15.023J / 12.848J / ESD.128J Global Climate Change: Economics, Science, and Policy
Spring 2008

Methods of Uncertainty Analysis

15.023 Lecture

9 April 2008

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(material from previous lectures by Mort Webster, Ian Sue Wing, Marcus Sarofim, and Travis Franck)

“Persons pretending to forecast the future shall be considered disorderly under subdivision 3, section 901 of the criminal code and liable to a fine of $250 and/or six months in prison.”

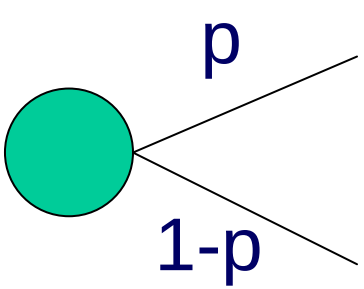
Section 889, New York State Code of Criminal Procedure

Lecture Highlights

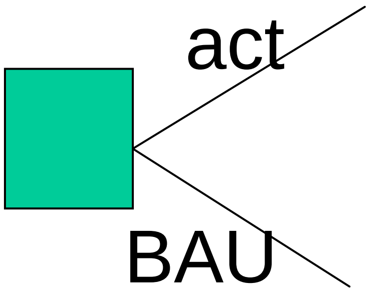
- How to use Decision Trees
- Understanding mean vs. median, standard deviations, etc.
- How to read PDFs (and make one from a histogram)
- The role of uncertainty in climate change science

Elements of a Decision Tree

Uncertainty Node: Shows different possible outcomes for an uncertain event, and the respective probabilities for each outcome



Decision Node: possible actions at a choice point



Example: Biking in the Rain

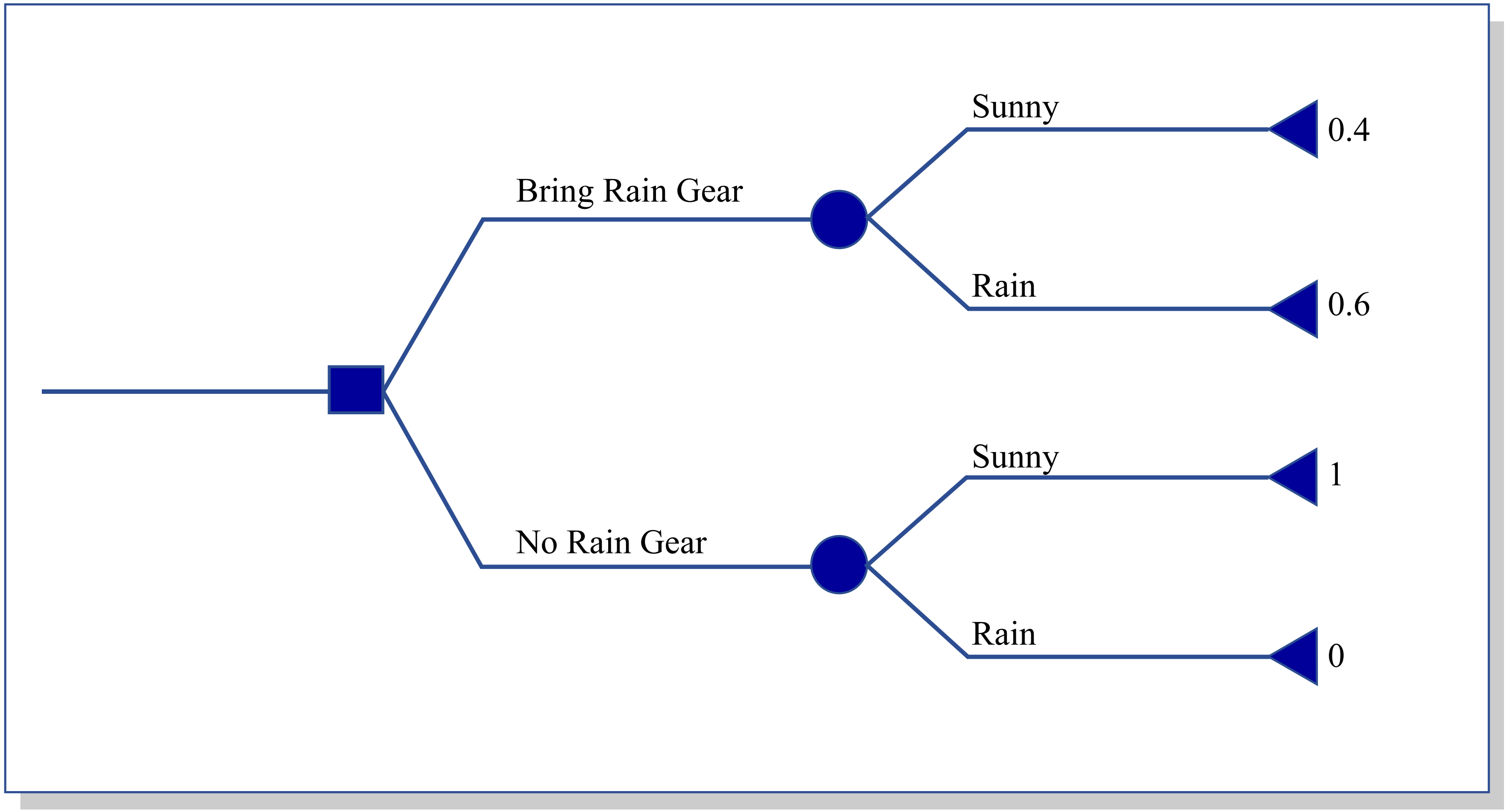
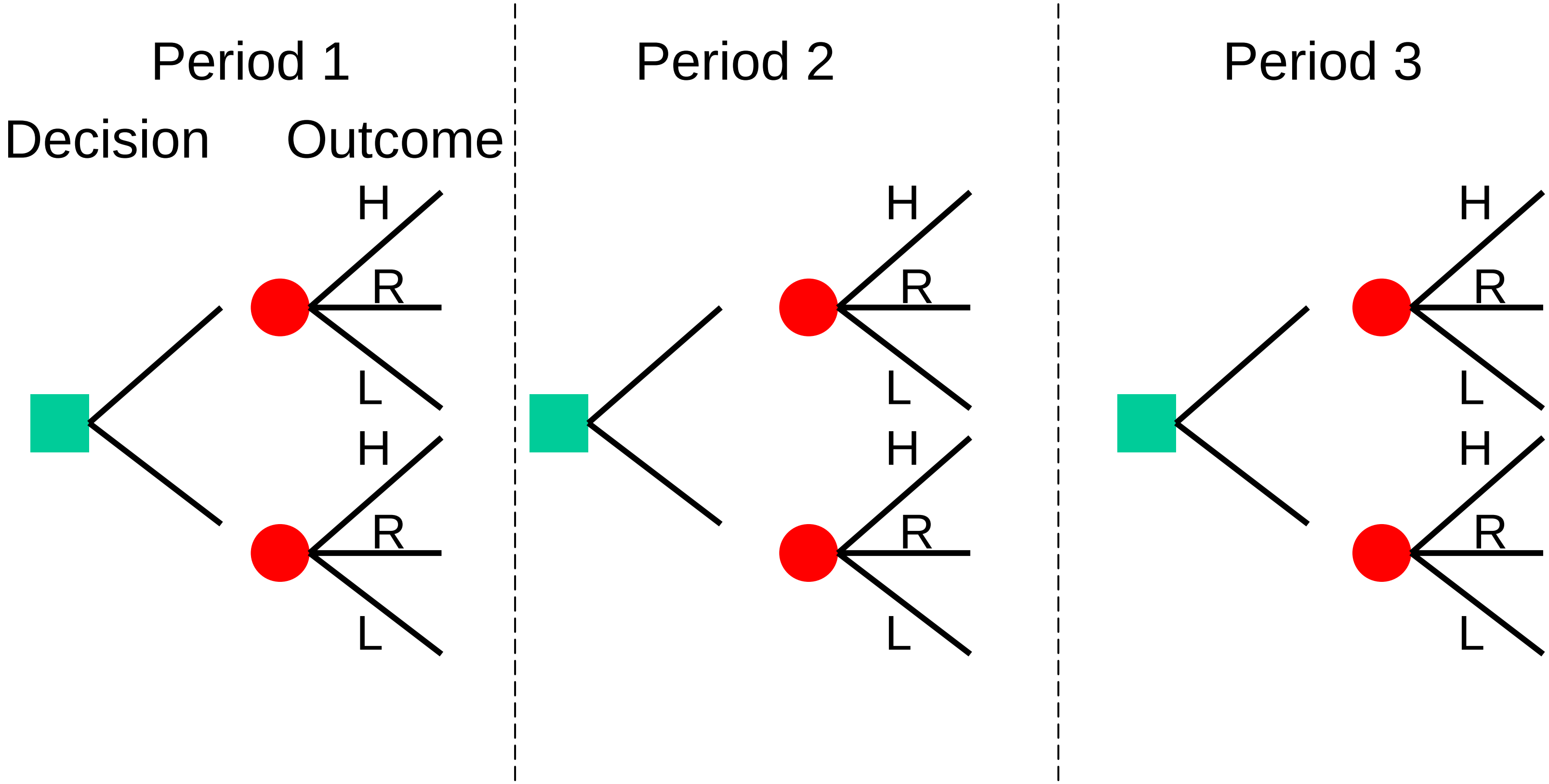


