts. The unit cost function is not continuous, i.e. if a customer purchases a volume of energy that puts them in the higher consumption bracket, they pay the lower unit cost for their total energy consumption. Table 2 shows the resulting Total Cost function for Option 1.

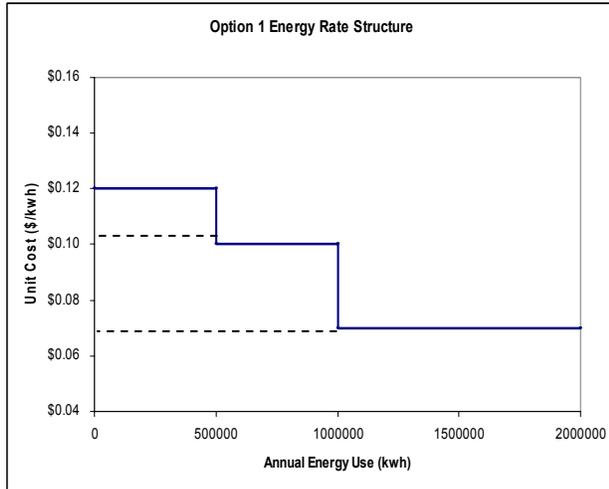


Table 1: Option 1 Unit Energy Costs

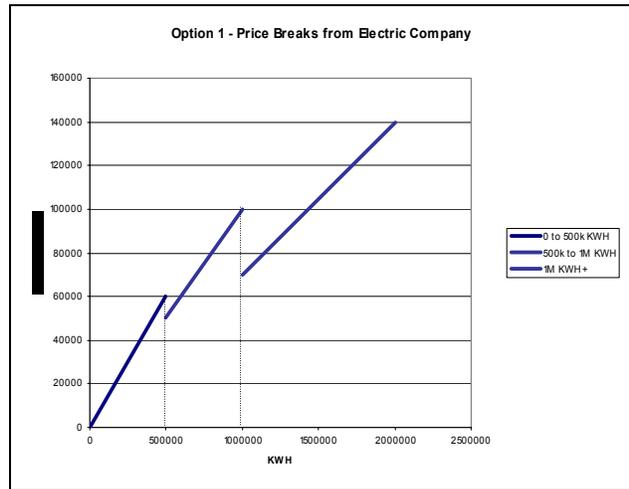


Table 2: Option 1 Total Energy Cost Function

Table 3 shows the Option 2 rate structure that increases in a step-wise fashion. This option represents an industrial facility that produces its own power on site. Their unit costs vary based on the assumed efficiency of their power boilers; the lowest unit cost represents power produced by their most efficient boiler; the mid-range unit cost represents power produced by a less efficient boiler; the highest unit cost assumes they must purchase energy from a public utility. Table 4 shows the resulting Option 2 total cost function.

