15.023J / 12.848J / ESD.128J Global Climate Change: Economics, Science, and Policy
Spring 2008

The Mathematics of Climate Modeling

12.848 / 15.023 / ESD.128

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OUTLINE

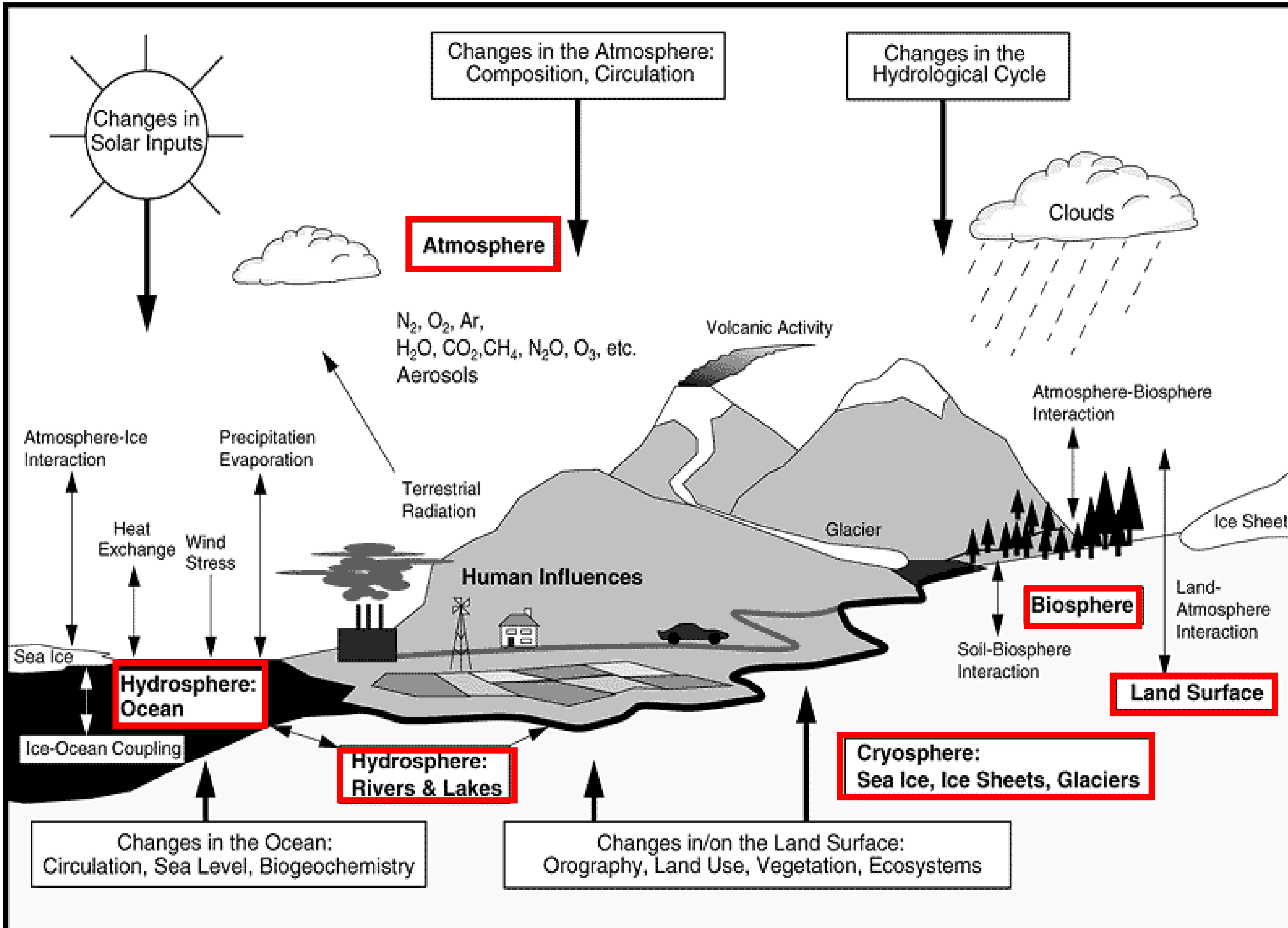
1. What is a **CLIMATE MODEL**?
2. Designing a model
   - Spatial Grid
   - Continuity equation
   - Time step and stability
3. Solving the equations
   - Reality… Computation time and parameterization

