



**RECITATION NOTES #2**

**Surplus Analysis with Government Intervention**

**Friday - September 17, 2004**

**OUTLINE OF TODAY'S RECITATION**

- 1. Review of Consumer and Producer Surplus:** brief review from last recitation
- 2. Government Intervention:** how can the Government intervene and what is the effect
- 3. Deadweight Loss:** what it is, why it is important and how it is calculated
- 4. Numeric Examples:** Two exercises to understand how all concepts work together

**1. REVIEW OF CONSUMER AND PRODUCER SURPLUS**

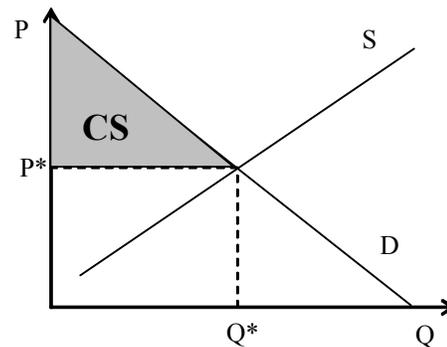
**1.1 Consumer Surplus**

**1.2 Producer Surplus**

**1.1 Consumer Surplus**

Consumer's surplus is the difference between what the consumer is *willing to pay*, and what he *actually pays*. Intuitively, it is "the amount left in the hands of the consumer".

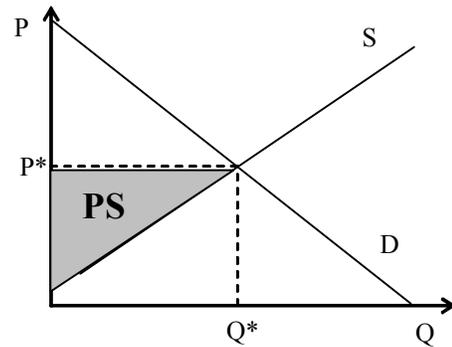
If a consumer has a demand curve  $D$ , like the one shown in the chart, and the Equilibrium is reached for a price  $P^*$ , then he will buy  $Q^*$  and will be left with a surplus equal to the shaded triangle.



## 1.2 Producer Surplus

Producer's surplus is the difference between the amount that the producer *is paid* and the amount he was *willing to accept*. Intuitively, it is "the amount left in the hands of the producer".

If a producer has a supply curve  $S$  and the Equilibrium is reached for a price  $P^*$ , then he will sell  $Q^*$  and will be left with a surplus equal to the shaded triangle.



## 2. GOVERNMENT INTERVENTION

### 2.1 Tax

### 2.2 Subsidy

### 2.3 Quota

### 2.4 Tariff

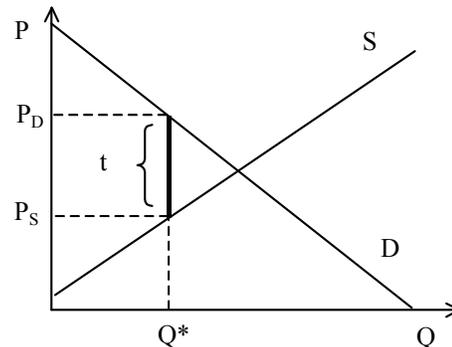
### 2.1 Tax

When a government applies a tax ( $t$ ) to a good, the price that consumers pay ( $P_D$ ) is **higher** than the price that suppliers receive for the good ( $P_S$ ):

$$P_D = P_S + t$$

The total amount of tax the Government will collect (Government's Surplus) is equal to the area:

$$t * Q^* = (P_D - P_S) * Q^*$$



### 2.2 Subsidy

When a government applies a subsidy ( $s$ ) to a good, the price that consumers pay for the good ( $P_D$ ) is **lower** than the price that suppliers receive for the good ( $P_S$ ):

$$P_D = P_S - s$$

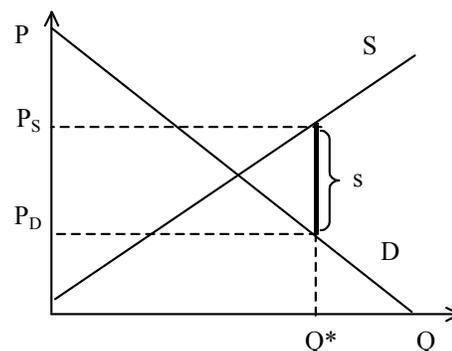
(NOTE: a subsidy is a negative tax!)