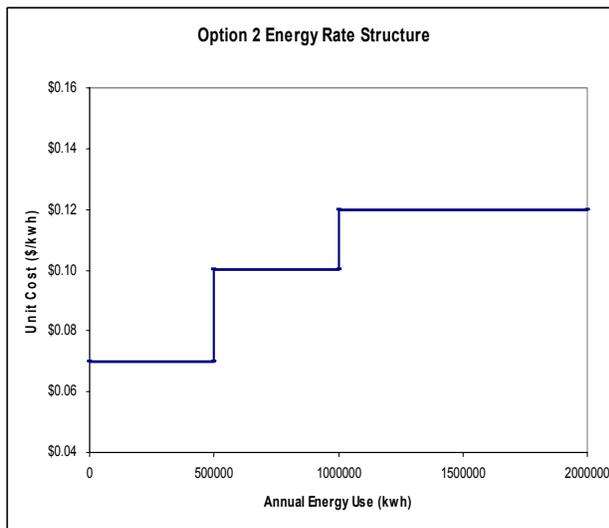


Table 3: Option 2 Unit Energy Costs

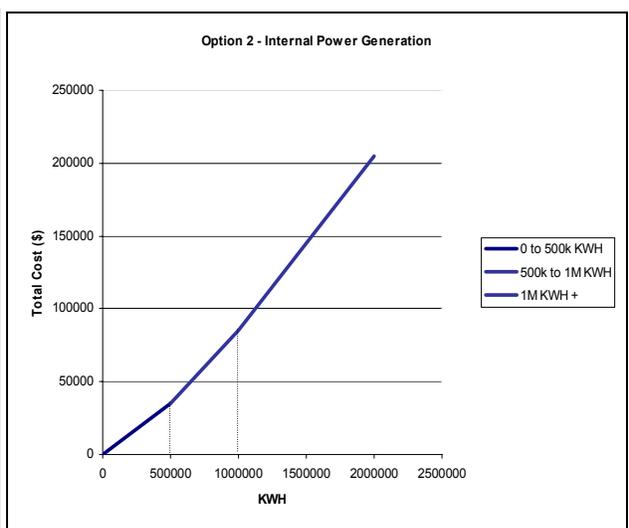


Table 4: Option 2 Total Energy Cost Function

The final energy rate options represent situations where the industrial facility must evaluate the probability that legislation implementing a carbon tax will be passed five years after the system is installed. These options assume that the facility purchases their power from a coal-fired power plant with a flat base rate or the base rate structure given in Option 1, or utilizes on-site coal-fired boilers as modeled in Option 2; in both cases, we assumed that the carbon tax costs will be directly absorbed by the energy consumer. Based on the 1999 Department of Energy report on Carbon Dioxide Emissions from the Generation of Electric Power in the United States, the average carbon emission per kwh produced at coal-fired power plants is 0.57 lbs C/kwh. Based on regional proposals to tax emissions, we modeled a probabilistic case where there is a 50% probability of no carbon tax being implemented, a 30% probability of a moderate carbon tax being implemented (\$5/1000 lb C), and a 20% probability of a more aggressive carbon tax being implemented (\$20/1000 lb C). In both cases, the probability of a carbon tax increases the unit cost rates.

### **Project Selection**

“Motors use three-fifths of the world’s electricity. Pumping systems use at least a fifth of their total output. In industrial pumping, most of the motors energy is actually spent in fighting against friction. Traditional optimization compares the cost of fatter pipe with only the value of the saved pumping energy. This comparison ignores the size, and hence the capital cost, of the equipment – pump, motor, motor-drive circuits, and electrical supply components – needed to combat the pipe friction.” (*Natural Capitalism*, Hawken et al, 1999)

We chose a project that expands on a recent engineering design idea that allows an industrial facility to minimize costs and reduce environmental impact simultaneously. Industrial pump system design has not changed significantly since the development of mechanical pumps. Appendix D is an excerpt from the book *Natural Capitalism* in which the authors refer to a simple, updated method of designing pumping systems that differs from the traditional method. Table X summarizes the differences between the traditional and proposed methods.

	Traditional	Proposed
Engineering Steps	<ol style="list-style-type: none"> <li>1. Design building based on major processes, equipment, and material flows.</li> <li>2. Locate pumps.</li> <li>3. Layout pipe runs.</li> <li>4. Select pipe diameters.</li> <li>5. Calculate frictional losses and TDH.</li> <li>6. Size pump based on prior decisions and calculations.</li> </ol>	<ol style="list-style-type: none"> <li>1. Design building based on major processes, equipment, and material flows including pipe runs.</li> <li>2. Locate pumps to minimize pipe length and bends.</li> <li>3. Select pipe diameters and size pumps as a system based on life cycle analysis.</li> </ol>
Cost Analysis	Consider operating costs (pumping energy) vs. capital costs to install pipe.	Optimize system costs given design life cycle. Consider operating costs (pumping energy) vs. capital costs to install pipe AND capital costs to install pump.

