



RECITATION NOTES #6

Price Discrimination and Two Part Tariff

Friday - October 29, 2004

OUTLINE OF TODAY'S RECITATION

1. **Conditions for applying price discrimination:** brief introduction to today's subject
2. **Perfect Price Discrimination:** definition and explanation
3. **Consumer Self-Selection:** definition and explanation
4. **Pricing to Observable Market Segments:** definition and explanation
5. **Two part Tariff: definition and calculations**
6. **Numeric Examples:** applying these concepts to exercises

1. CONDITIONS FOR APPLYING PRICE DISCRIMINATION

- 1.1 Definition of price discrimination
- 1.2 Conditions for applying price discrimination
- 1.3 Types of price discrimination

1.1 Definition of price discrimination

Up to this point we have been studying cases where only *one* price is charged by the producer to the consumers, even when the producer has market power. Now we will consider the strategies where different prices can be charged to different consumers. By doing so, the producer can capture even more consumer surplus and **earn even greater profits**. This is called **Price Discrimination**, as producers discriminate across consumers by charging higher prices to those willing to pay more and lower prices to those who are not willing to pay very much.

1.2 Conditions for applying price discrimination

In order to successfully be able to price discriminate, a firm must meet the following conditions:

1. **Have market power**
2. **Be able to prevent resale of the good (i.e. no secondary markets)**
3. **Be able to identify and distinguish between groups of customers**

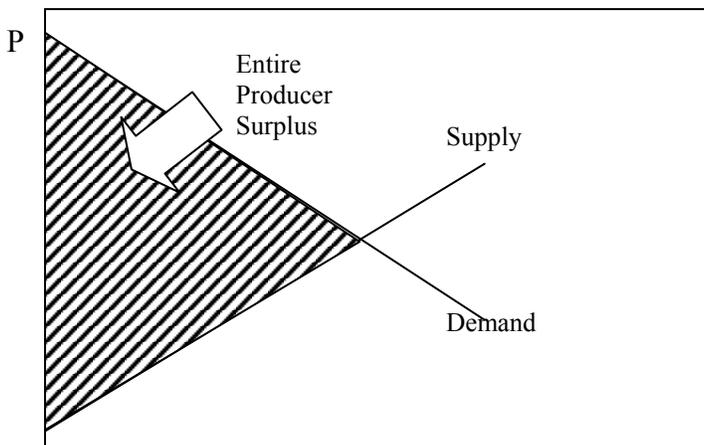
1.3 Types of price discrimination

There are three types of price discrimination strategies:

1. **Perfect Price Discrimination:** in this strategy, firms charge exactly each consumers' reservation prices (their maximum willingness to pay) for their products.
2. **Consumer Self-Selection:** in this case, being unable to determine the exact reservation price of the consumers, firms let consumers select different, predetermined levels of pricing that maximize the firms' profits
3. **Pricing to Observable Market Segments:** in this case, firms can not determine the exact reservation price of consumers. Therefore firms discriminate prices based upon objective criteria that distinguish consumers in different groups with different demand curves.

2. PERFECT PRICE DISCRMINATION

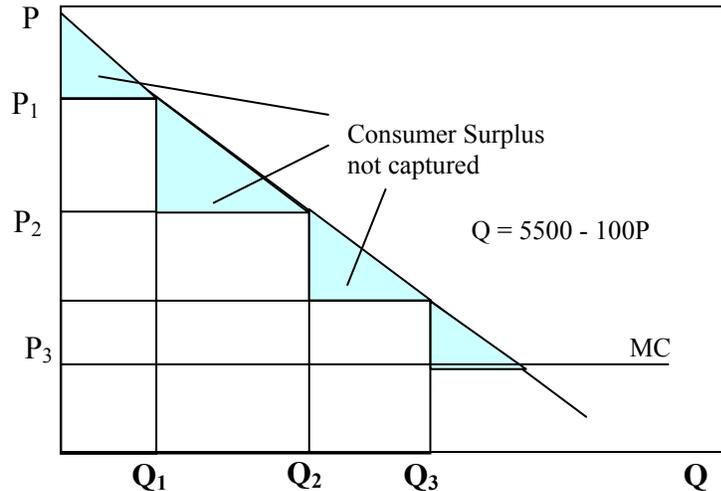
In this case the firm is able to charge the reservation price (i.e. the “willingness to pay” price) to *each* consumer. In this case, the firm is able to capture the entire consumer surplus. The diagram depicts this situation.



