15.053/8 February 5, 2013

Optimization Methods in Management Science and Operations Research

- Handout: Lecture Notes

Class website + more

- Class website
  - please log on as soon as possible
  - Problem Set 1 will be due next Tuesday
- Lots of class information on website
- Piazza.com used for Q&A and discussions
- No laptops permitted in class, except by permission

We will use clickers from Turning Technologies.

If you own one, please bring it to class with you, starting Thursday.

If you don't own one, we will lend you one for the semester.

Videotapes of classes

- **Current plan: start videotaping lectures starting Today (I think).**
- **In addition, PowerPoint presentations will all be available.**

Excel and Excel Solver

- During this semester, we will be using Excel Solver for solving optimization problems.
- We assume some familiarity with Excel, but no familiarity with Excel Solver.
- Homework exercises involve Excel.
  - Versions A and B (experiment starting this year).
  - Excel Solver tutorial this Friday

An optimization problem

- Given a collection of numbers, partition them into two groups such that the difference in the sums is as small as possible.

- Example: 7, 10, 13, 17, 20, 22

  These numbers sum to 89

I can split them into {7, 10, 13, 17} sum is 47

{20, 22} sum is 42

Difference = 5.

Can we do better?

Excel Example

What is Operations Research?
What is Management Science?

- World War II : British military leaders asked scientists and engineers to analyze several military problems
  - Deployment of radar
  - Management of convoy, bombing, antisubmarine, and mining operations.
- The result was called *Operations Research*
- MIT was one of the birthplaces of OR
  - Professor Morse at MIT was a pioneer in the US
  - Founded MIT OR Center and helped to found ORSA

What is Management Science (Operations Research)?

Operations Research (O.R.) is the discipline of applying advanced analytical methods to help make better decisions.

O.R. is an engineering and scientific approach for decision making.

Some Skills for Operations Researchers

- Modeling Skills
  - Take a real world situation, and model it using mathematics

Photo of female model removed due to copyright restrictions.

Not this

Adriana Lima

- Methodological Toolkit
  - Optimization
  - Probabilistic Models

Images removed due to copyright restrictions. See images of domino mosaic art at http://www.dominoartwork.com/.

Not this type of modeling

The Value of Operations Research and Management Science

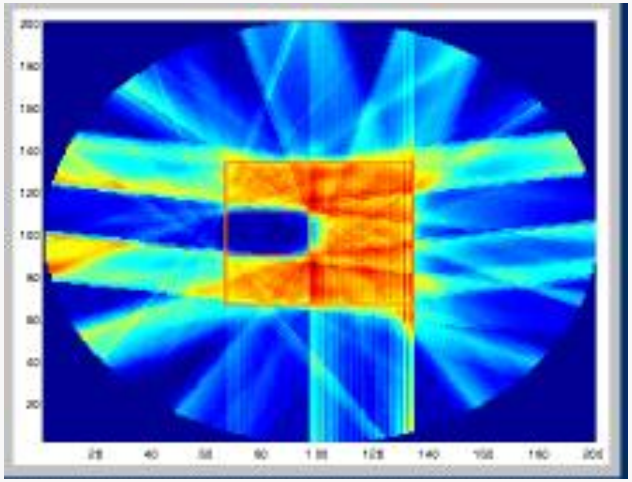
- Making sense of data
  - big data
  - social network info, transactional data, polls

- Dealing with complexity and uncertainty
  - understanding systems
  - making a good choice when there are billions of options (e.g., partitioning with 50 items)
  - making good choices in an uncertain world

- Using mathematical models to augment our own thinking.
  - develop insights
  - develop plans

Preview of Some Applications

Applying LP and NLP to optimal radiation therapy.



How to set prices.