Table X: Pumping System Design Methods

We wanted to develop a model that compared the traditional method of sizing pumps given the System Head (based on pipe layout and diameter) vs. the proposed system design method that integrates pump and piping system design to optimize life cycle costs. Additionally, our model considers the impact of different electricity rate structures on pumping system design decisions.

**Results**

In order to determine whether including the pump capital costs in the life cycle cost analysis had an impact on the results, we set the Option 1 model energy rates to a flat rate and optimized two cases, (1) pump capital costs included in the objective function, and (2) pump capital costs set to \$0. With the number of pumping systems (n) equal set to 5 and the life cycle set to 15 years, including the pump capital costs in the life cycle analysis changed shifted the results as shown in Table X.

Number of Systems - n	Decision Variable Design Results	Energy Consumption (kwh)	Life Cycle Cost	Pump Cost (add if not included in optimization)	Total System Life Cycle Cost	Life Cycle - t (yrs.)	Optimization includes pump capital costs	
5	P2D4	420095	\$ 641,099	\$ 177,000	\$ 818,099	15	No	
5	P1D5	376302	\$ 810,617	\$ -	\$ 810,617	15	Yes	
Delta		43793					\$ 7,482	
Delta lbs Carbon		24962						

Table X: Flat Rate (\$0.07/kwh) Results with and without Pump Capital Costs included in Optimization

We ran Option 1 a number of times varying number of pumping systems in order to observe the impact of the decreasing step power rate structure. At the points where energy consumption is near 500,000 kwh and 1,000,000 kwh, the optimal solution is to pick a less efficient pump and pipe combination to take advantage of the lower unit cost.

Number of Systems - n	Decision Variable Design Results	Energy Consumption (kwh)	Life Cycle Cost	Life Cycle (yrs)
1	P1D5	75260	\$ 263,725	20
2	P1D5	150521	\$ 527,450	20
3	P1D5	225781	\$ 791,175	20
4	P1D5	301042	\$ 874,275	20
5	P1D5	376302	\$ 1,318,625	20
6	P2D4	504111	\$ 1,460,627	20
7	P1D5	526823	\$ 1,635,346	20
8	P1D5	602083	\$ 1,868,967	20
9	P1D5	677344	\$ 2,102,587	20
10	P2D3	1012046	\$ 1,755,865	20
15	P1D5	1128906	\$ 2,526,969	20
20	P1D5	1505208	\$ 3,469,291	20

Table X: Option 1 Results

With a life cycle of 20 years, the optimal Option 2 result for any number of pumping systems is the most efficient pump and pipe combination even at the higher unit cost. If we set the life cycle to 15 years, and vary the number of pumping systems we can see the effects of the increasing unit cost power rate structure as shown in Table X.

Number of Systems - n	Decision Variable Design Results	Life Cycle (yrs)
1	P2D4	15
5	P2D4	15
10	P1D5	15
15	P1D5	15
1	P1D5	20

Table X: Option 2 Results

The probabilistic carbon tax results did not change the design decision at the tax rates and assumed system we modeled.

**Conclusions and Model Limitations**

Our program quantified the potential impact of (1) Integrated system design vs. sequential component design, and (2) Varying power rate structures given the assumed pumping system situation.

We can conclude that integrated pumping system design may yield lower life cycle by increasing pipe diameter and decreasing pump size. Including pump capital costs in the life cycle analysis may or may not impact design decisions depending on factors such as the length of the life cycle, and piping capital costs relative to pump capital costs. However, taking the proposed systems design approach has the potential to decrease life cycle costs and energy consumption and should be utilized particularly in situations where pipe diameter is not constrained by other criteria (i.e. minimum fluid velocity or average fluid time in system).

Given our assumed system, power rate structures have the potential to impact life cycle analysis and subsequent design decisions. The exact impact is specific to the situation and rate structure applied.