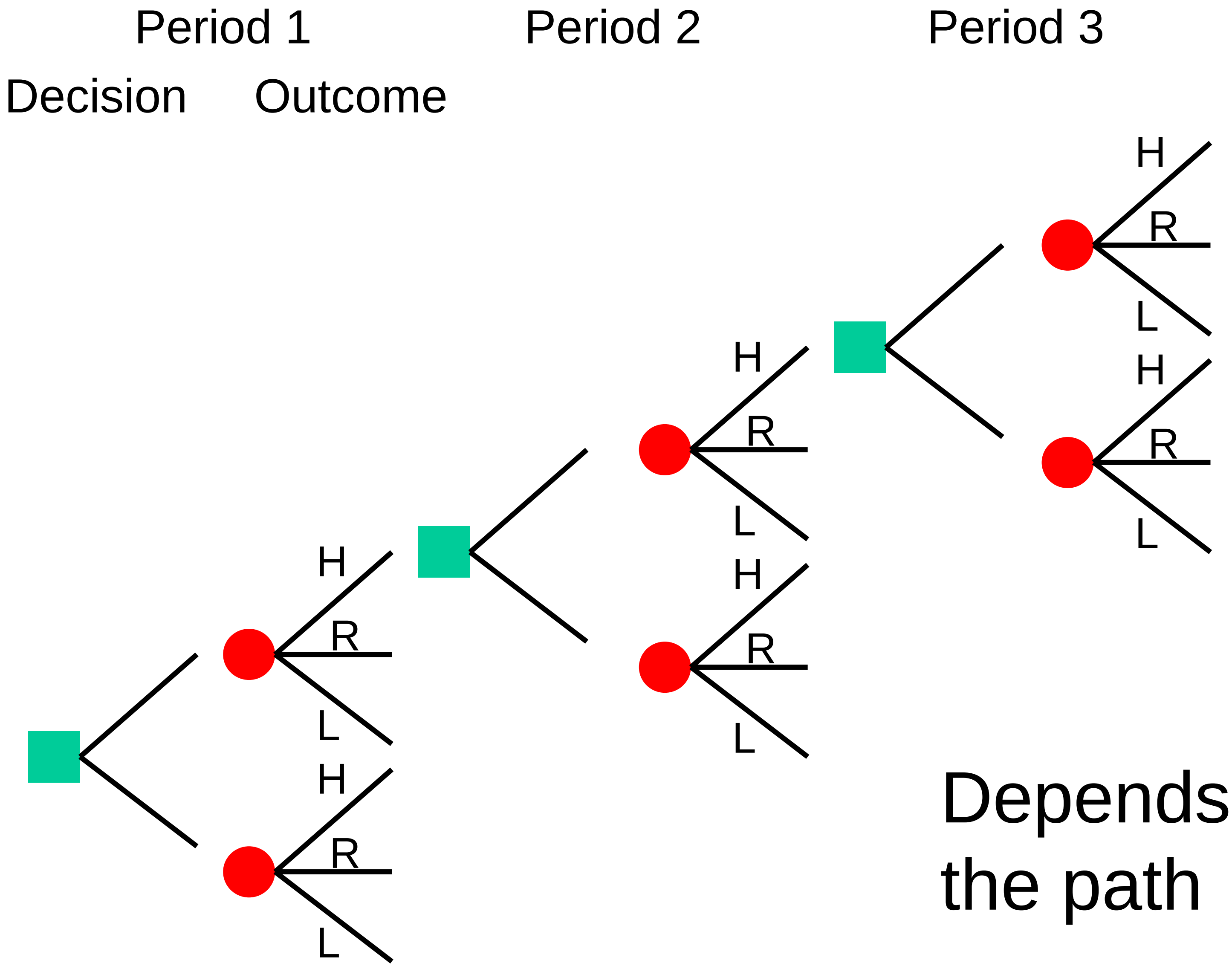
Figure by MIT OpenCourseWare.

