

Optimization Methods in Management Science

MIT 15.053, Spring 2013

PROBLEM SET 1

SECOND GROUP OF STUDENTS (WITH FIRST LETTER OF SURNAMES **A–H**)

DUE: FEBRUARY 12 , 2013

Problem Set Rules:

1. Each student should hand in an individual problem set.
2. Discussing problem sets with other students is permitted. Copying from another person or solution set is *not* permitted.
3. Late assignments will *not* be accepted. No exceptions.
4. The non-Excel solution should be handed in at the beginning of class on the day the problem set is due. The Excel solutions, if required, should be posted on the website by the beginning of class on the day the problem set is due. Questions that require an Excel submission are marked with EXCEL SUBMISSION . For EXCEL SUBMISSION questions, only the Excel spreadsheet will be graded.

Problem 1

(35 points total) The first problem asks you to formulate a 3-variable linear program in three different ways (four ways if you also count the algebraic formulation). Both of the first two ways are fairly natural. The third way is a bit obscure. And the algebraic formulation may seem overly complex. In practice, there are advantages to formulating linear programs in different ways. And there are huge advantages in the algebraic formulation. (One can express huge problems efficiently on a computer using a modeling language, which is based on the algebraic formulation.) In addition, formulating an LP in multiple ways provides insight into the LP models.

Accessories & co. is producing three kinds of covers for Apple products: one for iPod, one for iPad, and one for iPhone. The company's production facilities are such that if we devote the entire production to iPod covers, we can produce 6000 of them in one day. If we devote the entire production to iPhone covers or iPad covers, we can produce 5000 or 3000 of them in one day. The production schedule is one week (5 working days), and the week's production must be stored before distribution. Storing 1000 iPod covers (packaging included) takes up 40 cubic feet of space. Storing 1000 iPhone covers (packaging included) takes up 45 cubic feet of space, and storing 1000 iPad covers (packaging included) takes up 210 cubic feet of space. The total storage space available is 6000 cubic feet. Due to commercial agreements with Apple, Accessories & co. has to deliver at least 5000 iPod covers and 4000 iPad covers per week in order to strengthen the product's diffusion. The marketing department estimates that the weekly demand for iPod covers, iPhone, and iPad covers does not exceed 10000 and 15000, and 8000 units, therefore the company does not want to produce more than these amounts for iPod, iPhone, and iPad covers. Finally, the net profit per each iPod cover, iPhone cover, and iPad cover is \$4, \$6, and \$10, respectively.

The aim is to determine a weekly production schedule that maximizes the total net profit.

- (a) (5 points) Write a Linear Programming formulation for the problem. Start by stating any assumptions that you make. Label each constraint (except nonnegativity). For this first formulation, the decision variables should represent the proportion of time spent each day on producing each of the two items:

$$\begin{aligned}x_1 &= \text{proportion of time devoted each day to iPod cover production,} \\x_2 &= \text{proportion of time devoted each day to iPhone cover production,} \\x_3 &= \text{proportion of time devoted each day to iPad cover production.}\end{aligned}$$

(Different formulations will be required for parts (b) and (c).)

- (b) (5 points) Write a second Linear Programming formulation for the problem. Label each constraint (except nonnegativity). For this second formulation, the decision variables should represent the number of items of each type produced over the week:

$$\begin{aligned}y_1 &= \text{number of iPod covers produced over the week,} \\y_2 &= \text{number of iPhone covers produced over the week,} \\y_3 &= \text{number of iPad covers produced over the week.}\end{aligned}$$

The problem data is the same but you must make sure that everything matches the new decision variables.

- (c) (5 points) Write a third Linear Programming formulation for the problem. Label each constraint (except nonnegativity). Assume that each working day has 8 working hours. For this third formulation, the decision variables should be:

$$\begin{aligned}z_1 &= \text{number of hours devoted to the production of iPod smart covers in one week,} \\z_2 &= \text{number of hours devoted to the production of iPhone smart covers in one week,} \\z_3 &= \text{total number of production hours employed during the week.}\end{aligned}$$

Express the objective function in thousands of dollars. The problem data is the same but you must make sure that everything matches the new decision variables.

- (d) (5 points) What is the relationship between the variables z_1, z_2, z_3 of part (c) and the variables x_1, x_2, x_3 of part (a) of this problem? Give a formula to compute z_1, z_2, z_3 from x_1, x_2, x_3 .
- (e) (5 points) EXCEL SUBMISSION Solve the problem using Excel Solver, following the guidelines given in the Excel Workbook that comes with this problem set. Pay attention to the formulation in the Excel Workbook: it is similar to the one required for part (b), but it is not exactly the same.
- (f) (10 points) Write an algebraic formulation of the weekly production schedule problem described above using the following notation:
- n is the number of product types,
 - x_j is the number of days devoted to the production of products of type j ,
 - p_j is the number of items of type j that can be manufactured in one day, assuming that the process is devoted to products of type j .

- P is the number of production days in one week,
- s_j is the storage space required by *one* item of type j ,
- S is the total storage space available for the week's production,
- r_j is the unit profit for each product of type j ,
- d_j is the weekly maximum demand for an item of type j .
- b_j is the weekly minimum demand for an item of type j .

Problem 2

(10 points total). Problem 2 reviews the transformations from nonlinear constraints or objectives into linear constraints and objectives, as mentioned in the second lecture and discussed in the tutorial “LP Transformation Tricks”.