Preview Continued

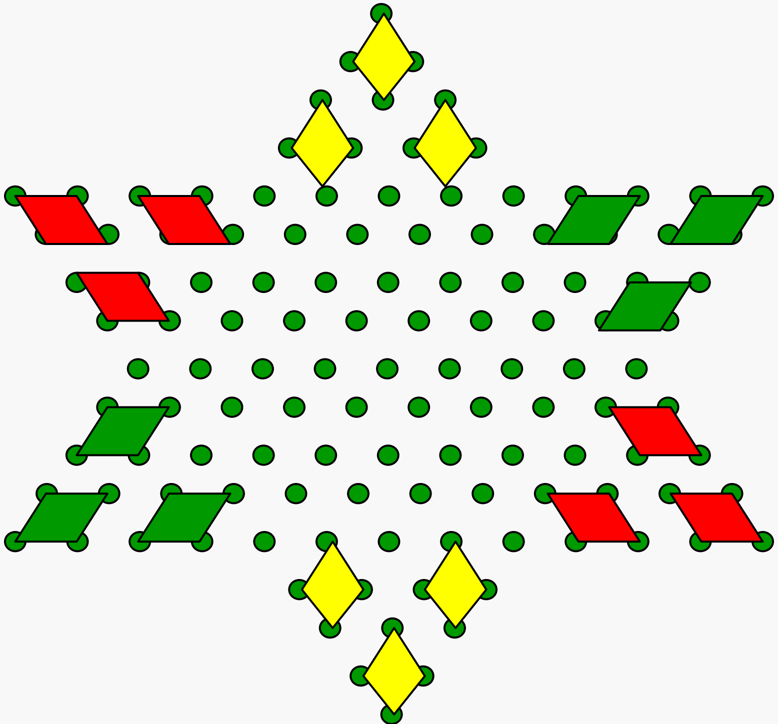
Find the shortest path in a network

More preview

Optimal strategies against adversaries.

Photo courtesy of Curtis Perry on Flickr.

How to solve some challenging puzzles



Some of the themes of 15.053/8

- Optimization is everywhere

- Data, Models, Algorithms, Insights, and Decisions
  - DMAID

Optimization is Everywhere

- **Personal choices**
  - best career choices,
  - best use of our time
  - best strategies,
  - best value for the dollar

- **Company choices**
  - maximize value to shareholders
  - determine optimal mix of products or services
  - minimize production costs
  - minimize cost of getting product to customers
  - maximize value of advertising
  - hire the best workers

Your class partner

- Introduce yourself to the person next to you (right or left), who we refer to as your "partner" for today.
- Those on aisle ends may be in a group of size 3.
- There will be a team project in which student groups will solve or attempt to solve an optimization that is useful in practice. This will involve collecting data, making a model and doing some analysis.
- Take 3 minutes with your partner to brainstorm on the type of problems you might be interested in doing a project on.

Some initial ideas for projects

What did you come up with?

Midclass break

- We searched Google for the number of pages with the expression “optimal X”
- There are at more than 10 expressions that have over 1 million hits. See if you can find them.

On 15.053/8 and Optimization Tools

- **Rest of the class**
  - **Introduce linear programming (LP) (also called linear optimization)**
  - **illustrates an important optimization tool for better decision making.**
  - **Efficiently solvable. LPs form the basis for solving more complex problems.**

The optimization paradigm

- Decision variables: the elements that are under the control of the decision maker.
  - The work schedules of each employee
  - The level of investments in a portfolio
  - what subjects a student should take in each semester

- A single objective function (of the decision variables)
  - minimize cost or …
  - maximize expected return or …
  - make the last semester as enjoyable as possible or …

The optimization paradigm

- Constraints: restrictions on the decision variables
  - "Business rules"
    - no worker can work more than 5 consecutive days
    - There is at most 2% investment in any stock in the portfolio
    - students must take a prerequisite of a subject before taking the subject
  - "Physical laws"
    - No worker can work a negative amount of time
    - The amount of a goods in inventory at the end of period j is the amount of goods arriving during period j plus the amount of goods in inventory in period j-1 minus the amount of goods that are sold in the period.

Generic optimization model
(usually called non-linear programming)

- Let x be the vector of decision variables:
- Suppose f, g_1, g_2, … , g_m are functions

max   f(x)

Maximize the objective

s.t. $g_i(x) \geq b_i$ for each i = 1 to m

Satisfy the constraints

$x \geq 0$

typically but not always the case.

Linear Programming

- minimize or maximize a linear objective
- subject to linear equalities and inequalities

Example. Max is in a pie eating contest that lasts 1 hour. Each torte that he eats takes 2 minutes. Each apple pie that he eats takes 3 minutes. He receives 4 points for each torte and 5 points for each pie. What should Max eat so as to get the most points?