The total amount of subsidy the Government will have to pay is equal to the area:

$$s * Q^* = (P_S - P_D) * Q^*$$

This is the Government's loss of surplus.

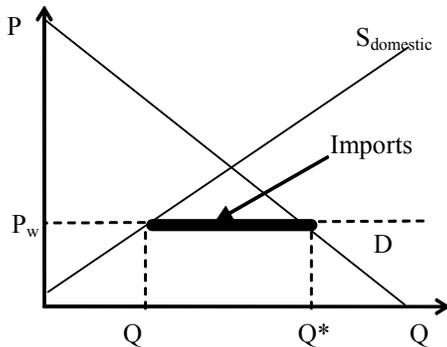
(In the examples section you can see what



happens if the Government sets a maximum price for the goods instead of a subsidy)

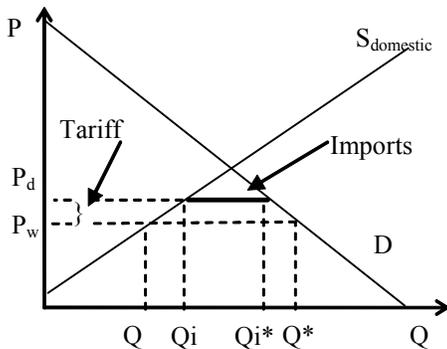
### 2.3 Tariff

Without government intervention, at world price ( $P_w$ ), domestic producers supply  $Q$  and foreign producers supply  $Q^* - Q$ .



When a government applies a tariff to a good, it is setting a tax on the quantities of good that foreign producers sell in the country. In effect, it is raising the price of foreign goods from  $P_w$  to  $P_d$ :

$$P_d = P_w + \text{tariff}$$



Domestic producers will produce up to a level  $Q_i$ , while foreign producers will produce  $(Q_i^* - Q_i)$

The total demanded amount will be:

$$Q_i^* = Q_i + \text{imports}$$

The total amount of tariff the Government will collect is equal to the area:

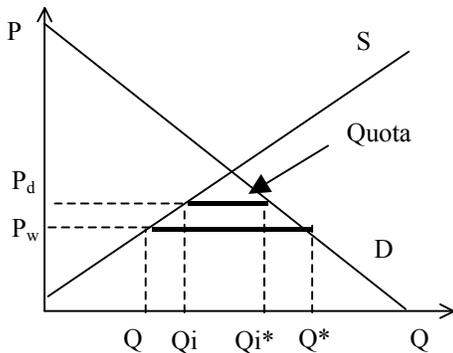
$$\text{imports} * \text{tariff} = (Q_i^* - Q_i) * (P_d - P_w)$$

This is the Government's surplus.

### 2.4 Quota

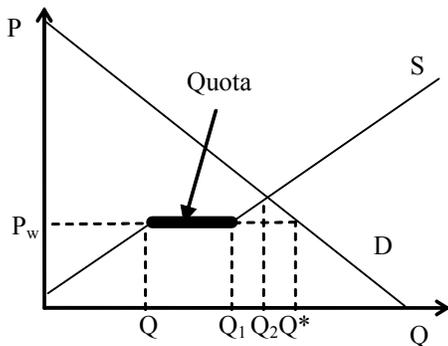
When a government applies a quota to a good, it is setting the maximum level of quantity that foreign producers can sell in the country. Domestic producers will produce up to a level  $Q_i$ , while foreign producers will produce  $(Q_i^* - Q_i)$ . The total demanded amount will be:

$$Q_i^* = Q_i + \text{quota}$$



NOTE: in a quota the Government does not collect any money! In the quota case the foreign producers receive the surplus, whereas in the tariff case the government does

This above graphic representation of a quota is equivalent to the one done in the lecture, where the market supply curve is derived more intuitively, as shown below. The only difference is that the supply curve for domestic producers is shifted, but the areas and quantities remain the same.