This strategy is applicable when a Firm has the ability to “read consumers’ minds” and determine exactly what each and every consumer in the market is willing to pay for the product sold. In reality, perfect price discrimination is not easily applied, as it would be far too expensive for firms to collect all the available information on all consumers.

3. CONSUMER SELF-SELECTION

Firms, however, do not usually know the reservation price of every customer. Therefore, to capture excess surplus, firms usually resort to consumers to self-select themselves into the different price segments. This is done through access fees, volume discounts, peak-price loading, etc. (e.g., airlines and first class vs. economy/coach class ticket pricing)



The diagram shows how volume discount would work – customers who do not purchase very much ($Q < Q_1$) pay a higher price (P_1), while consumers who purchase more ($Q_2 < Q < Q_3$) pay a lower price (P_3). Note that the firm captures only a subset of the total consumer surplus.

4. PRICING TO OBSERVABLE MARKET SEGMENTS

Many times consumers can be classified into two or more groups with *separate demand curves*. Using some objective characteristic to distinguish among consumers that belong to different groups firms can engage in selective pricing. *This is the most prevalent form of price discrimination because it is relatively easy to implement and far cheaper than the other methods.* Examples include student discounts and senior citizen discounts.

5. TWO PART TARIFF

5.1 Definition

5.2 Necessary conditions for utilizing a two part tariff

5.3 How to determine the optimal two-part tariff

5.1 Definition

The purpose of a two-part tariff is to extract more of the consumer surplus, by using a pricing scheme made up of two parts:

- A fixed, one-time fee charged to each user that entitles the person to make further purchases. It may be also called entry fee, set-up charge, or enrollment fee.
- A price per each unit purchased.

5.2 Necessary condition for utilizing a Two Part Tariff

Necessary conditions to take advantage of this strategy:

1. The supplier must have market power.
2. The producer must be able to control access.

NOTE: Preferably the individual consumers have similar demand curves.

5.3 How to determine the optimal two part tariff

5.3.1 A single kind of consumer

If there is one type of consumer and all consumers have the same demand curve, then you can capture all the consumer surplus by setting **price equal to marginal cost** and setting the **fixed fee equal to the consumer surplus** for an individual consumer.

The process needed to set up this profit-maximizing two-part tariff (a two-part tariff that extracts most available surplus from the consumers) is the following:

- 1 Let's assume we have N consumers, each with a demand curve $Q(P)$. First, we need to calculate each individual's Consumer Surplus, because this is the optimal Tariff that needs to be applied. This surplus is equal to the area below the demand curve and above the supply curve (or the marginal cost curve).
- 2 Second, we need to calculate how many units of output each consumer will demand for the price level equal to the Marginal Cost. In other words we need to calculate $Q(MC)$.
- 3 We now have the optimal Fee per consumer, the optimal quantity of output per consumer and the optimal price per unit of output. All we need is to calculate the profits, which are given by:

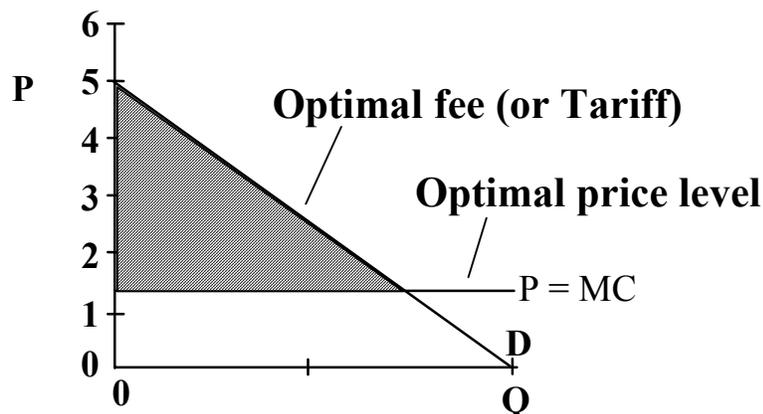
$$\Pi = N * T + N * (P * Q) - N * (MC * Q)$$

Note that if there are no fixed costs, because $P = MC$, the profits become:

$$\Pi = N * T + N * (MC * Q) - N * (MC * Q) = N * T$$

which means that the only profits will be given by the tariff portion.

