



RECITATION NOTES #8

The Beer Game, Collusion and Cartels.

Friday - November 12, 2004

OUTLINE OF TODAY'S RECITATION

1. **The Beer Game:** an in-class game to understand game theory in practice
2. **Collusion and cartels:** what is collusion, how it works, and what is a cartel.
3. **Numeric Example:** entry deterrence and cartel examples

1. THE BEER GAME

1.1 The Bidding

1.2 The Negotiation

1.1 Stage 1: Bidding

The class bids for a POSITION to play the game. People can bid in increments of \$0.25. When bidding closes, each winner pays the next highest bid. Each class member writes his or her name and sealed bid on a piece of paper and hands it into the TA.

E.g.: Winning bids of

Position	Bid	Has to pay
1	9.00	8.50
2	8.50	7.00
3	7.00	6.25
4	6.25	6.00
5	6.00	5.25
6	5.25	4.50
7	4.50	Doesn't get to play

Stage 2: Negotiating the split

There are six positions in the game. The person with the highest bid gets to choose his/her position first. Then the one with the second, third and so on.

Player 1 gets to propose a way to split the 6 beers. If a majority (4) agrees, then the game ends there. Otherwise, player 1 is eliminated and player 2 proposes a split. If 3 accept, then the game ends, else player 2 leaves and so on. The majority MUST be greater than half the number of players left.

2. COLLUSION AND CARTELS

2.1 Definition

2.2 Conditions for a cartel to work

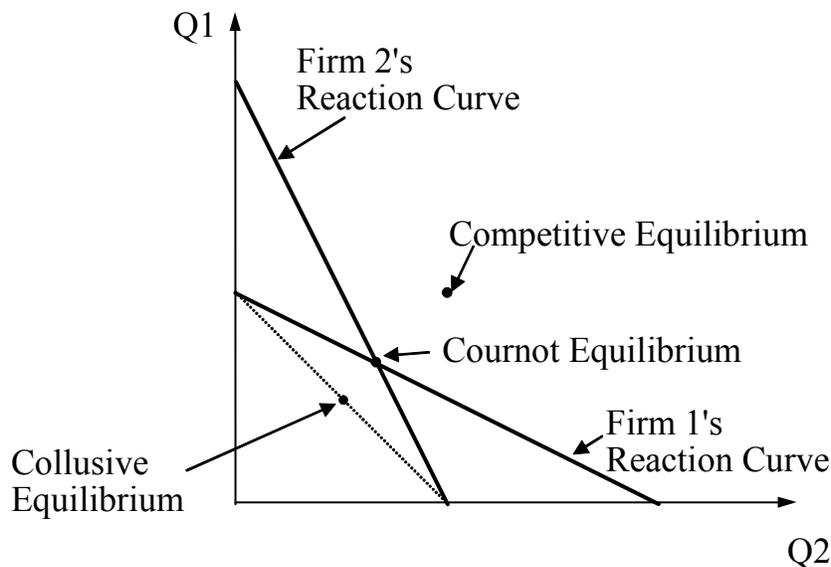
2.3 Optimizing profits in a cartel

2.1 Definition

Two firms collude when they agree to behave in a way that maximizes their profits as a whole and then split the returns. If two firms formally agree upon a certain level of output or a certain price of the goods in order to maximize profits as a whole, then they are forming a cartel.

This agreement, though, presents one very big downside: it is not easily sustainable since both players have a large incentive to cheat and not respect the collusive pact.

We easily can explain this graphically. Recalling the Cournot equilibrium in which two firms maximized their profits given the level of output of the opponent firm, we can represent the collusive equilibrium as follows:



For any given quantity produced by Firm2 Q_2 , the optimal level that Firm2 should produce is higher than the one on the dotted line representing the collusive equilibrium. Therefore, both companies will have an incentive to move to the quantity indicated by their reaction curves instead of what indicated by their collusive agreement.

2.2 Conditions for a cartel to work

In order for a cartel to work, the following three elements must be present:

1. **Agreement:** A clear agreement on the levels of output or price has to be formalized between players.
2. **Monitoring:** A system for precise monitoring of the effective quantities produced (or the effective prices applied) by each participant.
3. **Enforcement:** A credible system for punishing violations of the cartel agreement must be created. The punishment must be a large enough threat for the parties not to cheat.