Decision tree without Memory



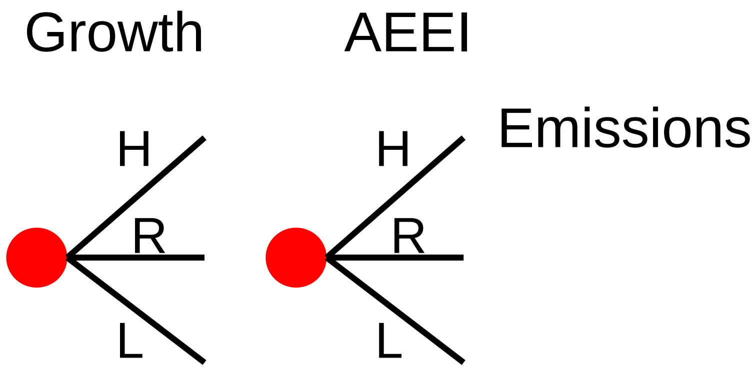
No connection between periods

Decision Tree with Memory

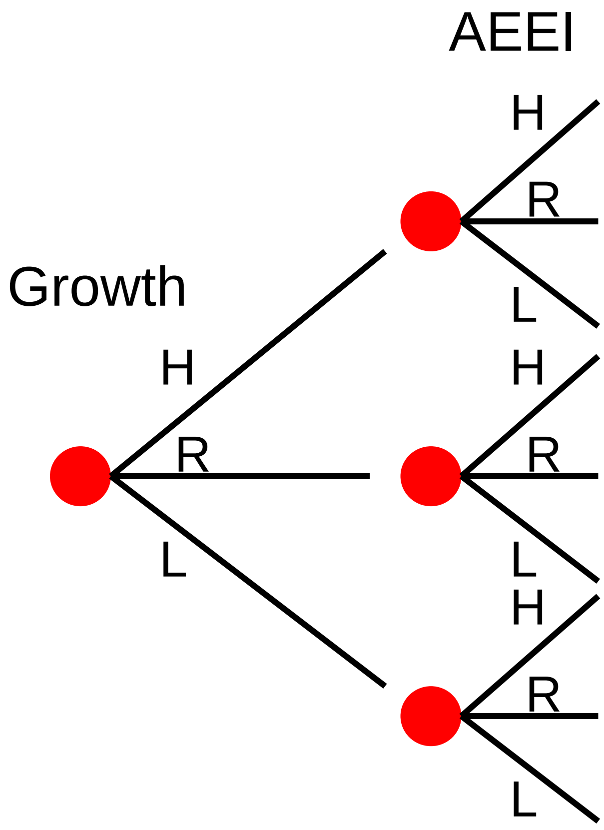


Depends on the path

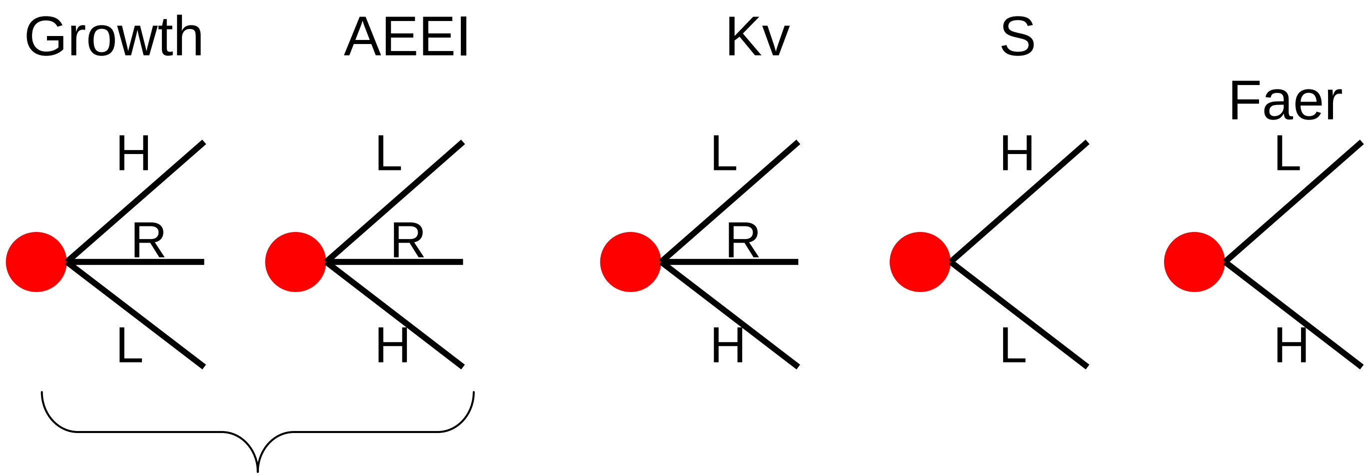
Emissions Scenarios



Shorthand for



Climate Scenarios



Collapse into RR, HL, LH

In Homework

Economic Uncertainty

Climate Uncertainty

Probability distribution

Handling scientific uncertainty in AR4

- In general, uncertainty ranges for results given in this Summary for Policymakers are 90% uncertainty intervals unless stated otherwise, that is, there is an estimated 5% likelihood that the value could be above the range given in square brackets and 5% likelihood that the value could be below that range. Best estimates are given where available. Assessed uncertainty intervals are not always symmetric about the corresponding best estimate. Note that a number of uncertainty ranges in the Working Group I TAR corresponded to 2 standard deviations (95%), often using expert judgment.

(IPCC, 2007: Summary for Policymakers. In: *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change)*

Why Worry about Uncertainty?