Graphically:



5.3.2 Two kinds of consumers

If there are two types of consumers — and all consumers within the same group have the same demand curve — then the way to capture all the available consumer surplus is by **maximizing the profit function with respect to price**. The reason for this to be true, is that this time we do not know which of the following solutions would award us more profits:

- 1) sell only to the high-yield customers: set $P = MC$ and the fee equal to the surplus of the high-yield customers. This is identical to the one-type-of-consumer case, discussed above.

2) sell to both types of consumers: set the fee equal to the surplus of the low-yield consumers and then choose P so as to maximize total profit (including the fees); this will result in $P > MC$.

The process needed to solve this second type of problem is the following:

- 1 Let's assume we have N consumers of one kind (high-yield) and M consumers of a different kind (low-yield), and that all consumers within a group share the same demand function. We then will have $Q_1(P)$ and $Q_2(P)$. In order to be attractive to both groups of consumers, we need to set the Fee equal to the surplus of the low yield consumers (by doing so we are sure we will attract both kinds of consumers) (NOTE: This is not necessarily the best strategy. Profits could indeed be maximized by focusing only on the high-yield consumers. In that case the calculations would be very similar to what we have see in 1.3.1). This time, though, the Marginal Cost is not necessarily the optimal price level for our product. We therefore have to express $T = \text{Surplus of low-yield consumers}$ as a function of P. (In other words, calculate the area below the demand curve and above a certain level of pricing P, writing the Tariff as $T = f(P)$).

- 2 Second, we need to write the total profit function in terms of P. This means that we need to put together all the sources of income and all costs into one formula:

$$\begin{aligned} \Pi &= N * T(P) + M * T(P) && \text{Total revenues from Fee (calculated on } Q_1) \\ &+ N * (P * Q_1(P)) && \text{Total revenues from per-unit sales to Low Yields} \\ &+ M * (P * Q_2(P)) && \text{Total revenues from per-unit sales to High Yields} \\ &- (N * Q_1) * MC && \text{Total costs from per-unit sales to Low Yields} \\ &- (M * Q_2) * MC && \text{Total costs from per-unit sales to High Yields} \\ &- \text{Fixed Costs} && \text{Fixed costs (if any)} \end{aligned}$$

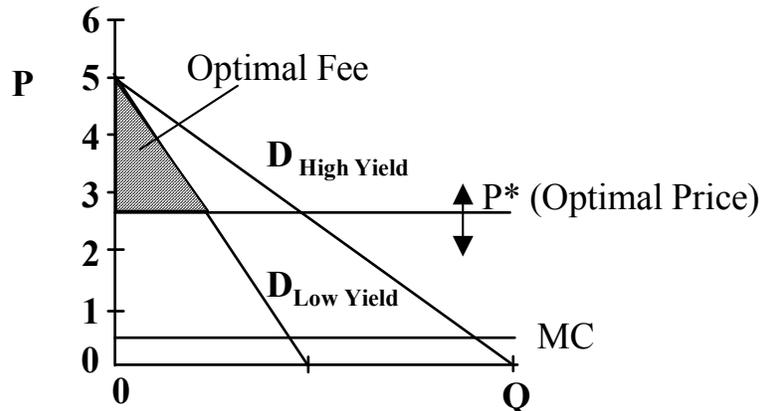
- 3 Third, we need to derive the first derivative of the previous profit function with respect to P: $\Delta \Pi / \Delta P$

- 4 Then, we need to set the first derivative of the profit equation (from step 3) equal to 0 and solve for P. This will give us the optimal level of P that will maximize profits. $\Delta \Pi / \Delta P = 0 \Rightarrow P^* = \text{optimal price level that maximizes profits}$

- 5 Once we have the optimal price level, we can plug this number in $Q_1(P)$ and in $Q_2(P)$ and figure out how many units of output each consumer will buy. Then we can plug P^* in the Tariff equation and we obtain the optimal Tariff Level T^* . We then are only missing the information about profits, that can be calculated as:

$$\begin{aligned} \Pi &= N * T(P) + M * T(P) && \text{Total revenues from Fee (calculated on } Q_1) \\ &+ N * (P * Q_1(P)) && \text{Total revenues from per-unit sales to Low Yields} \\ &+ M * (P * Q_2(P)) && \text{Total revenues from per-unit sales to High Yields} \\ &- (N * Q_1) * MC && \text{Total costs from per-unit sales to Low Yields} \\ &- (M * Q_2) * MC && \text{Total costs from per-unit sales to High Yields} \\ &- \text{Fixed Costs} && \text{Fixed costs (if any)} \end{aligned}$$