Amounts produced:

Local producers:  $Q + (Q_2 - Q_1)$

Foreign producers:  $(Q_1 - Q)$

### 3. DEADWEIGHT LOSS

#### 3.1 DWL with a Tax

#### 3.2 DWL with a Tariff or a Quota

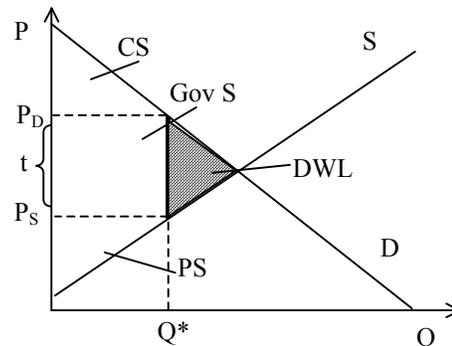
#### 3.3 Pass through formula

#### 3.1 DWL with a Tax

When the Government intervenes with a tax the total surplus available in an economy is smaller than the total surplus available when there is no intervention:

$$W/\text{Tax}(CS + PS + \text{Gov S}) < W/o \text{ Tax}(CS + PS)$$

The amount of surplus lost in the economy is called Deadweight Loss (DWL) and is represented by the shaded area in the chart.



#### 3.2 DWL with a Quota or Tariff

When the Government intervenes with a Tariff or a Quota the total surplus available in an economy is smaller than the total surplus available when there is no intervention:

$$\Delta \text{ Consumer Surplus} = -(A+B+D+C)$$

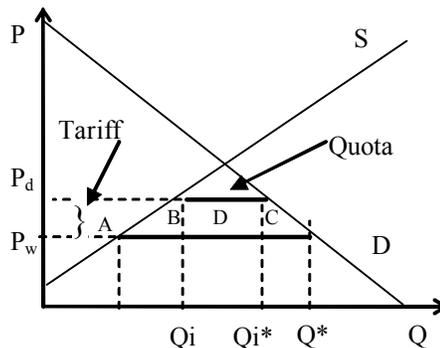
$$\Delta \text{ Producer Surplus} = A$$

$$\text{Quota: Gain to Foreign Producers} = D$$

$$\text{Tariff: Gain to Government} = D$$

$$\text{DWL} = -(B+C)$$

The amount of surplus lost in the economy is called Deadweight Loss (DWL) and is represented by the sum of the two triangles B and C.



#### 3.3 Pass through formula

When a tax is applied, it generally affects both producers and consumers and not necessarily in the same way.

We can graphically demonstrate that if the demand for a good is very elastic, producers will bear most of the tax burden. Instead, if the demand is very inelastic, the consumers will bear most of the tax burden.

In order to calculate the percentage of a tax that will be paid by the consumer, we can use the pass-through formula:

$$\frac{\% \text{ of tax paid by the consumer}}{Es - Ed} = \frac{Es}{Es - Ed}$$

where  $E_s$  is the price elasticity of supply, and  $E_d$  is the price elasticity of demand.

#### 4. NUMERIC EXAMPLES

##### 4.1 Gasoline at Market Equilibrium and with a Tax

##### 4.2 Dynamic Changes in Market Equilibrium in the Gasoline Market

##### 4.3 Government sets a Maximum Price

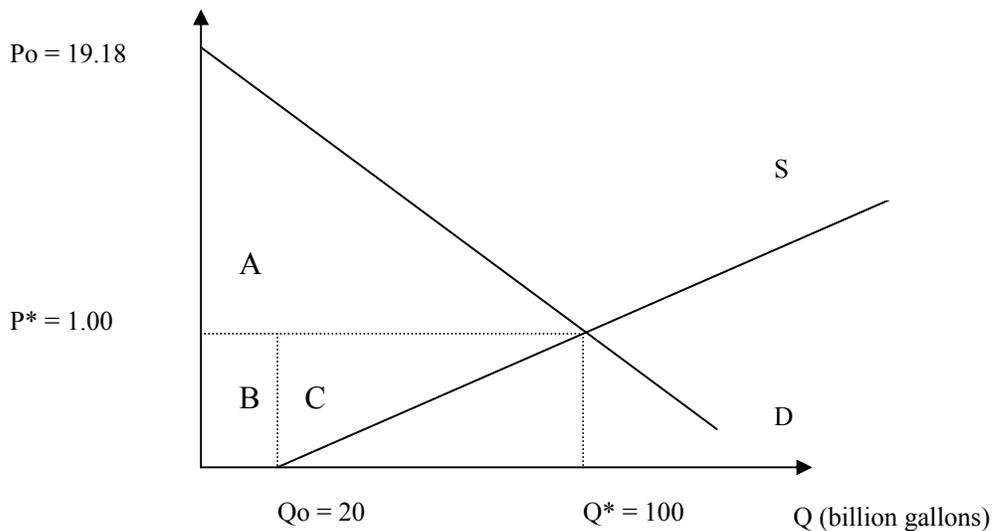
#### 4.1 Gasoline at Market Equilibrium with a Tax