- To identify important factors and assumptions underlying disagreements
- To know where more information is needed
- To attach a range to model forecasts
- To understand attitudes toward risk
- To account for learning over time

(Attaching uncertainty to predictions might be as natural as attaching cost/benefit to climate targets)

Types of Uncertainty

- Parametric Uncertainty
  - Uncertainty in the value of a quantity
- Model or Structural Uncertainty
  - Uncertainty in the form of a model
  - e.g. Linear vs. Quadratic relationship
- Surprise/Indeterminacy
  - Don’t know what we don’t know

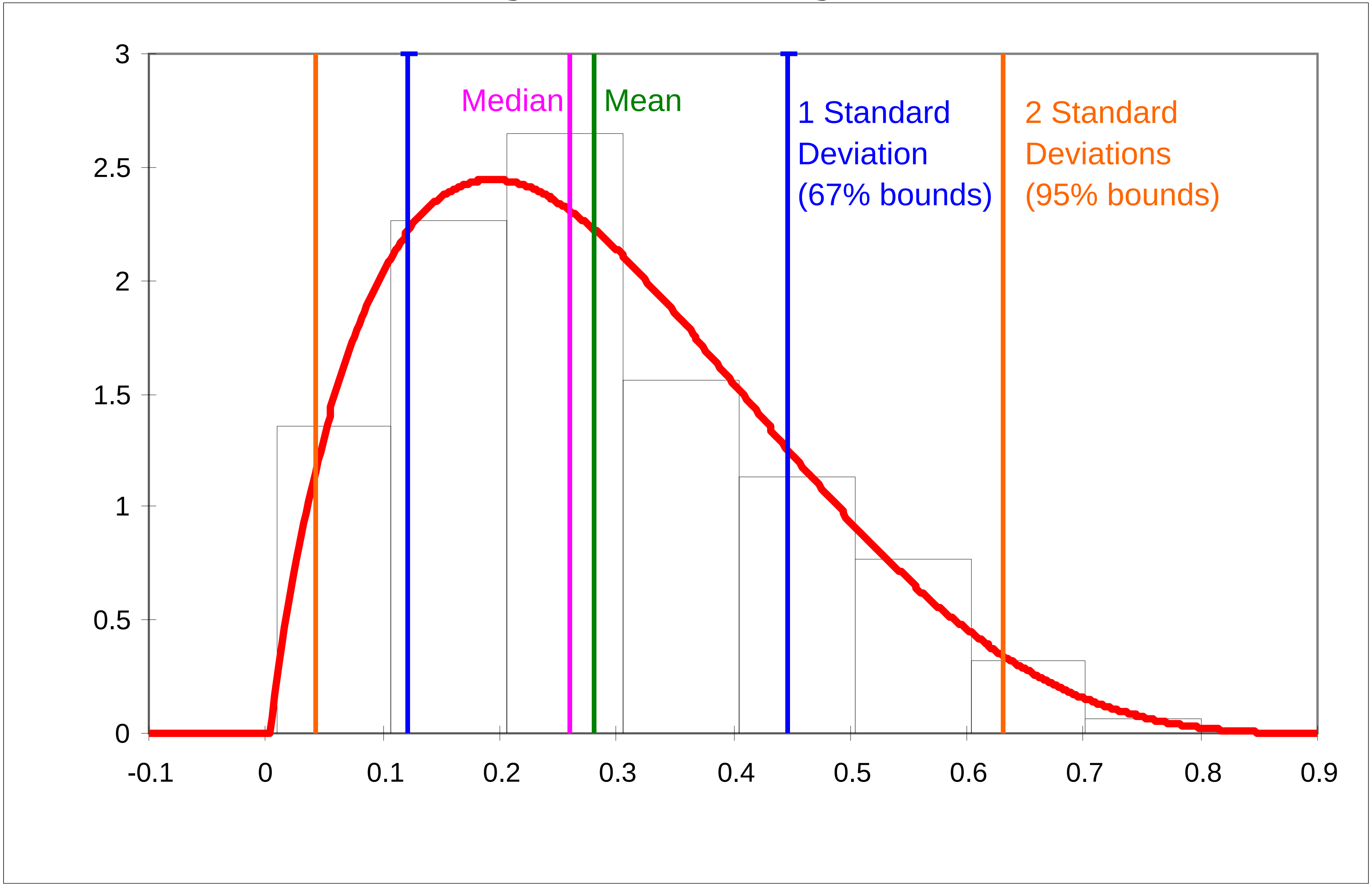
Describing an Uncertainty Quantity

- Mean $\mu_x = \int_{-\infty}^{\infty} x f_x(x) dx = E[x]$
- Variance $\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x) dx = Var[x]$
- Standard Deviation $\sigma_x = \sqrt{Var(x)}$
- Covariance $\text{cov}(X_1, X_2) = E\left[X_1 * X_2\right]$ (if E[x]=0)