1. What is a CLIMATE MODEL?

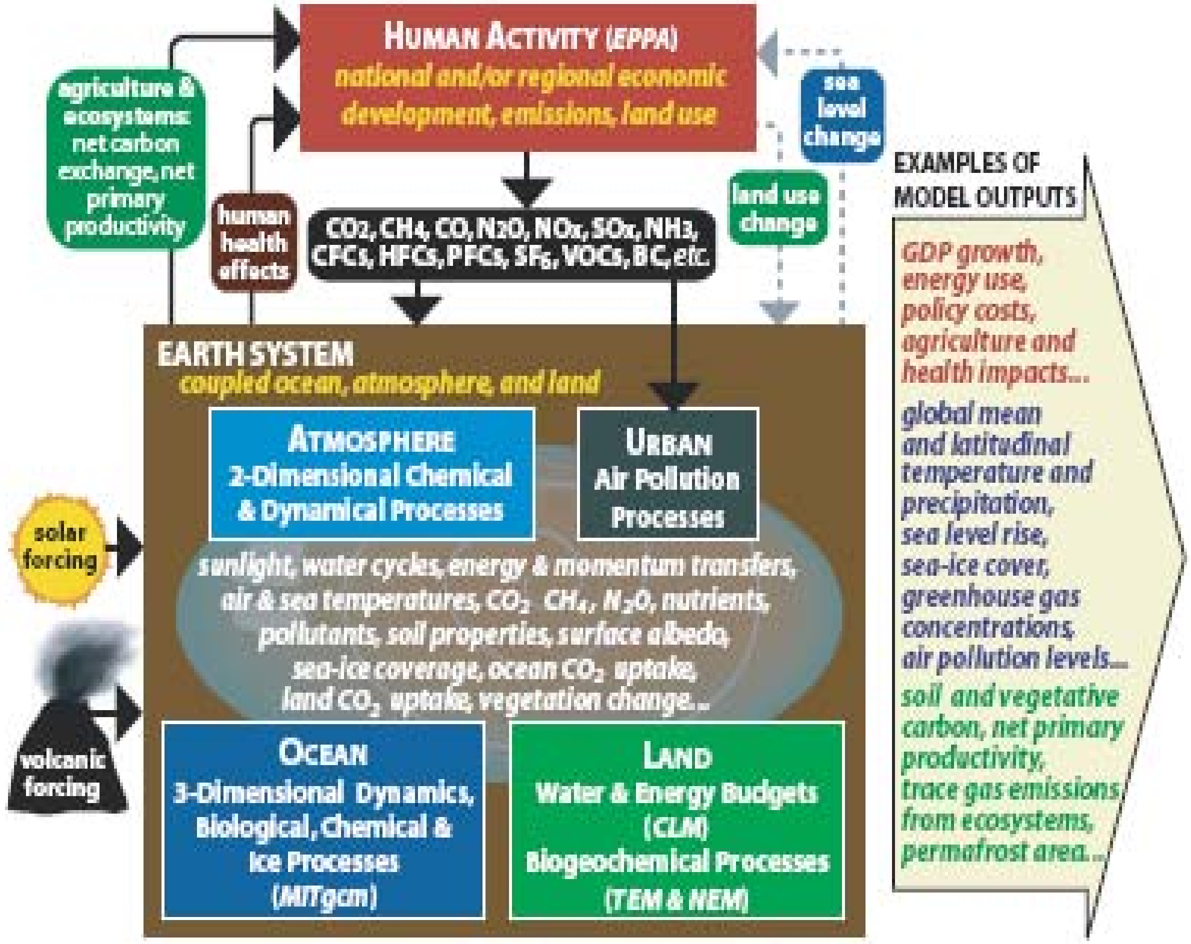
- A model that incorporates the principles of **physics**, **chemistry**, **biology** into a mathematical model of climate

  e.g. GCM (Global Circulation Model)

- Such a model has to answer what happens to temperature, precipitation, humidity, wind speed and direction, clouds, ice and other variables all around the globe over time


Changes in Solar Inputs
Changes in the Atmosphere: Composition, Circulation
Changes in the Hydrological Cycle
Atmosphere
Clouds
N2, O2, Ar,
H2O, CO2,CH4, N2O, O3, etc.
Aerosols
Volcanic Activity
Atmosphere-Biosphere Interaction
Atmosphere-Ice Interaction
Precipitatien Evaporation
Terrestrial Radiation
Heat Exchange
Wind Stress
Glacier
Ice Sheet
Human Influences
Biosphere
Land-Atmosphere Interaction
Sea Ice
Soil-Biosphere Interaction
Hydrosphere: Ocean
Land Surface
Ice-Ocean Coupling
Hydrosphere: Rivers & Lakes
Cryosphere: Sea Ice, Ice Sheets, Glaciers
Changes in the Ocean: Circulation, Sea Level, Biogeochemistry
Changes in/on the Land Surface: Orography, Land Use, Vegetation, Ecosystems

Example of a climate model : MIT-IGSM



A schematic figure of the MIT-IGSM Version 2

Sokolov *et al.*, 2005

2. DESIGNING a MODEL

Spatial grid

We divide the earth's atmosphere into a finite number of boxes (grid cells).