##### 4.1.1 Available Information

Let's assume that:

Demand curve for Gasoline:  $Q_d = 105.5 - 5.5 \cdot P$  (P in \$/gallon, Q in Bn gallons)

Supply curve for Gasoline:  $Q_s = 20 + 80 P$



##### 4.1.2 Determine the Equilibrium

In order to do so, we first determine the market price by setting:

$$Q_d = Q_s$$

$$105.5 - 5.5 P = 20 + 80 P$$

$$P^* = 1.0 \text{ \$/gal}$$

$$Q^* = 100 \text{ billion gal}$$

Therefore, the market equilibrium is  $P^* = 1.0$  \$/gal and  $Q^* = 100$  billion gallons.

#### 4.1.3 Calculate Consumer's and Producer's Surplus

##### Consumer's Surplus

Consumer Surplus is the area **A**. We calculate this area using geometry.

First we must obtain  $P_o$ , which is calculated by setting  $Q_d = 0$  in the demand curve. I.e.:

$$Q_d = 105.05 - 5.5 P_o = 0 \rightarrow P_o = 19.18 \text{ \$/gal}$$

$$\begin{aligned} \text{Area A} &= \frac{1}{2} (P_o - P^*) (Q^* - 0) \\ &= \frac{1}{2} (19.18 - 1.00) (100 - 0) \\ &= \$ 909 \text{ billion} \end{aligned}$$

Therefore, the **consumer surplus is \$ 909 billion.**

##### Producer's Surplus:

Producer Surplus is the area **B + C**. We calculate this area using geometry.

$$\begin{aligned} \text{Area B + Area C} &= (P^* - 0) (Q_o - 0) + \frac{1}{2} (P^* - 0) (Q^* - Q_o) \\ &= (1.0) (20) + \frac{1}{2} (1.00) (100 - 20) \\ &= \$ 60 \text{ billion} \end{aligned}$$

Therefore, the **producer surplus is \$ 60 billion.**

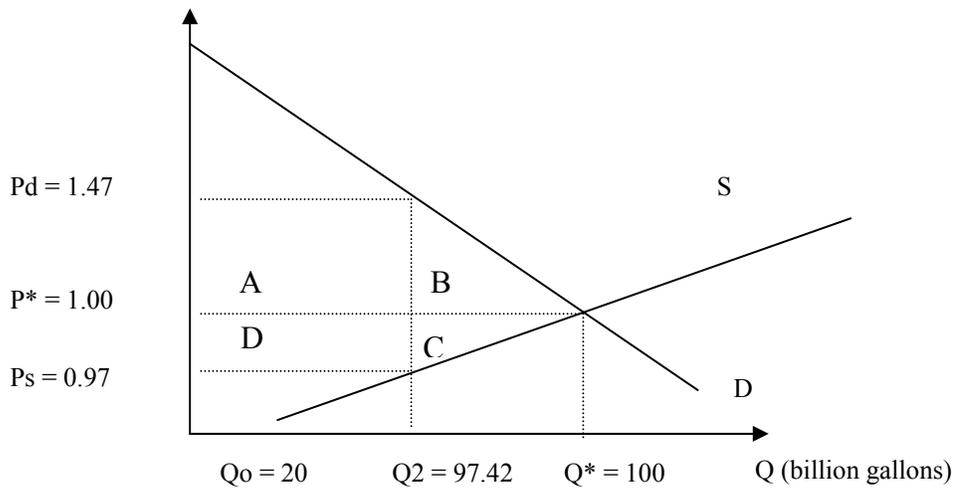
Alternatively, we can use the area of a tetrahedral to calculate the producer surplus:

$$\text{Area (B+C)} = \frac{1}{2} (Q_o + Q^*) (P^* - 0) = \$ 60 \text{ billion}$$

#### 4.1.4 Government imposes a tax

Now let's consider the case when the government imposes a 0.5 \$/gal tax on gasoline to raise revenue. Who are the winners and the losers with this policy?