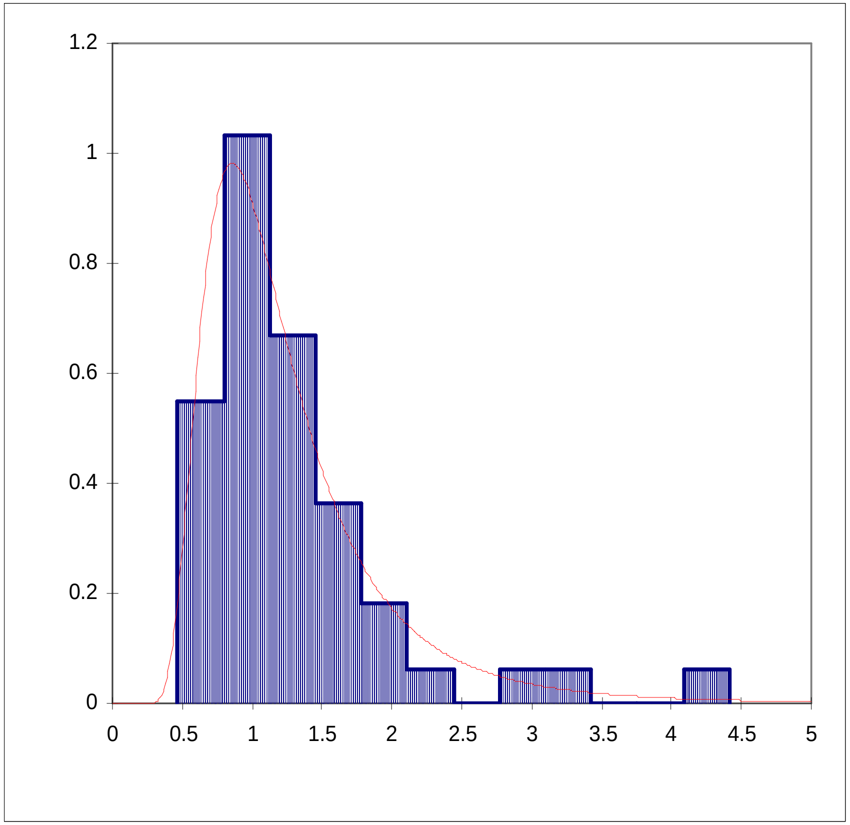
Describing an Uncertain Quantity II

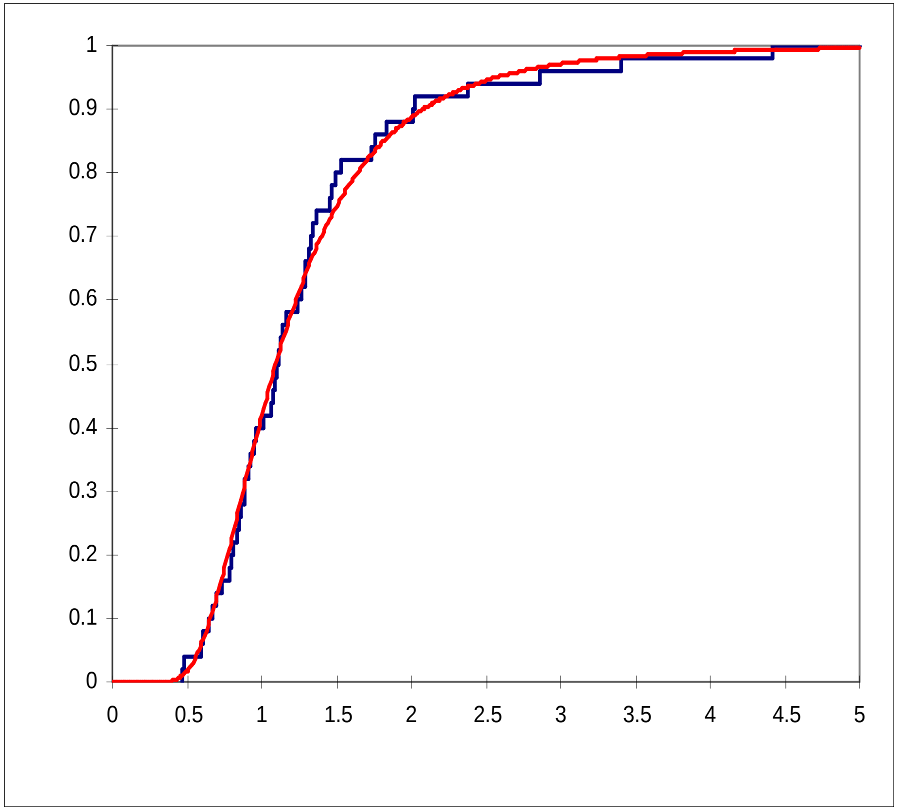
- Mode: Most likely value (peak)
- Median: Value of x such that
  - Prob $(x<x_0)$ = Prob $(x>x_0) = 0.50$
- Fractile: The *p* fractile is the value x_0 such that
  - Prob $(x<x_0) = p$
- Probability Density Function (pdf)
  - The integral under a portion of the function is the probability that the event will fall into that range.
- Cumulative Density Function (cdf)

