



RECITATION NOTES #1

Market Definition, Elasticities and Surpluses

Friday - September 10, 2004

OUTLINE OF TODAY'S RECITATION

- 1. Market Definition:** How to determine whether two products are in the same market or not and how to use the Market Definition Test
- 2. Elasticities:** Definition of own-price elasticity and cross-price elasticity
- 3. Supply and Demand:** Types of supply and demand curves; how to find the elasticities from a supply/demand curve; how to derive the curves from the elasticities
- 4. Surplus:** Definition of Consumer Surplus and Producer Surplus; how to calculate them
- 5. Numeric Example:** An example to put all the concepts together

1. MARKET DEFINITION

1.1 Definition

1.2 Market Check

1.1. Definition: A Market is a collection of buyers and sellers that, through their actual or potential interactions, determine the price of a product or set of products.

Markets can be defined in several ways, for instance:

- Products - which to include, coffee and tea, or just tea
- Geography – only Boston, or the whole world

1.2 Market Check: A convenient way to “test” whether a product is in the same market as another one is to ask the following question:

“Are there either demand substitutes or supply substitutes to this product?”

Demand Substitutes If the price of good A rises substantially (holding all else constant), will a significant number of buyers of good A instead purchase good B?

Supply Substitutes If the price of good A rises substantially (holding all else constant), will a significant number of producers of good B decide to produce good A instead?

NOTE: In general, demand substitutability is the more important factor in determining whether two products are in the same market.

2. ELASTICITIES

2.1 Definition

2.2 Different kinds

2.3 How to calculate them

2.1 Definition: The Elasticity is a measure of the sensitivity of one variable to a change in another.

Examples: How does the quantity demanded for good A change if the price of good A increases by 1%? (This is the **Price – elasticity of demand for good A**. It is also called own-price elasticity, because it refers to a change in demand driven by a change in its own price.)

How does the quantity demanded for good B change if the price in good A increases by 1%? (This is the **Cross-price elasticity of demand for goods A and B**, as it measures the cross-effects of a change in price on one of the goods on the other good’s demand)

2.2 There are many kinds of Elasticities:

	Price	Income	Cross Price
Demand	<i>Price-Elasticity of Demand, or Own-Price Elasticity of Demand</i>	<i>Income-Elasticity of Demand</i>	<i>Cross Price Elasticity of Demand</i>
Supply	<i>Price-Elasticity of Supply, or Own-Price Elasticity of Supply</i>		<i>Cross Price Elasticity of Supply</i>

2.3 How to calculate Elasticities

Here we will show how to calculate the different kinds of elasticities:

2.3.1 Price elasticity of demand:

Recall: It measures the percentage change in quantity demanded resulting from a 1% change in the price P

It is calculated with the following formula

$$E_p = \left(\frac{\Delta Q}{Q} \right) \bigg/ \left(\frac{\Delta P}{P} \right) = \frac{\Delta Q * P}{\Delta P * Q}$$

Example:

$$P_0 = \$5000 \quad Q_0 = 1000$$

$$P_1 = \$6000 \quad Q_1 = 900$$

$$\text{At } Q=1000, E_p = \frac{(900 - 1000)}{(6000 - 5000)} * \frac{5000}{1000} = -0.5$$

2.3.2 Cross Price elasticity of demand:

Recall: It measures the percentage change in quantity demanded Q resulting from a 1% change in the price P' of another good

It is calculated with the following formula:

$$E_{p'} = \left(\frac{\Delta Q}{Q} \right) \bigg/ \left(\frac{\Delta P'}{P'} \right) = \frac{\Delta Q * P'}{\Delta P' * Q}$$

2.3.3 Income elasticity of demand:

Recall: It measures the percentage change in quantity demanded Q resulting from a 1% change in the income I

It is calculated with the following formula:

$$E_I = \left(\frac{\Delta Q}{Q} \right) \bigg/ \left(\frac{\Delta I}{I} \right) = \frac{\Delta Q * I}{\Delta I * Q}$$

NOTE: For a given quantity, **demand** is classified as **elastic** or **inelastic** based on the following criteria:

- $|E_p| < 1$, demand is inelastic
- $|E_p| > 1$, demand is elastic

2.3.4 Price elasticity of supply:

Recall: It measures the percentage change in quantity supplied Q resulting from a 1% change in the price P

It is calculated with the following formula:

$$E_p = \left(\frac{\Delta Q}{Q} \right) \bigg/ \left(\frac{\Delta P}{P} \right) = \frac{\Delta Q * P}{\Delta P * Q}$$

Example

If Price elasticity of supply, $E_p=2$, then a price increase of 1% results in an increase of quantity supplied of $2 * 1\% = 2\%$

3. SUPPLY AND DEMAND

3.1 Definition

3.2 Supply curve characteristics

3.3 Types of demand curves and characteristics

3.4 Short-run versus Long-run elasticity of demand

3.5 Deriving supply / demand curves from the elasticities and a point

3.1 Definition: **Supply curves** represent the output firms will supply for given prices. **Demand curves** represent the quantity consumers will purchase for given prices. Supply curves are usually upward sloping (the higher the price, the more products a firm will produce) while demand curve are usually downward sloping (the higher the price, the fewer units are demanded).

Equilibrium. The equilibrium represents the price at which the quantity demanded equals the quantity supplied.

3.2 Supply curve characteristics

We usually focus on Linear Supply Curves: supply curves that can be approximated by a linear function of Q and P:

- **Supply Function:** $Q_s = c + d * P$

In **linear** supply functions, the price elasticity is given by the following formula:

- Price elasticity of Supply: $E_p = d * (P/Q)$

NOTE: This means that in different points of the curve, the elasticity of supply will be different.

Example

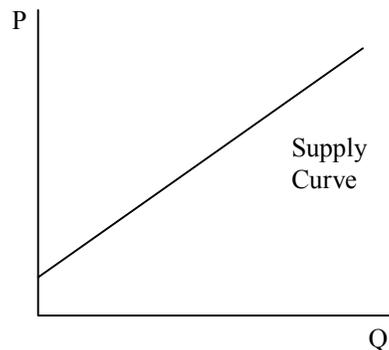
Let's assume the supply function is represented by $Q_s = 8 + 4 * P$. What is the price elasticity of supply at point $P = 4$; $Q = 24$?

- $E = d * (P/Q) = 4 * (4/24) = 16/24 = 2/3 = 0.666$

What about at point $P = 2$; $Q = 16$?

- $E = d * (P/Q) = 4 * (2/16) = 8/16 = 0.50$

At point $P=2$, $Q=16$ we have a lower price elasticity of supply than at point $P=4$; $Q=24$.



3.3 Types of demand curves and characteristics

We have two main types of demand curves:

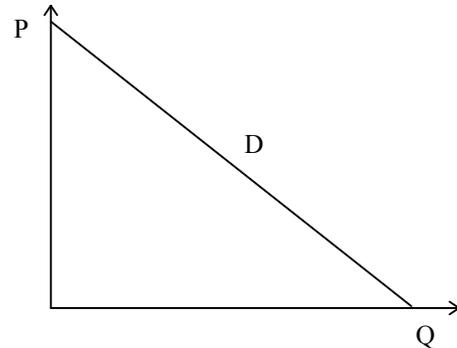
- Linear demand curves,
- Iso-elastic demand curves.

3.3.1 Linear Demand Curve

- **Demand Function:** $Q = a - b \cdot P$

Similarly to the case of linear supply curves, in **linear** demand functions, the price elasticity is given by the following formula:

- Price elasticity of demand $E = -b \cdot (P/Q)$



NOTE: Again, similarly to linear supply functions, elasticity on linear demand curves is not constant along the curve. Also, note that for linear demand and supply curves, Elasticity $E = (\Delta Q/Q)/(\Delta P/P)$ is **NOT** equal to the slope ($= \Delta P/\Delta Q = -1/b$ for demand or $= \Delta P/\Delta Q = 1/d$ for supply).