The model is effective in optimizing the assumed pumping system and illustrating that current engineering design practices may be improved, but it is not practical for repetitive or complicated hydraulic modeling. Although all of our assumptions are reasonable, they limit the analysis in order to be fit to a linear program. Alternatively, existing hydraulic modeling software could be modified to include the system life cycle analysis illustrated in this program or the linear program could be modified to further analyze the results of existing modeling software.

Appendix A: Model Formulation and Figures (CC)

Appendix B: Engineering Calculations

B1: System Head Equations and Spreadsheet Calculations (TG)

B2: System and Pump Performance Curves (TG)

B3: Energy Consumption Equations (TG)

B4: Piping Cost Estimates (CS)

B5: Pump Cost Estimates (CS)

Appendix C: Natural Capitalism Excerpt (TG)

## **APPENDIX A**

### **Model Formulation and Figures**

**Variables** $C_i$  - Capital Cost to install pipe (\$) $C_j$  - Capital Cost to install pump (\$) $C_{om}$  - Annual Operations and Maintenance Cost (per year) $D_iP_j$  - Pipe Diameter (inches) and Associated Pump Size (hp) $E$  - Total Annual Energy Use $E_k$  - Annual Energy Use within Each Pricing Range,  $k = A, B,$  or  $C$  $F_1$  - Annual Cost Savings when  $E_k$  Exceeds Energy Usage 'A' kwh $F_2$  - Annual Cost Savings when  $E_k$  Exceeds Energy Usage 'B' kwh $n$  - Design Life Cycle (years) $Y_m$  - Binomial Decision Variable for Determining Pricing Range**Energy Pricing** (actually cost range with unit price will be provided for final analysis)

Range	Unit Price
0-A (kwh)	C1
A-B (kwh)	C2
B-C (kwh)	C3

**Decision Variables** $D_iP_j, E_1, E_2, E_3, Y_1, Y_2$ **Constraints**

$$\sum D_iP_j = 1$$

 $D_iP_j = \text{Binary}$  $H_i < H_p$  (for all  $i, p$ ) $Y_1, Y_2,$  &  $Y_3 = \text{binary constraints}$ 

$$E_1 + E_2 + E_3 = E$$

**Energy Pricing Options 1 and 2**

$$A * Y_1 \leq E_1 \leq A$$

$$(B-A) * Y_2 \leq E_2 \leq (B-A) * Y_1$$

$$0 \leq E_3 \leq C * Y_2 \quad \text{where } C \text{ is very large}$$

$$F_1 = C_2 * A - C_1 * A \quad (\text{Note: Set } F_1 \text{ to zero for Option 2})$$

$$F_2 = C_3 * (A+B) - C_2 * (A+B) \quad (\text{Note: Set } F_2 \text{ to zero for Option 2})$$

$$C_{om} = C_1 E_1 + C_2 E_2 + C_3 E_3 + F_1 Y_1 + F_2 Y_2$$

*Note: A visualization of the Total Annual Costs can be found on page 3.*

**Objective Function**

$$\text{Minimize Total Cost} = \text{Min}(C_i + C_j + C_{om} * n)$$

$Y_1 = Y_2 = 0$	when $E \leq A$
-----------------	-----------------

$Y_1 = 1, Y_2 = 0$	when $A < E < B$
--------------------	------------------

$Y_1 = Y_2 = 1$	when $E > B$
-----------------	--------------

**Energy Pricing Options 1 and 2, with Taxes**

Probability Distribution for regulatory fee

No tax	with probability	0.5
Low tax	with probability	0.35
High tax	with probability	0.15

Above model is valid with the following adjustments:

$$\text{Low Tax (LT)} = (\$5/1000 \text{ lb C}) * (0.57 \text{ lb C/KWH}) = \$2.86 /1000 \text{ KWH}$$

$$\text{High Tax (HT)} = (\$20/1000 \text{ lb C}) * (0.57 \text{ lb C/KWH}) = \$11.43 /1000 \text{ KWH}$$

$$\text{Annual Operating Cost, No Tax (AOC(nt))} = C_1E_1 + C_2E_2 + C_3E_3 + F_1Y_1 + F_2Y_2$$

$$\text{Annual Operating Cost, Low Tax (AOC(lt))} = (C_1+LT)*E_1 + (C_2+LT)*E_2 + (C_3+LT)*E_3 + F_1*Y_1 + F_2*Y_2$$

$$\text{Annual Operating Cost, High Tax (AOC(ht))} = (C_1+HT)*E_1 + (C_2+HT)*E_2 + (C_3+HT)*E_3 + F_1*Y_1 + F_2*Y_2$$