Probability Density Functions

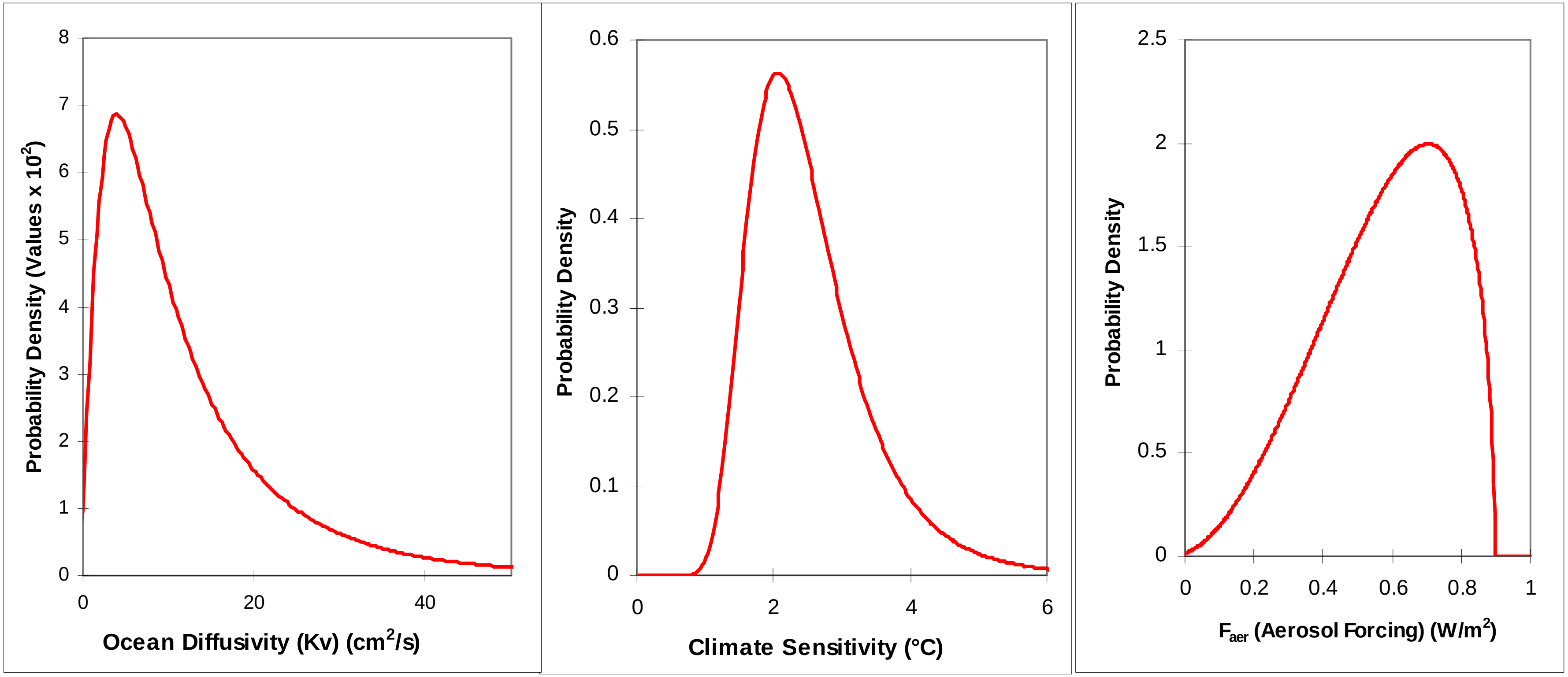


Cumulative Density Function





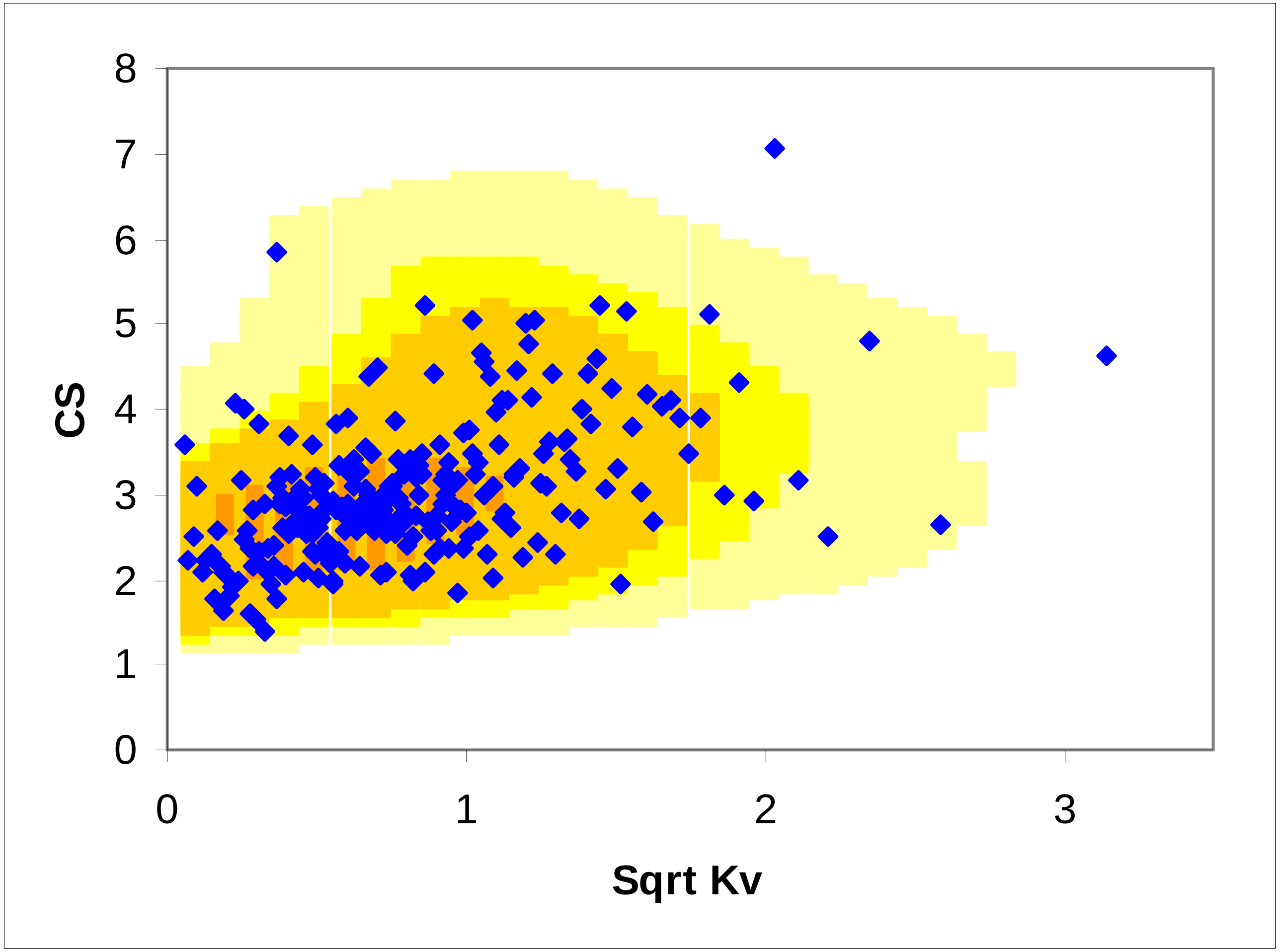
Input distributions



Methods of Input Distribution Acquisition:

Expert Elicitation, Historical Analysis, Modeling Ranges

Correlation between Distributions



Measuring Uncertainty - Global

- Range Sensitivity
  - Varying each x from low to high (tornado plots)
- Joint Parametric Analysis
  - Vary lows and highs for multiple variables
- Monte Carlo Simulation
  - Use pdfs of inputs to produce pdfs of outputs