Assume that each variable has the same value throughout the box.

Write a **budget** for each each box, defining the changes within the box, and the flows between the boxes.

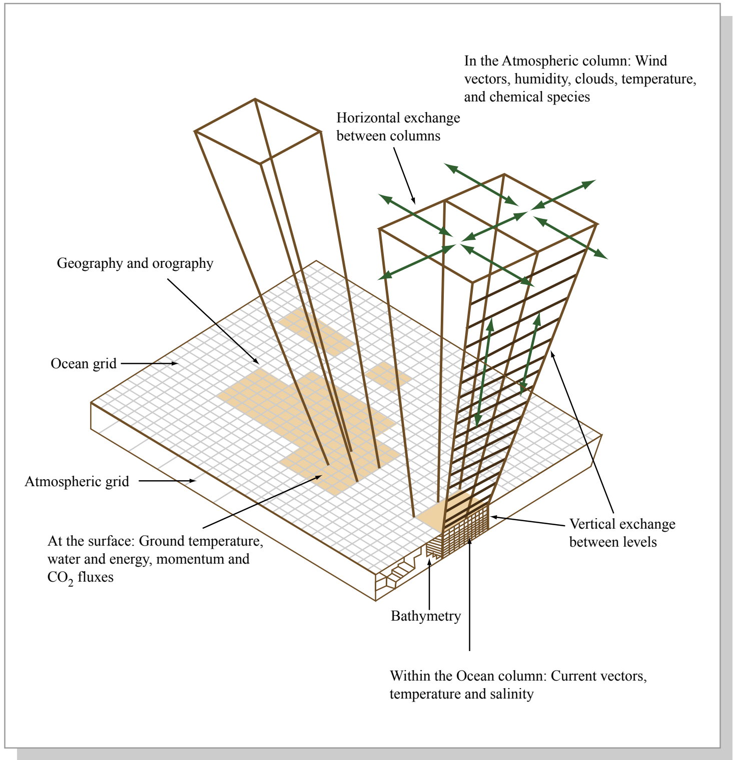
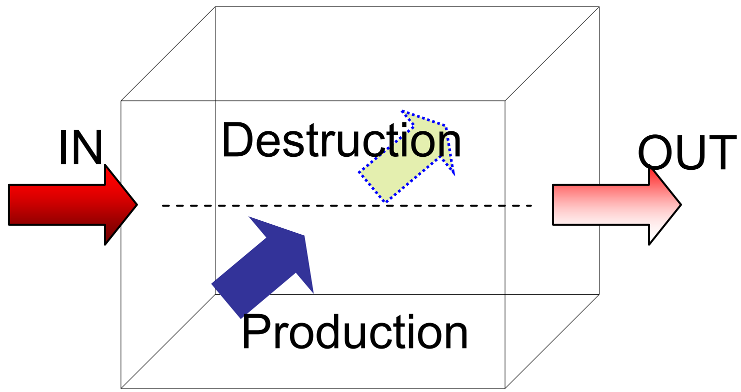


Figure by MIT OpenCourseWare, adapted from Henderson-Sellers: *A Climate Modeling Primer*.

Continuity Equations

We can express changes in a grid cell at a given time step



CHANGE = (Production - Destruction) +/- (Gain or Loss by advection)

$$\left(\frac{\partial \Phi}{\partial t}\right) = \frac{d\Phi}{dt} - \frac{\partial}{\partial x}(u\Phi) - \frac{\partial}{\partial y}(v\Phi) - \frac{\partial}{\partial z}(w\Phi)$$

The total change (rate of accumulation) of Φ in the box

Actual production or destruction of Φ within the box

Change in Φ due to loss to downstream boxes or arrival of Φ from an upstream box (called advection or convection)

Continuity Equations (Atmosphere/Oceans in 3-D)