*Note: A visualization of the Total Annual Costs follows on the next page.*

$$\text{Note: } F_1 = (C_2 + XT)*A - (C_1 + XT)*A = C_2*A - C_1*A, \text{ XT} = \text{LT or HT}$$

$$F_2 = (C_3 + XT)*(A - B) - (C_2 + XT)*(A - B) = C_3*(A - B) - C_2*(A - B), \text{ XT} = \text{LT or HT}$$

$$F_1 = F_2 = 0 \text{ for Option 2}$$

Because we assume that any tax will not go into effect for five years, the total expected operating costs of the system for each possible scenario are:

$$\text{Note: Years Before Tax (YBT)} = 5$$

$$\text{Expected Cost (No Tax)} = P(\text{No Tax}) * (t * \text{AOC(nt)})$$

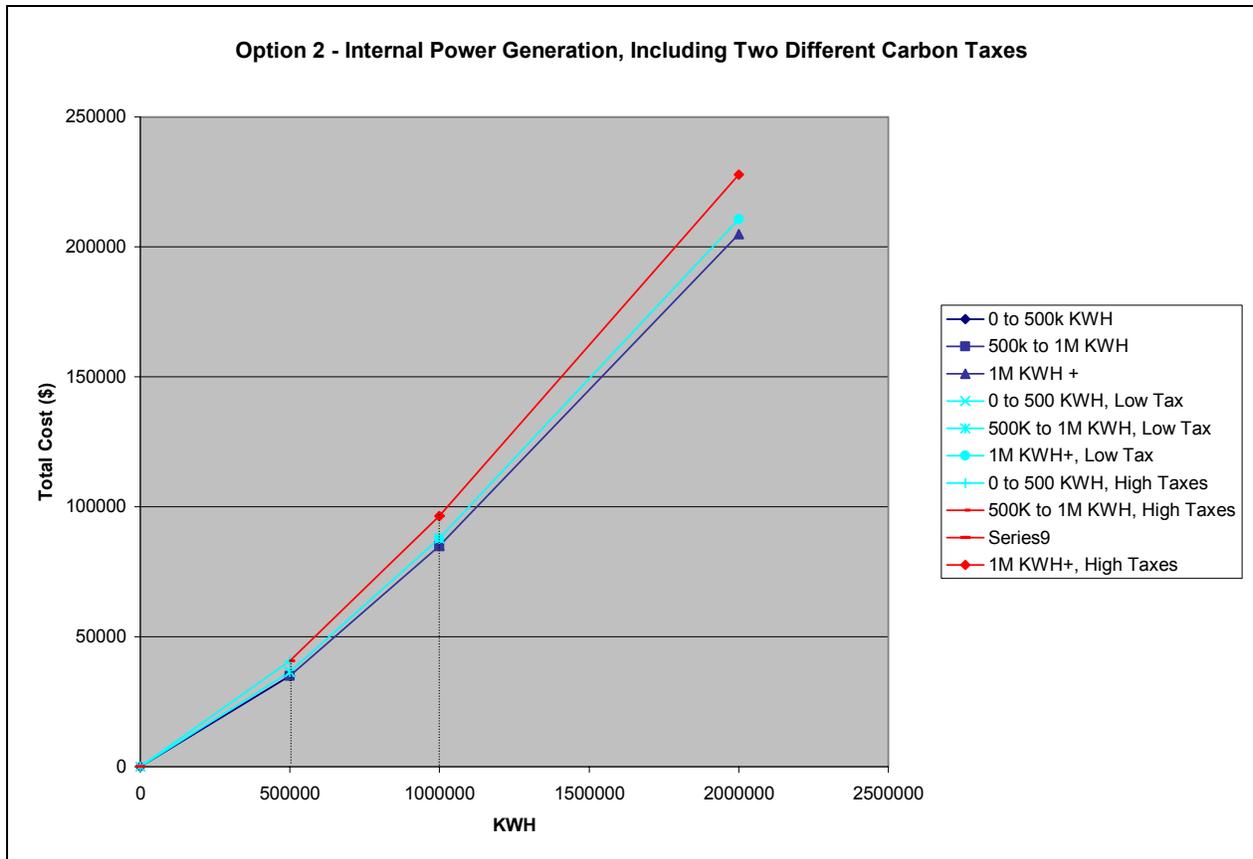
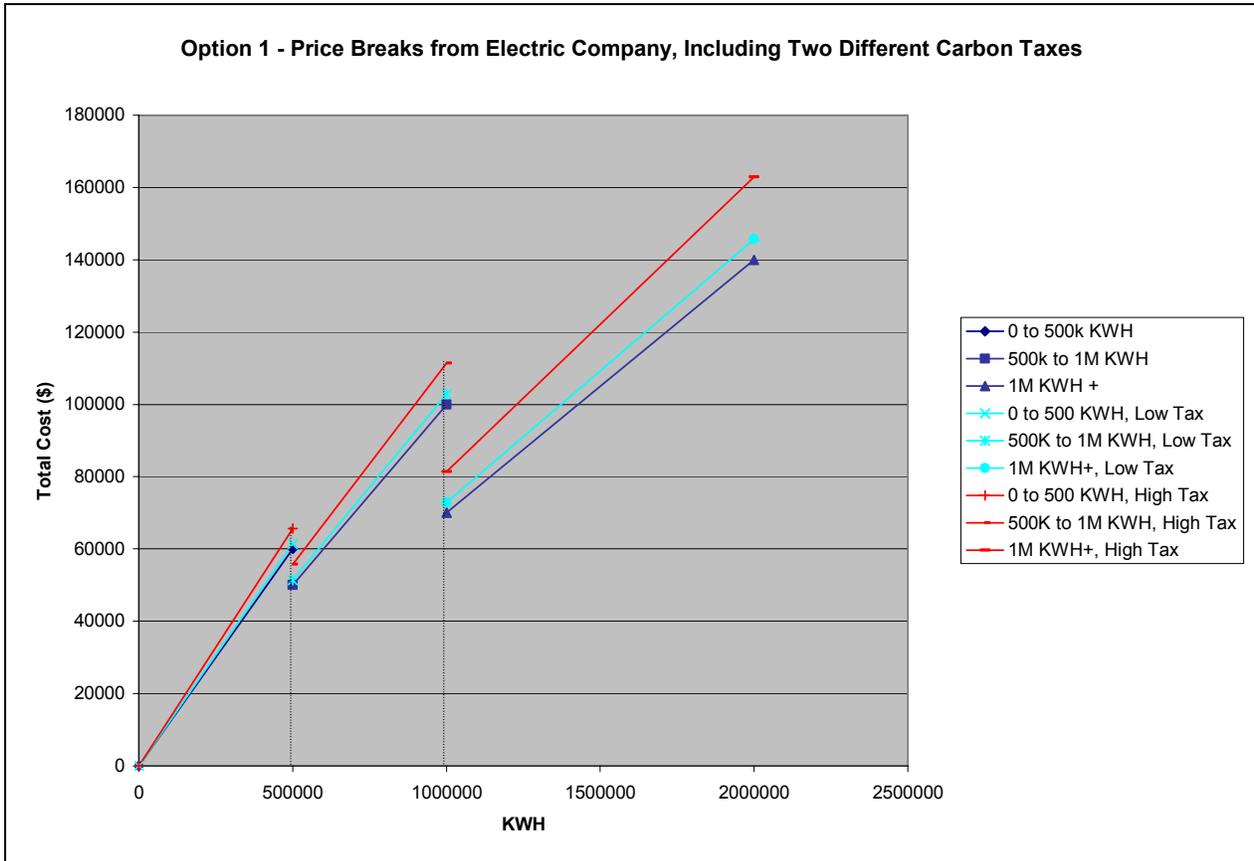
$$\text{Expected Cost (Low Tax)} = P(\text{Low Tax}) * (\text{YBT} * \text{AOC(nt)} + (t - \text{YBT}) * \text{AOC(lt)})$$