Graphically:



6. NUMERIC EXAMPLES

6.1 Example of Perfect Price Discrimination

6.2 Example of Pricing to Observable Market Segments

6.3 Comparison to Price Discriminating vs. Single price for all consumers

6.4 Summary of all price discrimination cases

6.5 Example of Two Part Tariff

6.1 Example of Perfect Price Discrimination

Jack, an ingenious Sloan student, develops a new personalized laser gun as part of the \$50K competition. After graduating, he starts his business. As part of his business plan:

- The lasers operate on user hand print recognition so *resale is not possible* (condition #2)
- Annual market demand curve *faced by the firm* is $5500 - 100P = Q$ (condition #1)
- Fixed costs are \$20,000 per year.
- Variable cost is \$15 per gun.

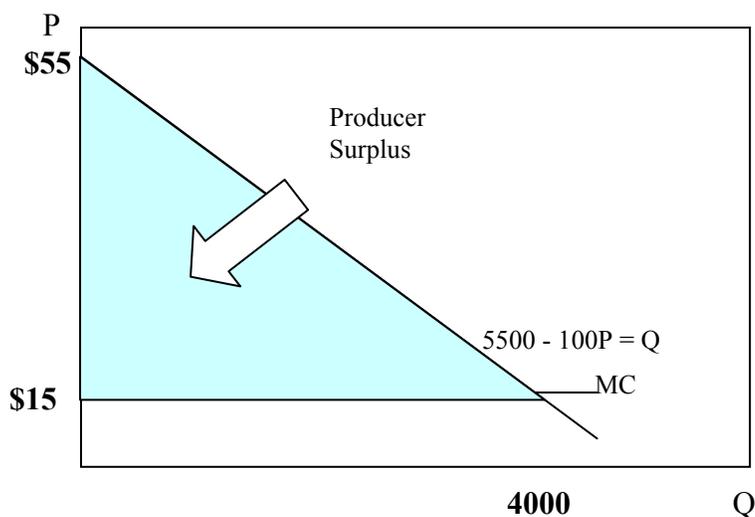
At Sloan, Jack did very well in his organizational/consumer behavior classes and is able to read people very well (condition #3). Thus, *he is able to determine and charge the reservation price to each customer*. How many guns will Jack sell and what will his total profit be?

Solution

Total Cost = Fixed Cost + Variable Cost = \$20,000 + \$15Q

Marginal Cost = $\partial TC / \partial Q = \15

Since Jack can charge the reservation price, he captures the entire consumer surplus as shown below:



He will sell 4000 guns. His annual profits are:

$$\pi = \text{Area of Producer Surplus} - \text{Fixed Costs}$$

$$\pi = .5 * (\$55 - \$15) * (4000) - \$20,000$$

$$\pi = \underline{\underline{\$60,000}}$$

6.2 Example of Pricing to Observable Market Segments

Let's consider the case where there are two customer segments. One group consists of the usual customers mentioned before and the other group is students. However, the students have a different buying pattern and have the following demand curve: $2000 - 50P = Q$. How should Jack price if he is trying to supply *both* customer segments and can easily segment the two types of customers? (note that in this case, he is *not* able to determine each customer's reservation price as before, but he is still able to tell if the customer is a student or not).

Solution

For the usual customers, Jack should do the following:

$$\text{Total Cost} = \text{Fixed Cost} + \text{Variable Cost} = \$20,000 + \$15Q$$

$$\text{Marginal Cost} = \partial \text{TC} / \partial Q = \$15 \text{ (same as before)}$$

$$\text{Demand: } 5500 - 100P = Q$$

Rearrange in terms of P:

$$P = (5500 - Q) / 100$$

$$P = 55 - .01Q$$

$$\text{Total Revenue} = P * Q$$

$$\text{Total Revenue} = 55Q - .01Q^2$$

$$\text{Marginal Revenue} = \partial \text{TR} / \partial Q = 55 - .02Q$$

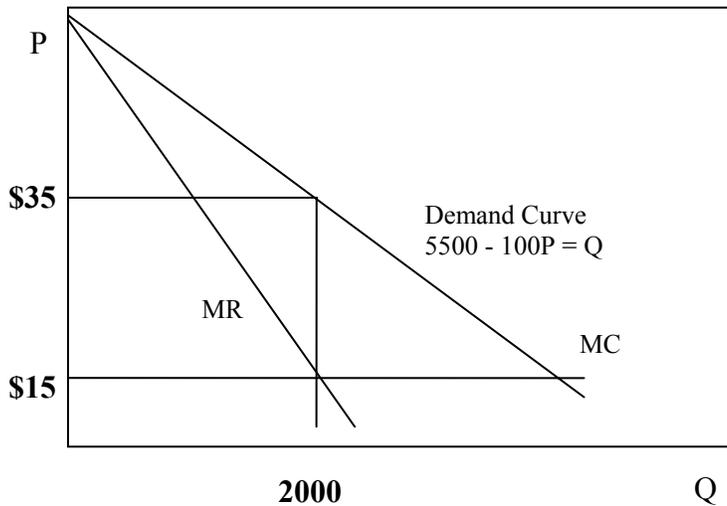
$$\text{MR} = \text{MC}$$

$$55 - .02Q = 15$$

$$Q = 2000 \text{ units}$$

$$\text{Price} = P = 55 - .01Q = 55 - .01*2000$$

$$\text{Price} = \$35$$



For the MIT students, Jack should do the following:

$$\text{Total Cost} = \text{Fixed Cost} + \text{Variable Cost} = \$20,000 + \$15Q$$

$$\text{Marginal Cost} = \partial \text{TC} / \partial Q = \$15 \text{ (same as before)}$$

$$\text{Demand: } 2000 - 50P = Q$$

Rearrange in terms of P:

$$P = (2000 - Q) / 50$$

$$P = 40 - .02Q$$

$$\text{Total Revenue} = P * Q$$

$$\text{Total Revenue} = 40Q - .02Q^2$$

$$\text{Marginal Revenue} = \partial \text{TR} / \partial Q = 40 - .04Q$$

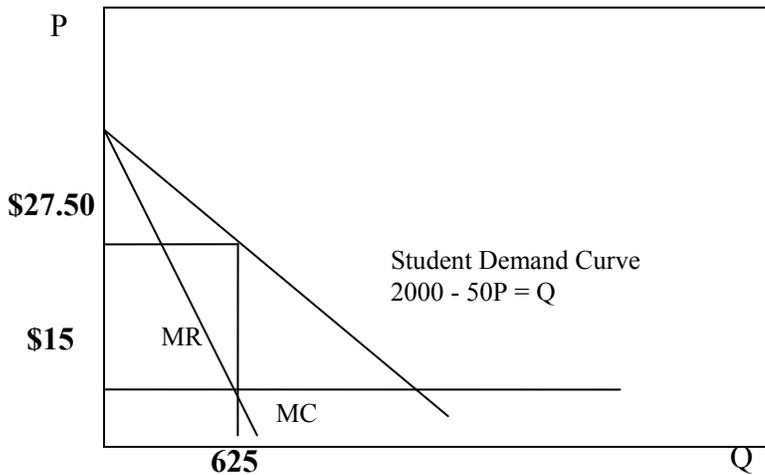
$$\text{MR} = \text{MC}$$

$$40 - .04Q = 15$$

$$Q = 625 \text{ units}$$

$$\text{Price} = P = 40 - .02Q = 40 - .02*625$$

$$\text{Price} = \$27.50$$



His annual profits from the usual customers and students are:

$$\pi = P_{\text{normal}} * Q_{\text{normal}} + P_{\text{student}} * Q_{\text{student}} - \text{Fixed Cost} - \text{Variable Cost}$$

$$\pi = \$35 * 2000 + \$27.50 * 625 - \$20,000 - \$15 * (2000 + 625)$$

$$\pi = \$27,812.50$$

Observations:

- The price charged for students is less than for usual customers. This is as expected since they are more price elastic.
- To charge different prices, Jack would need to be able to distinguish students from usual customer (for example, use student IDs).

6.3 Example: Comparison between price discrimination and single price for all consumers

Now suppose the government gets involved and forces Jack to charge the same price to all customers. What should that price and quantity be?