3.3.2 Iso-Elastic Demand Curve

- **Demand Function:** $\ln Q = \ln a + n \ln P + m \ln I$
Or $Q = a \cdot P^n \cdot I^m$

Iso-elastic Demand Curves have two particular properties:

1. Along their curves, elasticities are constant for any given combination of Price and Quantities. (The prefix “Iso” in Greek means “Equal”)
2. The elasticities can be read directly from the demand curve as they coincide with the coefficients (or the powers) of the function:

$$\text{Price Elasticity: } E_p = n$$

$$\text{Income Elasticity } E_I = m$$

3.4 Short-run vs. Long-run elasticity and Durable vs. Non-Durable Goods

Non-durable goods: **Short run demand is usually less elastic than long run demand**

Why? In the short run, there are often fewer substitutes to choose from, and consumers may find it difficult to adjust their behavior. For example, if the price of gasoline suddenly doubles, because we still have the same car and we still need to go to work, we will consume in the short run the same amount of gasoline (despite the change in price, our quantity demanded won't change significantly). In the long-run, consumers can turn to more fuel efficient cars or change their driving behavior in ways that are not possible in the short run.

Durable goods: **Long run demand is usually less elastic than short run demand**

Why? While we can hold durable goods in the short run if market conditions are not ideal, we cannot hold these goods forever. For example, if we want to change our car, but prices today suddenly jump up by 100%, we can decide not to change in the short run and to hold on to our old car. In the long run though, our car won't be able to run any more and – despite the price change – we will be forced to buy a new one.

3.5 Deriving supply /demand curves from elasticities and a point

If we know an equilibrium point and the supply- and demand-elasticities at that point, we can easily derive the linear supply and demand curves from this information.

Example

Let's assume we know that the point $P=2$ and $Q=8$ is an equilibrium. Furthermore, let's assume that Price elasticity of supply at that point is $E_P = 0.5$ and that Price elasticity of demand in the same point is $E_P = -1.0$. What are the Supply - and Demand Curve equations?

Supply Curve: $Q_S = c + d P$

From the definition of Elasticity we know that $E_S = d*(P/Q)$, therefore we can rearrange as:

$$d = E_S / (P/Q) = E_S * (Q/P) = .5 * 8/2 = 2$$

And from $Q_S = c + d P$ again rearranging we obtain:

$$c = Q_S - d P = 8 - 2 * 2 = 4$$

Therefore, the **Supply curve is: $Q_S = 4 + 2 P$** . Similarly:

Demand Curve: $Q_D = a - b P$

From the definition of Elasticity we know that $|E_D| = b^*(P/Q)$, therefore we can rearrange as:

$$b = - E_D / (P/Q) = - E_D * (Q/P) = -1 * 8/2 = - 4$$

And from $Q_D = a - b P$ again rearranging we obtain:

$$a = Q + b P = 8 + 4 * 2 = 16$$

Therefore, the **Demand curve is: $Q_D = 16 - 4 P$**

4. SURPLUS

4.1 Definition

4.2 How to calculate it

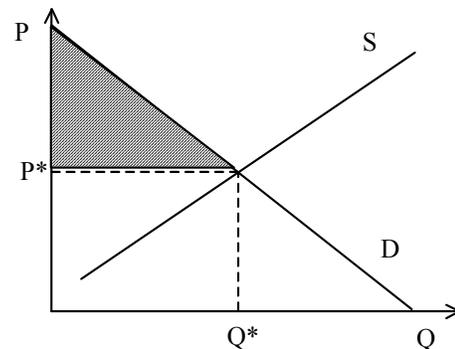
4.1 Definition: **Consumer's surplus:** the amount a buyer is willing to pay for a good minus the amount the buyer actually pays for the product. (Intuitively: "What is left in the consumers pockets after having bought the product").

Producer's surplus: The amount a producer is paid for selling a product minus the amount the producer is willing to accept to produce the product, which is measure of its costs. (in other words, the profits of the producer, ignoring fixed costs)

4.2 How to calculate surplus

4.2.1 Consumer's surplus

Consumer's surplus is represented by the amount left in the hands of the consumer. If a consumer has a demand curve D like the one shown in the chart and the Equilibrium is reached for a price P^* , then he will buy Q^* and will be left with a surplus equal to the shaded triangle.