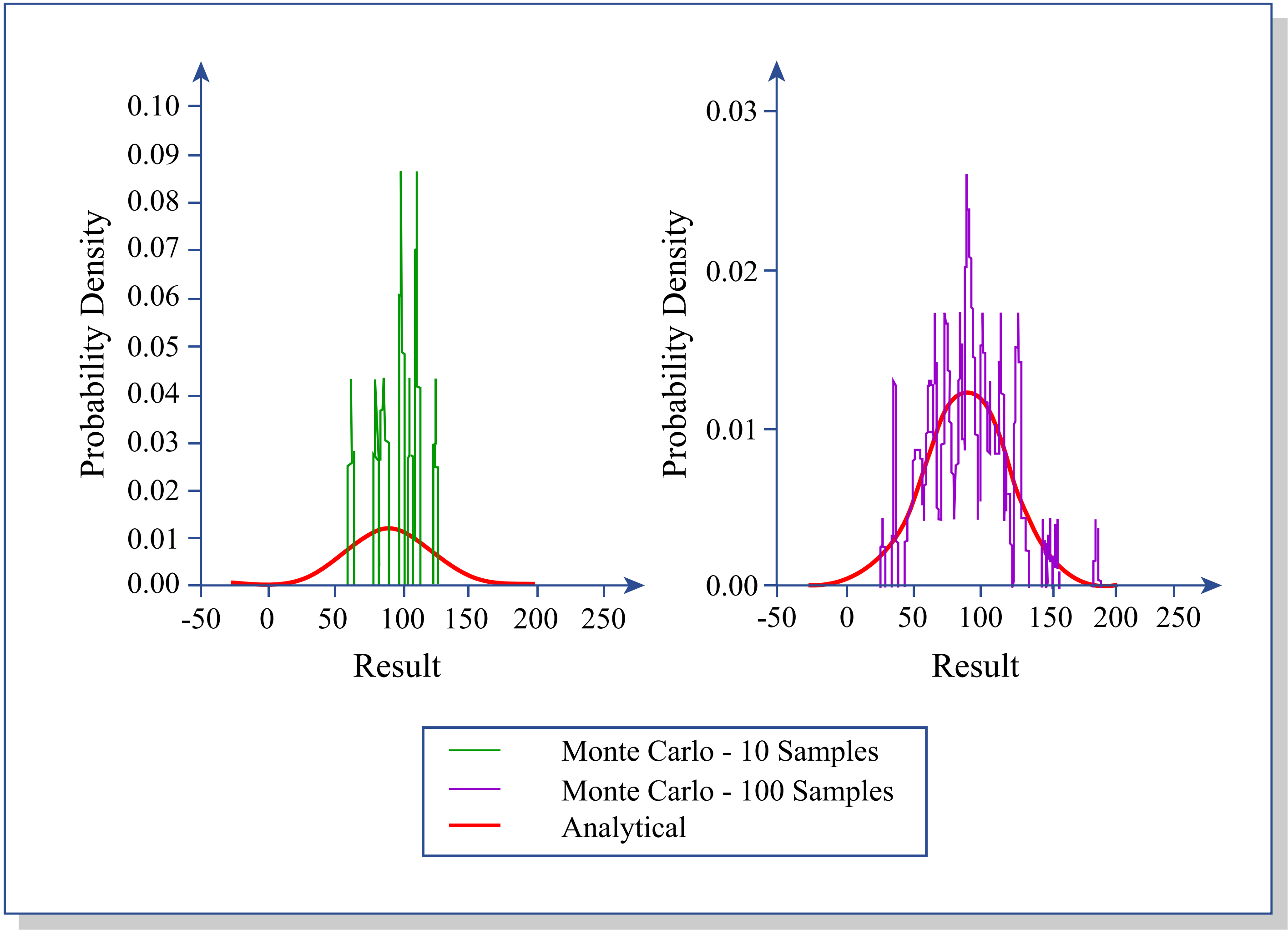
Monte Carlo Simulation

- Crude Monte Carlo
- Stratified Sampling Methods
  - Latin Hypercube Sampling
- Importance Sampling Methods
- Response Surface Methods
  - Probabilistic Collocation Method

Monte Carlo Simulation

- Example 1: Throwing Dice
- Example 2:
  - $Y = X1 + X2$
  - $X1 = N(50,20)$
  - $X2 = N(40,25)$
  - Analytic Analysis: $Y = N(90,32)$
  - Compare to Monte Carlo samples of size n=10,100,1000,10000

Monte Carlo, n=10,100



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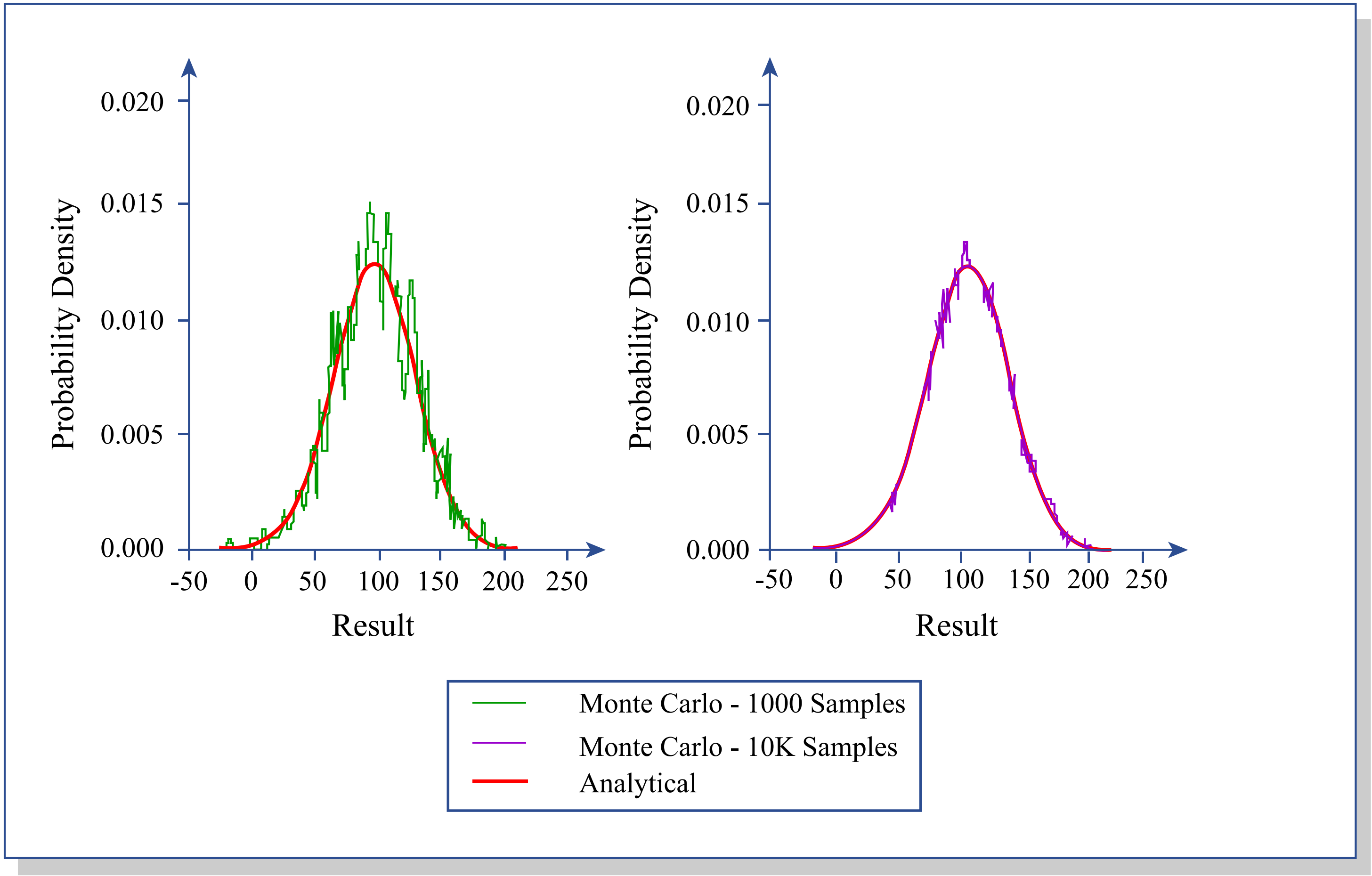
Monte Carlo, n=1000,10000