Solution

Without price discrimination, Jack must charge a single price to all his customers. In this case, he has to add the demand equations together and then determine the total marginal revenue curve.

Normal Demand: $5500 - 100P = Q$

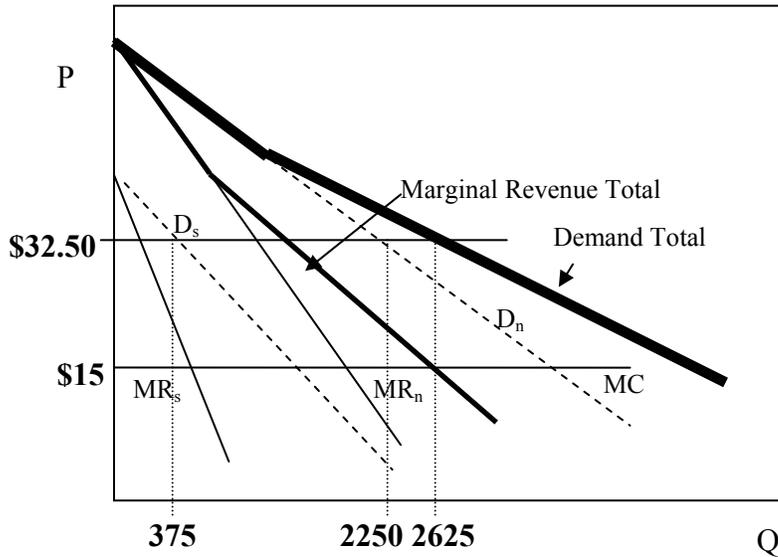
Student Demand: $2000 - 50P = Q$

If $P > \$40$, only look at Normal Demand curve $5500 - 100P = Q$

If $P \leq \$40$, add demand curves. $Q = 5500 - 100P + 2000 - 50P$

Therefore $Q = 7500 - 150P$

In P form: $P = 50 - .0067Q$
 $TR = P*Q = 50Q - .0067 Q^2$
 $MR = \partial TR/\partial Q = 50 - .0133Q$



Set $MR = MC$
 $50 - .0133Q = 15$
 $Q = 2625$
 $P = 50 - .0067*2625 = \$32.50$

$Q_n = 5500 - 100*32.50 = 2250$
 $Q_s = 2000 - 50*32.50 = 375$

His annual profits in the case of a single price scenario is:

$\pi = P*(Q_{normal} + Q_{student}) - \text{Fixed Cost} - \text{Variable Cost}$
 $\pi = \$32.50*2625 - \$20,000 - \$15*2625$
 $\pi = \$25,937.50$

Observations:

Note the following results from this portion of the example:

- Students bought less and normal customers bought more.
- Notice that the new price is between \$35 and \$27.50, the prices we charged when we could discriminate.
- Jack lost \$1,875 in profits by not being able to discriminate.

6.4 Summary of price discrimination cases

The following table offers a summary view of all possible pricing strategies seen so far and the implications on price, quantities and profits.

	Price(s)	Quantity(ies)	Profits
One Demand, Single price (*)	\$35	2,000	\$20,000
Perfect Price Discrimination	All the range from \$55 to \$15	4,000	\$60,000
Consumer Self Selection (<i>not in the example</i>)	P ₁ , P ₂ and P ₃ (see chart page 3)	Q ₁ , (Q ₂ - Q ₁) and (Q ₃ - Q ₂) (see chart page 3)	Less than \$60,000
Pricing to observable Market Segments	\$35 and \$27.50	2,000 and 625	\$27,812.50
Uncle Sam gets involved	\$32.50	2,250 and 375	\$25,937.50

(*) Like pricing to the first kind of customers only in observable market segments (MR=MC).