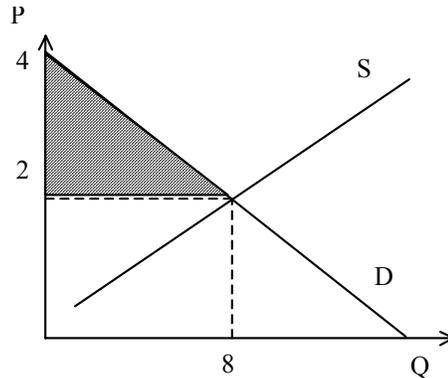
Example

Let's assume again, that the demand curve is: $Q_D = 16 - 4P$ and Equilibrium is reached for $P^* = 2$ and $Q^* = 8$.

In order to draw the chart we need to rearrange the equation in terms of P: $P = 4 - \frac{1}{4}Q$.

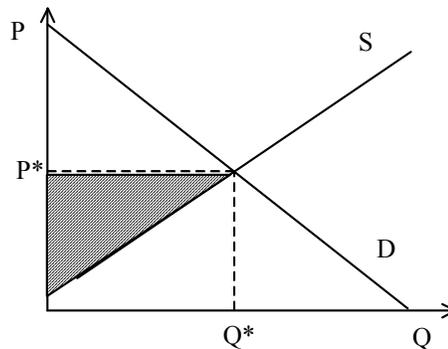
The Consumer's surplus – the shaded area shown in the chart – will therefore be given by:

$$CS = 0.5 * (4 - 2) * 8 = 8$$



4.2.2 Producer's surplus

Producer's surplus is represented by the amount left in the hands of the producer after he has sold his products. If a producer has a supply curve S like the one shown in the chart and the Equilibrium is reached for a price P^* , then he will sell Q^* and will be left with a surplus equal to the shaded triangle.



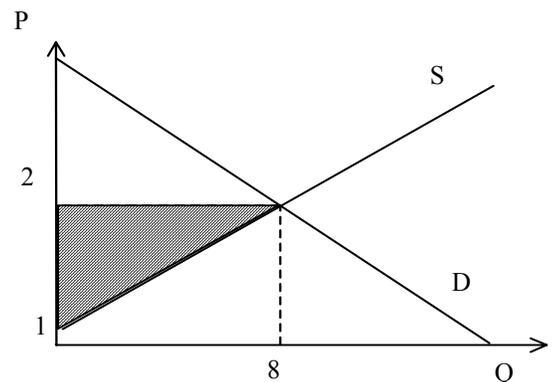
Example

Let's assume that the supply curve is: $Q_S = -8 + 8P$ and Equilibrium is reached for $P^* = 2$ and $Q^* = 8$.

In order to draw the chart we need to rearrange the equation in terms of P: $P = 1 + \frac{1}{8}Q$.

The Producer surplus – the shaded area shown in the chart – will therefore be given by:

$$PS = 0.5 * (2 - 1) * 8 = 4$$



5. EXAMPLE: PUTTING IT ALL TOGETHER

Market for Palm Pilots

5.1 Calculate Supply and Demand Functions

5.1.1 Available information

Suppose that the market equilibrium for PDAs is given by:

$$P_0 = \$250/\text{unit} \quad Q_d, Q_s = 100 \text{ Mln units /year}$$

Now suppose that one of the suppliers – Palm – offers the largest portion of PDA in the market and that the rest of the market follows a competitive supply dynamic. Also assume that Palm offers its supply at any given price level.

$$Q_s(\text{Palm}) = 70 \text{ mln units/year}, Q_s(\text{Competitive}) = 30 \text{ mln units/year}$$

A market analysis consultant comes up with the following information about the price elasticities for PDAs:

Price Elasticities	Short Run	Long Run
World Demand	-1.0	-0.2
Competitive Supply	0.4	0.1

Calculate the demand and supply curves:

5.1.2 Short run demand curve:

$$\text{Linear Demand Function: } Q = a - b \cdot P \quad \text{Eqn. (I)}$$

$$E_D = (\Delta Q/Q) / (\Delta P/P) = (\Delta Q/\Delta P) \cdot (P/Q) \quad \text{Eqn. (II)}$$

$$\text{Substitute } \Delta Q/\Delta P = -b \text{ into Eqn (II)} \Rightarrow E_D = -b \cdot (P/Q) \quad \text{Eqn. (III)}$$

$$\text{Substitute } E_D = -1.0, Q_D = 100, P_0 = 250 \text{ into Eqn. (III)}$$

$$\Rightarrow -1.0 = -b \cdot (250/100)$$

$$\Rightarrow \mathbf{b = 0.4}$$

$$\text{Substitute } Q_D = 100, P_0 = 250, b = 0.4$$

$$\Rightarrow \mathbf{a = 100 + 0.4 \cdot 250 = 200}$$

Substitute the values for a and b into Eqn. (I) for the short-run demand function:

$$\text{Short run demand function: } \quad \mathbf{Q_D = 200 - 0.4P} \quad (Q \text{ in mln PDAs/yr, } P \text{ in \$/unit})$$

5.1.3 Short run supply curve

By a similar approach, you can derive the competitive PDA supply:

$$\text{Linear Supply Function: } Q = c + d \cdot P \quad \text{Eqn. (I)}$$

$$E_S = (\Delta Q/Q) / (\Delta P/P) = (\Delta Q/\Delta P) \cdot (P/Q) \quad \text{Eqn. (II)}$$

$$\text{Substitute } \Delta Q/\Delta P = d \text{ into Eqn (II)} \Rightarrow E_S = d \cdot (P/Q) \quad \text{Eqn. (III)}$$

Substitute $E_S = 0.4$, $Q_S = 30$, $P_0 = 250$ into Eqn. (III)

$$\Rightarrow 0.4 = d \cdot (250/30)$$

$$\Rightarrow \mathbf{d = 0.048}$$

Substitute $Q_S = 30$, $P_0 = 250$, $d = 0.048$

$$\Rightarrow \mathbf{c = 30 - 0.048 \cdot 250 = 18}$$

Substitute the values for c and d into Eqn. (I) for the short-run supply function:

Competitive Short run supply function: $\mathbf{Q_S = 18 + 0.048P}$ (Q in mln PDAs/yr, P in \$/unit)

Total PDA supply is given by the sum of the competitive and Palm's supply:

$$\mathbf{Q_S(\text{Total}) = Q_S(\text{Palm}) + Q_S(\text{competitive}) = 70 + 18 + 0.048P}$$

Total Short run supply function: $\mathbf{Q_S = 88 + 0.048P}$ (Q in mln PDAs/yr, P in \$/unit)

NOTE: To **check your answers**, you can plug the equilibrium Price (\$250/unit) into the demand and supply curves and make sure that the result is the equilibrium quantity (\$100 mln units/yr).

5.1.4 Long run supply and demand curves

Similarly, you can derive the long-run demand and supply curves:

Long run demand function: $\mathbf{Q_D = 120 - 0.08P}$ (Q in mln PDAs/yr, P in \$/units)

Long run competitive supply: $Q_S(\text{competitive}) = 27 + 0.833 P$

Long run supply function: $\mathbf{Q_S = 97 + 0.012P}$ (Q in mln PDAs/yr, P in \$/units)

5.2 What happens if Palm cuts production by 10 mln units/year?

Short Run

1. Demand remains the same: $\mathbf{Q_D = 200 - 0.4P}$
2. Subtract 10 mln PDA's/year from supply: $Q_S(\text{Total}) = \mathbf{Q_S = 78 + 0.048P}$
3. Solve for P by setting $Q_d = Q_S(\text{Total})$

$$200 - 0.4P = 78 + 0.048P$$

$$0.448P = 122$$

$$\Rightarrow P = \underline{\$272.32/\text{PDA's}}$$

This is \$272/unit is higher than before.

5.3 Will this price increase hold in the long run?

Long Run:

1. Use Long run demand: $Q_d = 120 - 0.08P$
2. Subtract 10 mln PDAs/year from long run supply: $Q_s(\text{Total}) = 87 + 0.012P$
3. Set $Q_d = Q_s(\text{Total})$

$$120 - 0.08P = 87 + 0.012P$$

$$0.092P = 33$$

$$\Rightarrow P = \underline{\$358.70/\text{unit}}$$

This long run equilibrium price is even higher than the short run equilibrium price. This makes intuitive sense given that the long run price elasticity of demand is less elastic than in the short run. ($0.2 < 1.0$)