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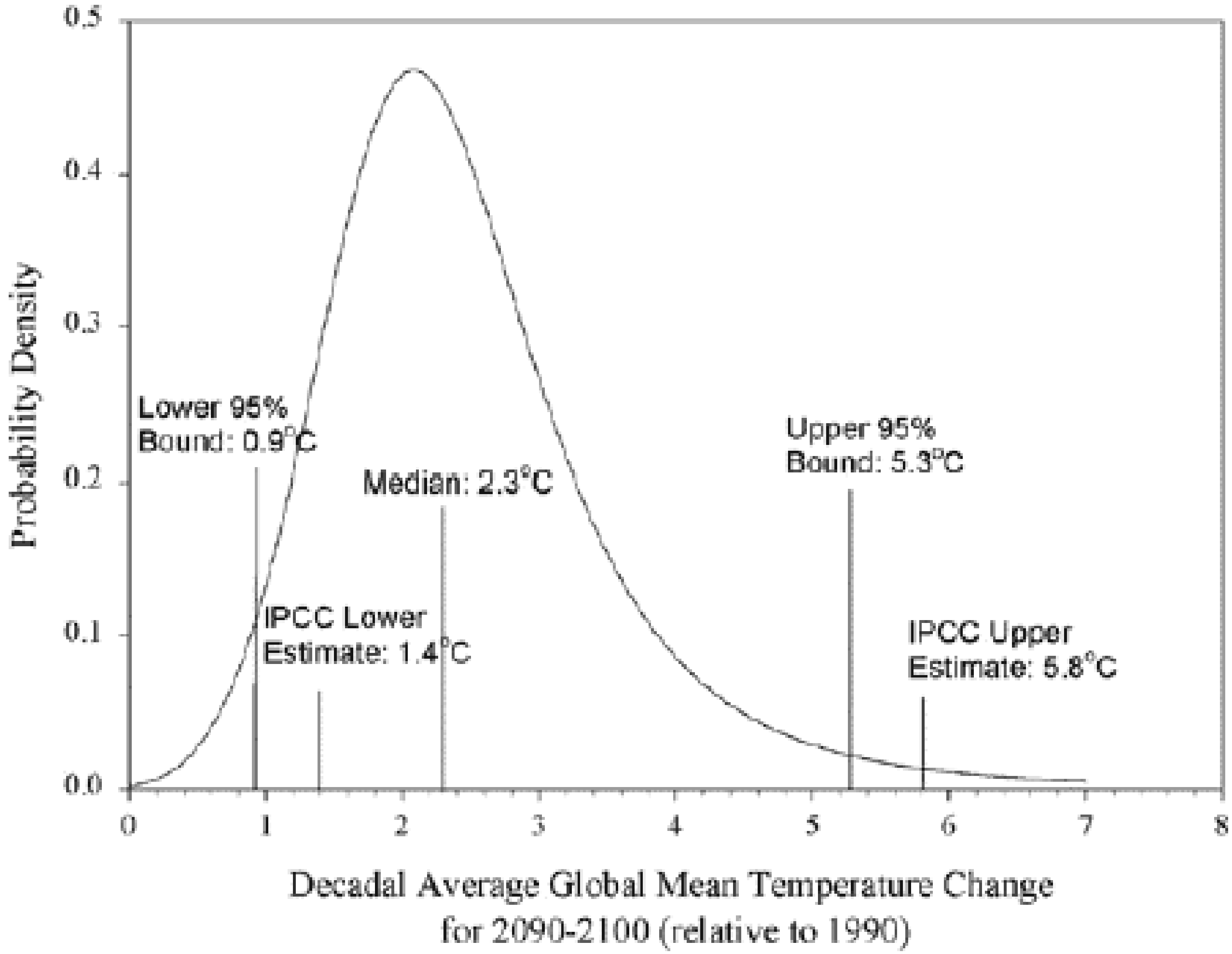
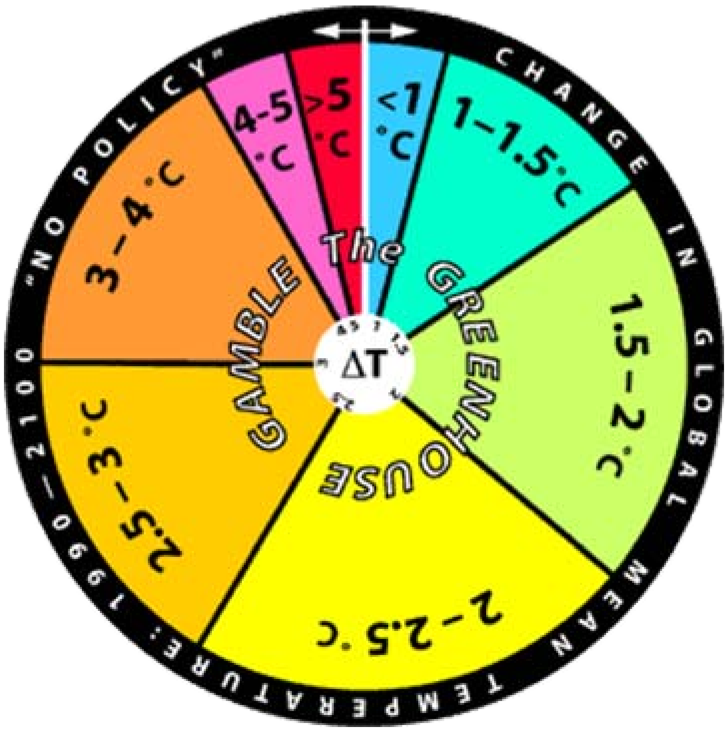
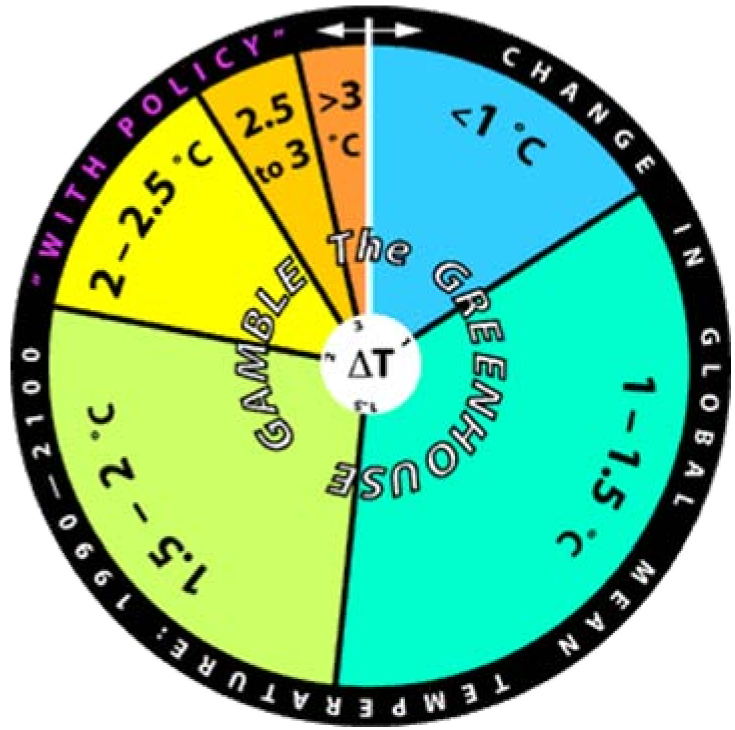


Source: Webster et al (2001). *Uncertainty Analysis of Global Climate Change Projections*, MIT Joint Program Report No. 73.

The Greenhouse Gamble



Business As Usual



Policy

Questions?