Step 1. Determine the ***decision variables***

- Let x be the number of tortes eaten by Max.
- Let y be the number of pies eaten by Max.

Max's linear program

Step 2. Determine the *objective function*

Step 3. Determine the *constraints*

Maximize $z = 4x + 5y$ **(objective function)**

subject to $2x + 3y \le 60$ **(constraint)**

$x \ge 0$; $y \ge 0$ **(non-negativity constraints)**

A *feasible solution* satisfies all of the constraints.

$x = 10$, $y = 10$ is feasible; $x = 10$, $y = 15$ is *infeasible*.

An *optimal solution* is the best feasible solution.

The optimal solution is $x = 30$, $y = 0$, $z = 120$.

Terminology

- ***Decision variables:*** e.g., x and y.
  - In general, these are quantities you can control to improve your objective which should completely describe the set of decisions to be made.
- ***Constraints:*** e.g., $2x + 3y \leq 60$, $x \geq 0$, $y \geq 0$
  - Limitations on the values of the decision variables.
- ***Objective Function.*** e.g., $4x + 5y$
  - Value measure used to rank alternatives
  - Seek to maximize or minimize this objective
  - examples: maximize NPV, minimize cost

Maximize x_1 (A)

subject to $3x_1 + 4x_2 \geq 7$ (B)

$x_1 - 2x_5 = 7$ (C)

$x_1 \geq 0, x_2 \geq 0$ (D)

(A) is referred to as

1. Nonnegativity constraints
2. An equality constraint
3. The objective function
4. An inequality constraint

David's Tool Corporation (DTC)

- **Motto: "We may be no Goliath, but we think big."**
- **Manufacturer of slingshots kits and stone shields.**

Public domain image (Wikimedia Commons)

Public domain image (painting by Osmar Schindler, 1888)

Data for the DTC Problem

	Slingshot Kits	Stone Shields	Resources
Stone Gathering time	2 hours	3 hours	100 hours
Stone Smoothing	1 hour	2 hours	60 hours
Delivery time	1 hour	1 hour	50 hours
Demand	40	30	
Profit	3 shekels	5 shekels	

Formulating the DTC Problem as an LP

Step 1: Determine Decision Variables

K = number of slingshot kits manufactured

S = number of stone shields manufactured

Step 2: Write the Objective Function as a linear function of the decision variables

Maximize Profit =

Step 3: Write the constraints as linear functions of the decision variables

subject to

The Formulation Continued

Step 3: Determine Constraints

Stone gathering:

Smoothing:

Delivery:

Slingshot demand:

Shield demand:

We will show how to solve this in Lecture 3.

Linear Programs

- A *linear function* is a function of the form:

$$f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$
$$= \sum_{i=1 \text{ to } n} c_ix_i$$

e.g., $3x_1 + 4x_2 - 3x_4$.

- A mathematical program is a *linear program (LP)* if the objective is a linear function and the constraints are linear equalities or inequalities.

e.g., $3x_1 + 4x_2 - 3x_4 \geq 7$
$x_1 - 2x_5 = 7$

- Typically, an LP has non-negativity constraints.
- Strict inequalities are not permitted. ($x > 0$ is not allowed.)

More on Linear Programs

- A linear program must have linear objectives and linear equalities and inequalities to be considered a linear program.

Maximize x_1

subject to

$$3x_1 + 4x_2 \geq 7$$
$$x_1 - 2x_5 = 7$$
$$|x_1| \geq 0$$

Not a linear program.

Maximize x^2

subject to $x = 3$

Not a linear program.

A ***non-linear program*** is permitted to have a non-linear objective and constraints.

- maximize $f(x,y) = xy$
- subject to $x - y^2/2 \geq 10$

  $3x - 4y \leq 2$

  $x \geq 0, y \geq 0$

Minimize x
subject to $x \geq 3$

Both a linear and a non-linear program.

An *integer program* is a linear program plus constraints that some or all of the variables are integer valued.

- **Maximize** $3x_1 + 4x_2 - 3x_3$

  $3x_1 + 2x_2 - x_3 \leq 17$

  $3x_2 - x_3 = 14$

  $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ and

  x_1, x_2, x_3 are all integers

Summary

- Mathematical models
- Optimization as a paradigm
- Linear programming

15.053 Optimization Methods in Management Science
Spring 2013