**Calculate the new equilibrium**



Let's assume that the demand and supply curves stay the same.

Due to the tax, the price that the consumer pays is higher than the price that the supplier receives.

$P_d$  = consumer price

$P_s$  = suppliers price

$t$  = tax = 0.5 \$/gal

$$P_d = P_s + t$$

Let's find the new equilibrium point:

$$Q_s = Q_d$$

$$20 + 80 P_s = 105.5 - 5.5 P_d = 105.5 - 5.5 (P_s + 0.5)$$

$$P_s = 0.97 \text{ \$/gal}$$

$$P_d = 1.47 \text{ \$/gal}$$

$$Q_s = Q_d = 20 + 80 (0.97) = 97.4 \text{ billion gallons}$$

With the tax, producers will sell gasoline at 0.97 \$/gal but consumers will buy it at 1.47 \$/gal. With the tax, producers will produce 97.42 billion gallons ( $Q_2$ ).

Let's look again at the change in the consumer surplus, producer surplus and deadweight loss:

### **Change in the Consumer Surplus**

Consumers lose area A due to taxes, and area B due to higher prices.

$$\begin{aligned}\Delta \text{ Consumer Surplus} &= - (\text{Area A} + \text{Area B}) \\ &= - (P_d - P^*) (Q_2 - 0) - \frac{1}{2} (P_d - P^*) (Q^* - Q_2) \\ &= - (1.47 - 1.00) (97.42 - 0) - \frac{1}{2} (1.47 - 1) (100 - 97.42) \\ &= - 45.78 - 0.61 \\ &= - \$46.4 \text{ billion}\end{aligned}$$

The change in the consumer surplus is - **\$ 46.4 billion**.

### **Change in the Producer Surplus**

Producers lose area D due to taxes, and also area C due to less production.

$$\begin{aligned}\Delta \text{ Producer Surplus} &= - (\text{Area D} + \text{Area C}) \\ &= - (P^* - P_s) (Q_2 - 0) - \frac{1}{2} (P^* - P_s) (Q^* - Q_2) \\ &= - (1.00 - 0.97) (97.42 - 0) - \frac{1}{2} (1 - 0.97) (100 - 97.42) \\ &= - 2.92 - 0.04 \\ &= - \$2.96 \text{ billion}\end{aligned}$$

The change in the producer surplus is - \$2.96 billion.

### **Government's Revenues**

Government revenue is the sum of Area A and Area D:

$$\begin{aligned}\text{Government revenue} &= \text{Area A} + \text{Area D} \\ &= 45.78 + 2.92 \\ &= \$48.71 \text{ billion}\end{aligned}$$

### **Deadweight loss due to Tax:**

Overall, the total deadweight loss is the Area B and C:

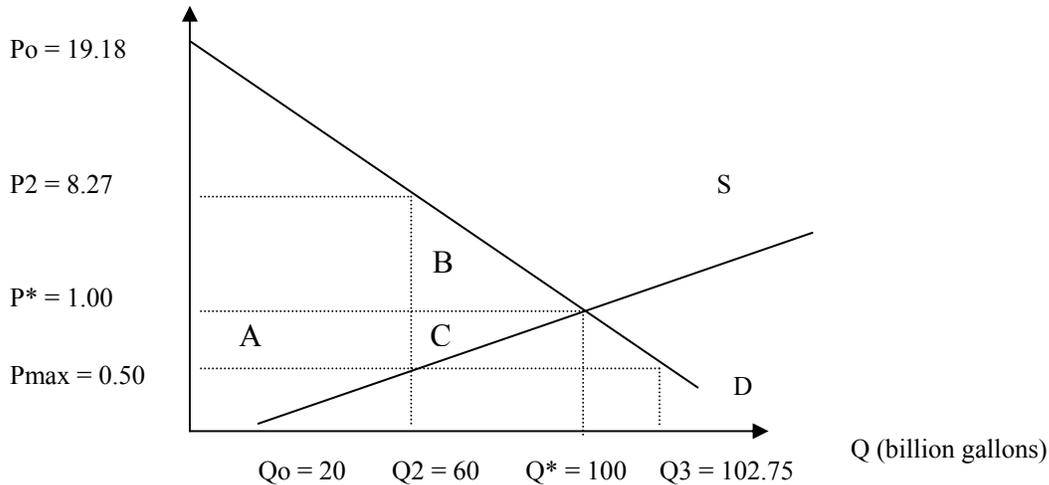
$$\begin{aligned}\text{Deadweight loss} &= - (\text{Area B} + \text{Area C}) \\ &= - (0.61 + 0.04) \\ &= - \mathbf{\$0.65 \text{ billion}}\end{aligned}$$

## 4.2 Government sets a Maximum Price for gasoline:

### 4.2.1 Equilibrium reviewed

Let's say that the government is concerned with the high price of gas and sets a maximum price of 0.5 \$/gal. Who are the winners and losers with this policy?

Let's assume that demand and supply stays the same.



Let's see what suppliers do:

$$Q_s = 20 + 80(0.5) = 60 \text{ billion gallons}$$

With the new price being set at 0.5 \$/gal, suppliers will only produce 60 billion gallons ( $Q_2$ ) instead of 100 billion.

Let's now look at the consumers:

$$Q_d = 105.5 - 5.5(0.5) = 102.75 \text{ billion gallons}$$

With the new price being set at 0.5 \$/gal, consumers will want 102.75 billion gallons ( $Q_3$ ). The result will be shortage and long lines for gasoline.

Now, let's look at the change in the consumer surplus, the producer surplus and the deadweight loss.

#### 4.2.2 Change in the Consumer Surplus:

Consumers gain area A due to lower prices, but they lose area B due to shortages.

$$\begin{aligned}\Delta \text{ Consumer Surplus} &= \text{Area A} - \text{Area B} \\ &= (P^* - P_{\max}) (Q_2 - 0) - \frac{1}{2} (P_2 - P^*) (Q^* - Q_2) \\ &= (1.00 - 0.50) (60 - 0) - \frac{1}{2} (8.27 - 1) (100 - 60) \\ &= - \$115.4 \text{ billion}\end{aligned}$$

Note: P2 results from plugging 60 billion gallons into the demand curve.

The change in the consumer surplus is - \$ 115.4 billion. Therefore consumers as a whole are not as well off as the government hoped!

#### 4.2.4 Change in the Producer Surplus:

Producers lose area A due to lower prices, and also area C due to less production.

$$\begin{aligned}\Delta \text{ Producer Surplus} &= - (\text{Area A} + \text{Area C}) \\ &= - (P^* - P_{\max}) (Q_2 - 0) - \frac{1}{2} (P^* - P_{\max}) (Q^* - Q_2) \\ &= - (1.00 - 0.50) (60 - 0) - \frac{1}{2} (1 - 0.5) (100 - 60) \\ &= - \$40 \text{ billion}\end{aligned}$$

The change in the producer surplus is - \$40 billion.

#### 4.2.5 Deadweight loss due to Maximum Price Policy

Overall, the total deadweight loss is the Area B and C:

$$\begin{aligned}\text{Deadweight loss} &= - (\text{Area B} + \text{Area C}) \\ &= - (145.4 + 10) \\ &= - \$155.4 \text{ billion}\end{aligned}$$

### 4.3 Dynamic Changes in Market Equilibrium in the Gasoline Market

(This example is based on example 9.6 of the Pindyck Rubinfeld textbook.)

During the 1980 presidential campaign, John Anderson, an independent candidate, proposed a 50 cent per gallon tax on gasoline. The idea of a gasoline tax, both to increase government revenue and to reduce oil consumption and the US dependence on oil imports, has been widely discussed since then. It even became a part of the Clinton Administration's 1993 budget plan.

You have been asked to study the effects that such a tax would have on gasoline demand, price, consumer surplus, producer surplus and deadweight loss.