Identify which of the following optimization problems can be reformulated as a single Linear Program, and write the corresponding equivalent LP formulation. For those problems that cannot be reformulated, the answer should identify all the troublesome constraint(s) and/or objective.

(a) (5 points) Problem formulation:

$$\begin{array}{rcl}
 \min & \max\{2.3x_1 + x_2, 4.3x_1 - 0.5x_2, 2.5x_1 + 3.5x_2\} & \\
 \text{s.t.} & & \\
 \text{Constr1 :} & x_1/(x_1 + x_2) \leq 0.5 & \\
 \text{Constr2 :} & 10x_1 + 28x_2 = 3.4 & \\
 \text{Constr3 :} & x_1 + x_2 \geq 0 & \\
 & x_1 \text{ free} & \\
 & x_2 \text{ free} &
 \end{array} \quad (1)$$

(b) (5 points) Problem formulation:

$$\begin{array}{rcl}
 \min & |0.8x_1 + 0.9x_2| & \\
 \text{s.t.} & & \\
 \text{Constr1 :} & |0.9x_1 + 1.2x_2| \leq 10 & \\
 & x_1 \geq 0 & \\
 & x_2 \text{ free} &
 \end{array} \quad (2)$$

Problem 3 (First group of students)

(55 points total) This problem is based on Problem 11 of Applied Mathematical Programming, Chapter 1. This is a more complex model than the one in Problem 1. For many product mix examples, each product uses a fixed set of materials. In this example, if one makes one gallon Deluxe, one does not know a priori how much of Additive A and Additive C are used. In fact, the decision maker must determine the optimal amounts of A and C to be added to Deluxe.

A corporation that produces gasoline and oil specialty additives purchases four grades of petroleum distillates, A, B, C, and D. The company then combines the four according to specifications of the maximum or minimum percentages of grades A, C, or D in each blend, given in Table 1.

| Mixture | Max % allowed for Additive A | Min % allowed for Additive C | Max % allowed for Additive D | Selling price \$/gallon |
|----------|------------------------------------|------------------------------------|------------------------------------|-------------------------------|
| Deluxe | 60% | 20% | 10% | 7.9 |
| Standard | 15% | 60% | 25% | 6.9 |
| Economy | – | 50% | 45% | 5.0 |

Table 1: Specifications of the three mixtures.

Supplies of the three basic additives and their costs are given in Table 2.

| Distillate | Max quantity available per day (gals) | Cost \$/gallon |
|------------|---------------------------------------------|-------------------|
| A | 4000 | 0.60 |
| B | 5000 | 0.52 |
| C | 3500 | 0.48 |
| D | 5500 | 0.35 |

Table 2: Supplies and costs of petroleum grades.

- (a) (15 points) Formulate a Linear Program to determine the production policy that maximizes profits.
- (b) (10 points) EXCEL SUBMISSION Write a spreadsheet for the problem and solve the problem using Excel Solver, following the guidelines given in the Excel Workbook that comes with this problem set. (Hint: The optimal value is \$102986.7 . In the optimal solution, there are 12500 gallons of mixture Deluxe and 1666.667 gallons of mixture Standard. No mixture Economy is produced.)
- (c) Use the Excel spreadsheet to answer the following questions:
- i) (4 points) No mixture Economy is produced in the optimal solution. What would the minimum selling price need to be in order for Deluxe to be worth producing? (Be accurate to within 5 cents).
 - ii) (4 points) There are constraints on the maximum amount of distillate D that is used in producing the different mixtures. If you could violate any of these constraints on distillate A, would you? Which constraint would be most advantageous to delete?
 - iii) (4 points) Suppose that you can increase the selling price of Deluxe. What would be the increase in the profit if the selling price of mixture Deluxe per gallon increases to p for $p = \$ 7.95, \$8.00,$ and $\$8.05$. (Assume that the selling price of mixtures Standard and Economy remain $\$6.9$ and $\$5.0$ per gallon, respectively.) The increase is the difference between the new profit and the profit from Part (b).
 - iv) (4 points) Based on your answer in Part (iv), what do you think will be the profit if the selling price of mixture Deluxe increases by 5%? (Verify that you are correct.) What is the formula for the optimum profit if the selling price of Deluxe per gallon increased by p ? (You may assume that p is between 1% and 10%).

- v) (4 points) Based on your formula in part (iii), what is the contribution if the selling price of Deluxe per gallon increases to 8.5. Use Excel solver to see if the formula is correct. (It wont be.) Use Excel solver to determine the maximum value of p for which your formula is correct. (Be accurate to within 5 cents).
- vi) (5 points) The quantity available of distillate A is 4000 gals per day which is entirely used up in an optimal mixture. If you could buy (just a little bit) more, would you do so? At what price would be worthwhile to buy more?
- (d) (10 points) Write an algebraic formulation for the problem using the following notation:
- m is the number of mixtures (final products),
 - n is the number of basic petroleum grades (additives),
 - p_j is the selling price of mixture j ,
 - c_i is the cost of additive i ,
 - a_i is the maximum availability of additive i ,
 - r_{ij} is the minimum percentage of additive i that is required in mixture j ,
 - q_{ij} is the maximum percentage of additive i that can be blended in for mixture j ,

MIT OpenCourseWare
<http://ocw.mit.edu>

15.053 Optimization Methods in Management Science
Spring 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.