$$\text{Expected Cost (High Tax)} = P(\text{High Tax}) * (\text{YBT} * \text{AOC(nt)} + (t - \text{YBT}) * \text{AOC(ht)})$$

**Objective Function**

**$C_p + C_i + \text{Total Expected Operating Costs}$**

**Where Total Expected Operating Costs = Expected Cost (No Tax) + Expected Cost (Low Tax) + Expected Cost (High Tax)**



**Appendix B1:**  
**System Head Equations and Spreadsheet Calculations**

**Sample System Head Calculation**

$$\mathbf{P_A = \Delta Z + H_f}$$

Where:  $P_A$ (or TDH) = System Head (ft)  
 $\Delta Z$  = change in elevation (ft)  
 $H_f$  = energy loss due to friction (ft)

$$\mathbf{H_f = H_{fittings} + H_{pipe}}$$

$$\mathbf{H_{fittings} = kV^2/2g}$$

Where:  $k$  = dimensionless constant for particular fitting of a specific diameter  
 $V$  = fluid velocity in pipe (ft/sec)  
 $g$  = gravitational constant (ft/sec<sup>2</sup>)

$$\mathbf{H_{pipe} = fLV^2 / 2gD}$$

Where:  $f$  = dimensionless constant for particular type of pipe of a specific diameter  
 $L$  = length of pipe (ft)  
 $V$  = fluid velocity in pipe (ft/sec)  
 $g$  = gravitational constant (ft/sec<sup>2</sup>)  
 $D$  = pipe diameter (ft)

$$\mathbf{V = Q / A}$$

Where:  $Q$  = Flow in pipe (gal/min)  
 $A$  = Cross-sectional area of pipe

$$\mathbf{f = 0.25 / (\log ((\epsilon / D) / (37 + 5.74 / R^{0.9})))^2}$$

Where:  $\epsilon$  = Absolute roughness of pipe (ft)  
 $D$  = Pipe diameter (ft)  
 $R$  = Reynolds number – dimensionless ratio

$$\mathbf{R = VD / \nu}$$

Where:  $V$  = Fluid velocity in pipe (ft/sec)  
 $D$  = pipe diameter (ft)  
 $\nu$  = kinematic viscosity of fluid (ft<sup>2</sup>/sec)

# **Appendix B2**

## **System and Pump Curves**

## **Appendix B3**

### **Energy Consumption Equations**

**Energy Consumption Equations**

$$\mathbf{BHp = (TDH \times Q) / (3960 \times \eta_p)}$$

Where: BHp = Brake horsepower used by pump  
Q = Flow in pipe (gal/min)  
 $\eta_p$  = Hydraulic pump efficiency from pump manufacturer curve

$$\mathbf{Input\ Power = BHp \times 0.7457 / \eta_m}$$

Where: Input Power = Power required by pump (kW)  
 $\eta_m$  = Motor efficiency from pump manufacturer

$$\mathbf{E = Input\ Power \times Operating\ Hours / year}$$

Where: E = Annual Energy Use (kWh)

**Appendix B4**  
**Piping Cost Estimates**

**Appendix B5**  
**Pump Cost Estimates**

## **Appendix C**

**Excerpt from:**

**Natural Capitalism,**  
**Creating the Next Industrial Revolution**