Solving the Basic Equations for the Atmosphere in 3-D

Mass Continuity

$$\frac{\partial \rho}{\partial t} = -\frac{d(u\rho)}{dx} - \frac{d(v\rho)}{dy} - \frac{d(w\rho)}{dz}$$

Equations of Motion (momentum continuity)

$$\frac{\partial u}{\partial t} = -\frac{d(uu)}{dx} - \frac{d(vu)}{dy} - \frac{d(wu)}{dz} + \ldots$$

$$\frac{\partial v}{\partial t} = -\frac{d(uv)}{dx} - \frac{d(vv)}{dy} - \frac{d(wv)}{dz} + \ldots$$

$$\frac{\partial w}{\partial t} = -\frac{d(uw)}{dx} - \frac{d(vw)}{dy} - \frac{d(ww)}{dz} + \ldots$$

+ pressure gradient
+ Coriolis force
+ gravity
+ friction

Thermodynamic Equation (energy continuity)

$$\frac{\partial T}{\partial t} = -\frac{d(uT)}{dx} - \frac{d(vT)}{dy} - \frac{d(wT)}{dz} + \frac{1}{c_v}\left(J - p\frac{D(1/\rho)}{Dt}\right)$$

J : radiation, conduction, latent heat release, etc

$D(1/\rho)/Dt$: conversion between thermal and mechanical energy in fluid system

Chemical Continuity Equation

$$\frac{\partial \chi}{\partial t} = -\frac{d(u\chi)}{dx} - \frac{d(v\chi)}{dy} - \frac{d(w\chi)}{dz} \quad \textit{+ Chemical Production} \; \textit{– Chemical Loss}$$

Source: MIT OCW, 10.571J, lecture note by R.Prinn

The Equations for Chemistry and Biology

- For ocean chemistry, air-sea CO_2 flux is:

Air-sea CO_2 exchange flux
$= k_s \, (pCO_{2,\, ocean} - pCO_{2,atmosphere})$

- For land biology and chemistry, atmosphere-land CO_2 flux is:

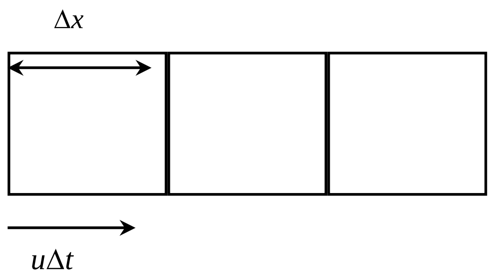
Atmosphere-land CO_2 exchange flux
= Photosynthesis – Respiration – Decomposition
$= (GPP - R_A) - R_H$

Time stepping and stability

- Time is also treated in discrete units.
- Time intervals depend on the size of the boxes:

<u>General Rule for stability: the CFL condition</u>

$$\frac{u\Delta t}{\Delta x} \leq 1$$



Intuitively don't want to transport more than a grid cell over a time step.

Eg. In atmosphere max u = 100m/s; grid spacing = 300 km;
Constraint: Δt < 3000 seconds (less than 1 hour)

3. How to Solve the equations

- We want to solve for the values of the variables described by these equations over time.

  i.e. to integrate the set of differential equations

- Essentially we have seven (or more) variables described by the same number of equations that describe change with respect to time. (T,p, ρ, u, v, w, water, etc.). So we should be able to solve for the values of the variables through time…

- BUT these equations **cannot be solved analytically**; there is no closed form solution

- So need to use numerics: discretize in time and space…

Demand on Computation

Total Computation Time:

For example, for a 2.8° x 2.8 ° degree atmospheric model

How Many Grid Cells?

	128	Longitudes
	64	Latitudes
*	18	Vertical Levels

~ 150,000 Grid Cells

What Happens at each Grid Cell?

	10	Variables
*	100	Computations Each

~ 1,000 Computations per Grid Cell per Time Step

How Many Time Steps Per Year?