6.5 Example of Two Part Tariff

6.5.1 You decide to open a bar!!

You decide to open a bar. For any given night you will have fixed cost of \$1,000 plus a variable cost of \$0.50 per drink (drinks are the only thing you sell at the bar.)

$$TC = \$1,000 + 0.5Q$$

$$MC = \$0.50 \quad (Q \text{ is number of drinks, costs in } \$)$$

6.5.2 Only one type of customer - the party animal.

So you look to the market and find a set of party animals (or Sloan students after midterms). There are 500 of these customers in a given night. They *each* have the same demand curve for drinks:

$$Q_{\text{animal}} = 10 - 2P \quad (P \text{ is the price of each drink in } \$)$$

We will start with the simple case where we don't charge a cover charge or entrance fee (no two-part tariff). How should we price the drinks?

$$P = 5 - Q/2$$

$$TR = (5 - Q/2) \times Q = 5Q - Q^2/2$$

$$MR = 5 - Q$$

$$\text{Set } MR = MC$$

$$5 - Q = 0.5$$

$Q = 4.5$ drinks/person per night, so $P = \$2.75$ /drink (from demand curve)

Each of the 500 people consumes 4.5 drinks in a night at a price of \$2.75.

$$\begin{aligned}
 Q_{\text{total}} &= 500 \times 4.5 = 2,250 \text{ drinks} \\
 \text{Profits} &= TR - TC = (P \times Q) - (1,000 + (0.5Q)) \\
 &= 2,250 \times 2.75 - (1,000 + .5 \times 2,250) = \underline{\underline{\$4,062.5 \text{ profit per night}}}
 \end{aligned}$$

6.5.3 Using a two-part tariff with a single kind of consumers

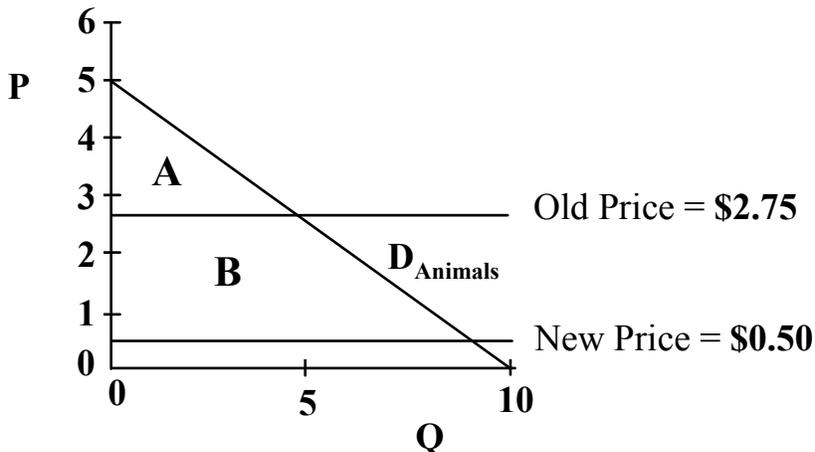
You did very well with this strategy but there is some consumer surplus that is slipping through your hands. So you decide to add a cover charge at the door while setting a new price for each drink. Notice that there will be a trade-off: a high cover charge means fewer entrants and less profits from the drinks, but also more profits from the cover. As it usually occurs when there are trade-offs, the optimum solution is somewhere in between.

How can we find the optimal price for the cover charge and the drinks? Ideally we want to capture the entire consumer surplus. Consumers have the most surplus when Price = MC (the lowest price at which producer will still offer the good). In this case, if you charged \$0.50 per drink, each of the customers would purchase:

$$Q = 10 - 2P = 9 \text{ drinks/night.}$$

The consumer surplus (seen below) would be

$$CS = A + B = 0.5 \times (5 - 0.5) \times (9) = \$20.25/\text{person}$$



If we charge the customer this amount as the cover charge then they will "break even" when they drink their beers at \$0.50 each. So the customer would visit the club but you would get the entire consumer surplus.

What does the Profit equal? As above: Cover Charge = A + B = $0.5 \times (5 - 0.5) \times (9 - 0) = \20.25 (pretty stiff for a cover charge). So now 500 people come to the bar, pay the fee, and have 9 drinks each (and get pretty tipsy, too!)

$$\begin{aligned}
 Q_{\text{total}} &= 4,500 \\
 \Pi &= TR - TC = (500 \times \text{Fee} + 0.5Q) - (1,000 + 0.5Q) \\
 &= 500 \times \text{Fee} - 1,000 = \underline{\underline{\$9,125 \text{ profit per night.}}}
 \end{aligned}$$

6.5.4 Using a two-part tariff with two kinds of consumers

You decide that you could do better! You notice that there are also some Latin folks who like to dance. They are currently not coming because the cover charge is too high. You want to get this customer to come to your bar also.