A detailed study has determined that demand elasticity for gasoline to be the following:

1 year elasticity	-0.0550
2 year elasticity	-0.1100
3 year elasticity	-0.1600
4 year elasticity	-0.2025
5 year elasticity	-0.2450

The current price of gasoline is 1.00 \$/gal and the total consumption is 100 billion/year. Assume that the clearing price remains the same for five years.

Your estimate of the supply elasticity is 0.8.

1. Calculate the demand curves for the next five years
2. Calculate the supply curve
3. Determine the total gasoline consumption and prices for the next five years
4. Calculate the changes in consumer surplus, producer surplus and deadweight loss for these years. What percentage of the tax is paid by consumers?

#### 4.3.1 Demand curves:

$$Q_d = a - b P$$

$$E_d = (\Delta Q_d / \Delta P) P / Q_d \rightarrow$$

$$\rightarrow \Delta Q_d / \Delta P = -b$$

$$E_d = -b (P/Q_d)$$

$$\rightarrow b = - E_d (Q_d/P)$$

$$b = -(-0.0550)(100/1) = 5.5 \text{ billion}$$

$$\rightarrow a = Q_d + b P$$

$$a = Qd (1 - Ed)$$

$$a = 100 (1 + 0.0550) = 105.5 \text{ billion}$$

$$Qd = 105.5 - 5.5P$$

#### 4.3.2 Supply curve:

$$Qs = c + d P$$

$$Es = (\Delta Qs / \Delta P) P / Qs$$

$$\rightarrow \Delta Qs / \Delta P = d$$

$$\rightarrow Es = d (P / Qs)$$

$$\rightarrow d = Es (Qs / P)$$

$$d = (0.8)(100) / 1 = 80 \text{ billion}$$

$$\rightarrow c = Qs - d P$$

$$c = Qs (1 - Es)$$

$$c = (100)(1 - 0.8) = 20 \text{ billion}$$

$$Qs = 20 + 80P$$

#### 4.3.3 Total gasoline consumption and prices for the next five years:

Because there is a tax, we know that  $Pd$  is no longer equal to  $Ps$ . Their relation is  $Pd = Ps + t$ .

We can rewrite the demand equations as follows:

$$Qd = a - b Pd$$

$$Qs = c + d Ps$$

What is now the quantity demanded?

We know that at equilibrium the quantity demanded and the quantity supplied are equal, then:

$$Qd = Qs$$

$$a - b Pd = c + d Ps$$

$$\text{but } Pd = Ps + t$$

Then:

$$a - b (Ps + t) = c + d Ps$$

$$Ps = \frac{a - b t - c}{d + b} = \frac{105.5 - 5.5(0.5) - 20}{80 + 5.5} = 0.968$$

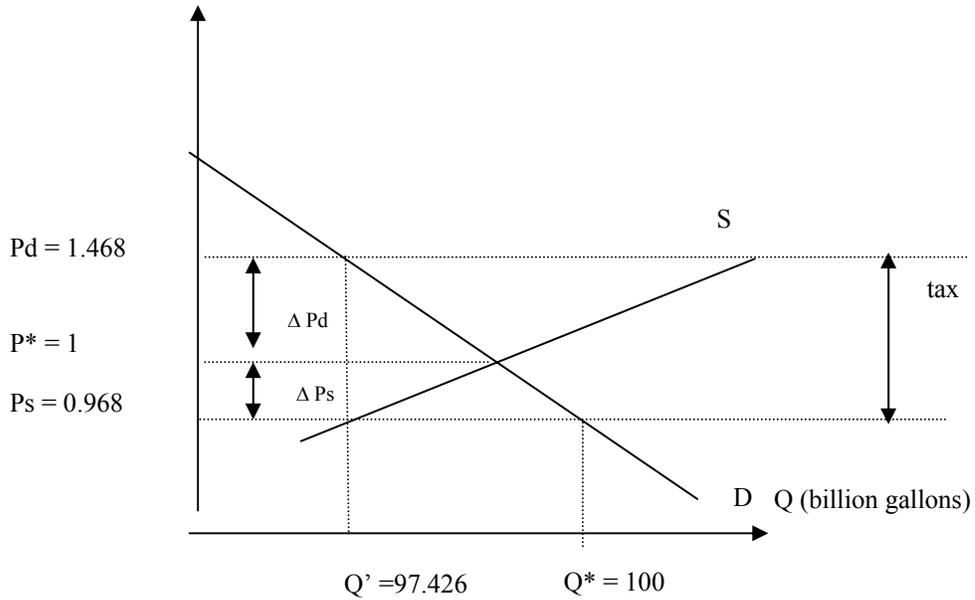
We now can calculate

$$Pd = Ps + t = 0.968 + 0.5$$

and the total consumption:

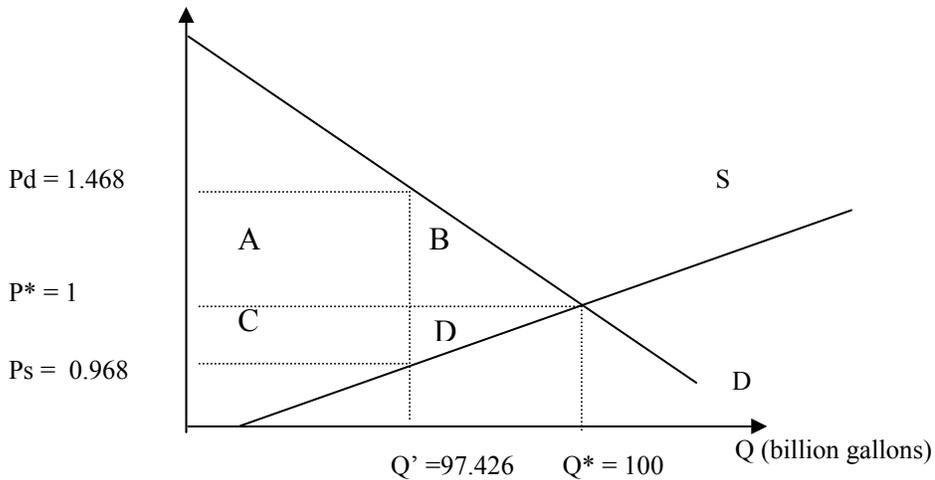
$$Q_d = a - b P_d = 105.5 - 5.5 (1.4678) = 97.426$$

This is shown graphically in the following figure:



**4.3.4 Changes in the consumer surplus, producer surplus and deadweight loss for the next five years. What percentage of tax is paid by consumers?**

P (\$/gal)



Area A =  $(P_d - P^*) Q' = 45.576$   
 Area B =  $\frac{1}{2} (P_d - P^*) (Q^* - Q') = 0.60189$   
 Area C =  $(P^* - P_s) Q' = 3.1336$

$$\text{Area D} = \frac{1}{2} (P^* - P_s) (Q^* - Q') = 0.04138$$

$$\Delta \text{ Consumer Surplus} = - (A+B) = -46.178$$

$$\Delta \text{ Producers Surplus} = - (C+D) = -3.17498$$

$$\text{Government Revenue} = A+C = 48.7096$$

$$\text{DWL} = - (B+D) = -0.64327$$

The percentage of the tax that is paid by consumers is given by the pass-through formula:

$$\text{Buyer's share} = \frac{E_s}{E_s - E_d}$$

Year	Ed	Qs = Qd	C Share	Δ CS	Δ PS	GR	DWL
1	-0.0550	97.43	94%	-46.18	-3.17	48.71	-0.64
2	-0.1100	95.16	88%	-42.89	-5.90	47.58	-1.21
3	-0.1600	93.33	83%	-40.28	-8.06	46.67	-1.67
4	-0.2025	91.92	80%	-38.29	-9.69	45.96	-2.02
5	-0.2450	90.62	77%	-36.48	-11.17	45.31	-2.34

Here we can observe the evolution of quantities, consumer share of the tax and changes in the surpluses, in the case of a non-durable good. As time passes, the demand elasticity increases and the quantity that is being demanded decreases – because consumers become more sensitive to price increments –. For the same reason, the consumers' share of the tax goes down and so does the change in their surplus. On the contrary, the change in the producers' surplus increases, as their tax share increases. Government revenues also fall over time because the quantity goes down.