24+	Time Steps per Day
365	Days per Year

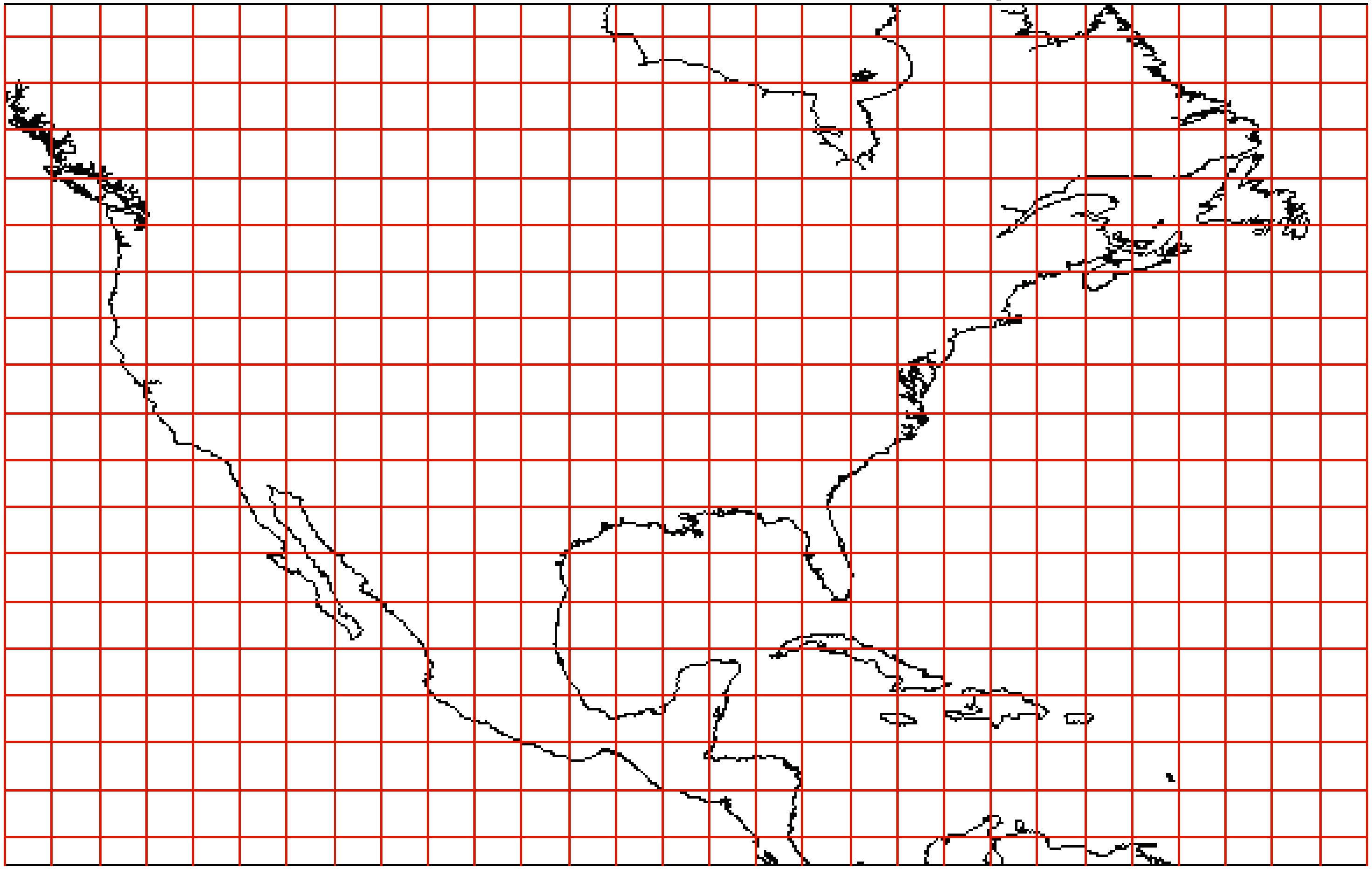
~ 10,000 Time Steps per Year

$$150{,}000 \text{ (Grid Cells)} * 1{,}000 \frac{\text{Computations}}{\text{(Grid Cell) (Time Step)}} * 10{,}000 \frac{\text{Time Steps}}{\text{Year}} \approx 1.5 \text{ Trillion} \frac{\text{Calculations}}{\text{Year}}$$

With a 1 GHz machine, a 1 year simulation takes about three hours

And, remember, this is just about the simplest possible model and we generally want to run the model for decades or centuries…

CCM3 Horizontal Resolution (2.8 x 2.8 degrees)



Parameterizations

For GCMs, grid cells are typically hundreds of miles across and often there are thirty vertical layers for the atmosphere.

Many processes happen at smaller scales and must be approximately included (a.k.a., parameterized), including:

- Convection
- Cloud Cover
- Ice Cover: sea and land (glaciers)
- Snow Cover
- Rainfall
- Emissions of Pollutants
- River Runoff into Oceans
- "Eddy Fluxes"
- Sharp weather fronts
- "Gravity Waves"
- Mountains
- Cities (heat islands, emissions, etc)