There are 500 Dancers each with the exact same demand curve for drinks:

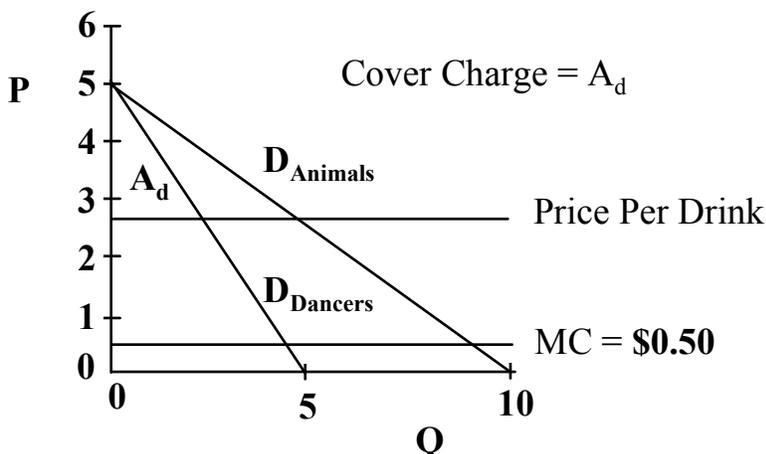
$$\begin{aligned}
 Q_{\text{dancer}} &= 5 - P \\
 P &= 5 - Q_{\text{dancer}}
 \end{aligned}$$

Remember that there are still 500 Party Animals all with an individual demand curve for drinks:

$$\begin{aligned}
 Q_{\text{Animal}} &= 10 - 2P \\
 \text{or } P &= 5 - Q_{\text{animal}}/2
 \end{aligned}$$

We will keep the cost of doing business the same (i.e. we have the same cost equation). We want to maximize profit but we must decide what the cover charge and price per drink should be.

Our first strategy will be to try to get both customers (it is not necessarily the best strategy). Therefore the cover charge can not be larger than the consumer surplus of the customer with the smallest consumer surplus. In this case the Dancers will always have smaller consumer surplus. This can be seen in the diagram below — if the cover charge is larger than A_{dancer} , then the Dancers will not go.



The cover charge is a function of P (as we move the horizontal line, the area of the triangle A_d changes)

$$\text{Cover Charge} = T(P) = 0.5 \times (5 - P) \times (5 - P) = 12.5 - 5P + P^2/2$$

$$\Pi = TR - TC = [1000 \times T(P) + 500 \times P \times (10 - 2P) + 500 \times P \times (5 - P)] - [1000 + 0.5(500 \times (10 - 2P) + 500 \times (5 - P))]$$

$$\Pi = 1000 \times (12.5 - 5P + P^2/2) + 5000P - 1000P^2 + 2500P - 500P^2 - (1000 + 3750 - 750P)$$

$$\Pi = 12500 - 5000P + 500P^2 + 5000P - 1000P^2 + 2500P - 500P^2 - 4750 + 750P$$

$$\Pi = -1000P^2 + 3250P + 7750$$

In order to maximize the profits we take the derivative $d\Pi/dP$ and set it equal to 0, which results in:

$$P = \$1.625 \text{ per drink.}$$

$$\text{Cover charge} = T(P) = \$5.70$$

$$Q_{\text{animal}} = 10 - 2P = 10 - 2(1.625) = 6.75 \text{ drinks per party animal per night.}$$

$$Q_{\text{dancer}} = 5 - P = 5 - 1.625 = 3.375 \text{ drinks per dancer per night.}$$

What is the new profit? From the last expression for Π , we obtain:

$$\Pi = -1000 \times (1.625)^2 + 3,250 \times (1.625) + 7,750 = \underline{\underline{\$ 10,391 \text{ per night}}}$$

We said that this was not necessarily the best strategy. Compare it to the strategy of keeping the price high and only having party animals. This is what happened in the first example and the profit was 9,125 per night. Therefore it is more profitable to lower the cover charge and get the Dancers at your bar also.

Two-Part Tariffs – Summary

	Cover charge	Price/drink	Drinks/person	Profits
No two-part tariff	None	$\$2^{75}$	4.5	\$4,063
One kind of consumer	$\$20^{25}$	$\$0^{50}$	9.0	\$9,125
Two kinds of consumers	$\$5^{70}$	$\$1^{63}$	6.75 (party animals) 3.38 (dancers)	\$10,391