2.3 Optimizing profits in a cartel

In a cartel, firms must behave as if they were a single (large) monopolist. Therefore, they have to calculate their profit function as a whole and maximize it (by setting the first derivative of the profit function equal to zero). This will provide the optimal total quantity to produce and the total profit of the cartel.

3. NUMERIC EXAMPLE

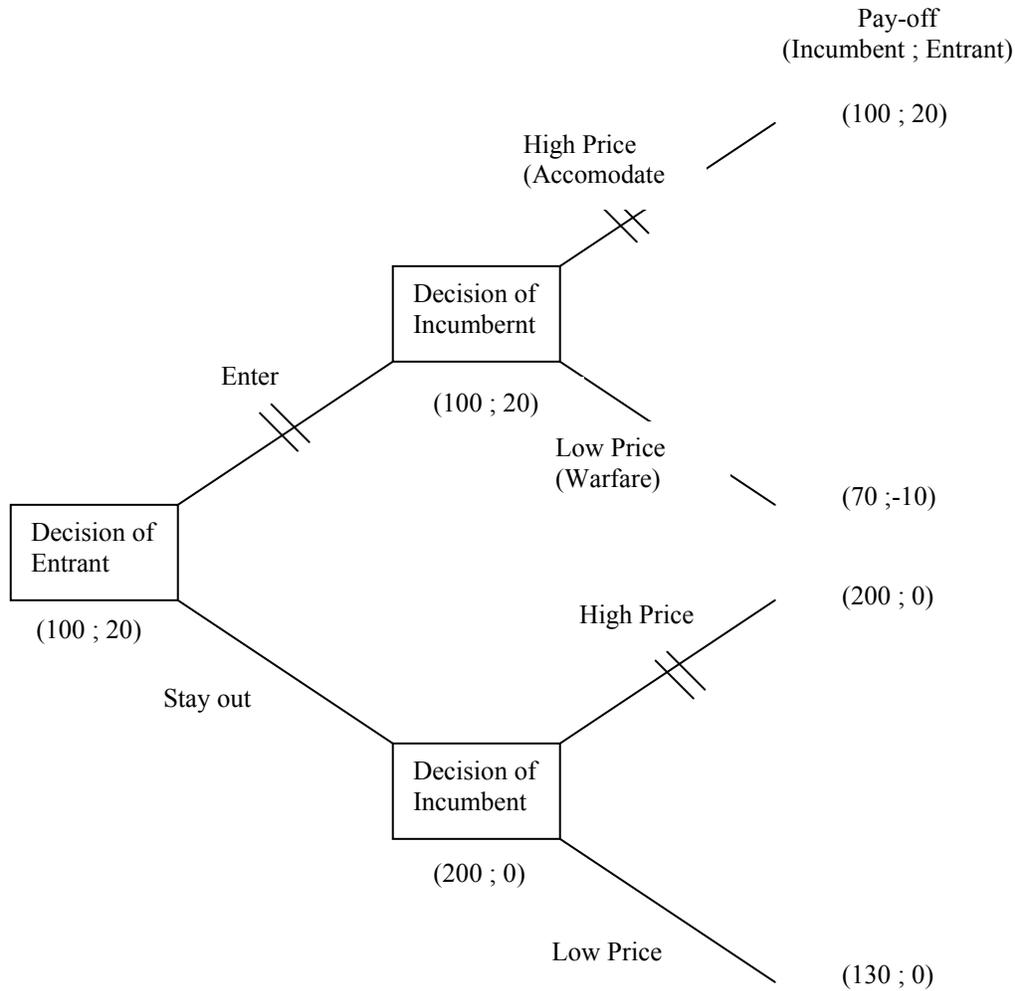
3.1 Example of Entry Deterrence Game

3.2 Example of a Cartel

3.1 Example of Entry Deterrence

This problem is explained in section 13.7 of P&R, though we will discuss the problem using a tree instead of a matrix. *Firms can create Barriers to Entry through patents, licenses, and economies of scale. They can also deter entry by making the payoffs of entry unprofitable for new firms.*

Suppose you are an incumbent (earning profits of 200) and you know that there is an entrant contemplating your market (sunk entry cost of 80). You can either keep your prices high and accommodate the entrant (reducing your earnings by 100) or increase production capacity, lower prices and compete when the entrant enters the market (lower prices will increase revenues by 20, but the additional capacity will cost 50). Either case, you prefer to remain as the sole player, charge a high price and earn nice monopoly profits. Let's take a look at game tree representation:



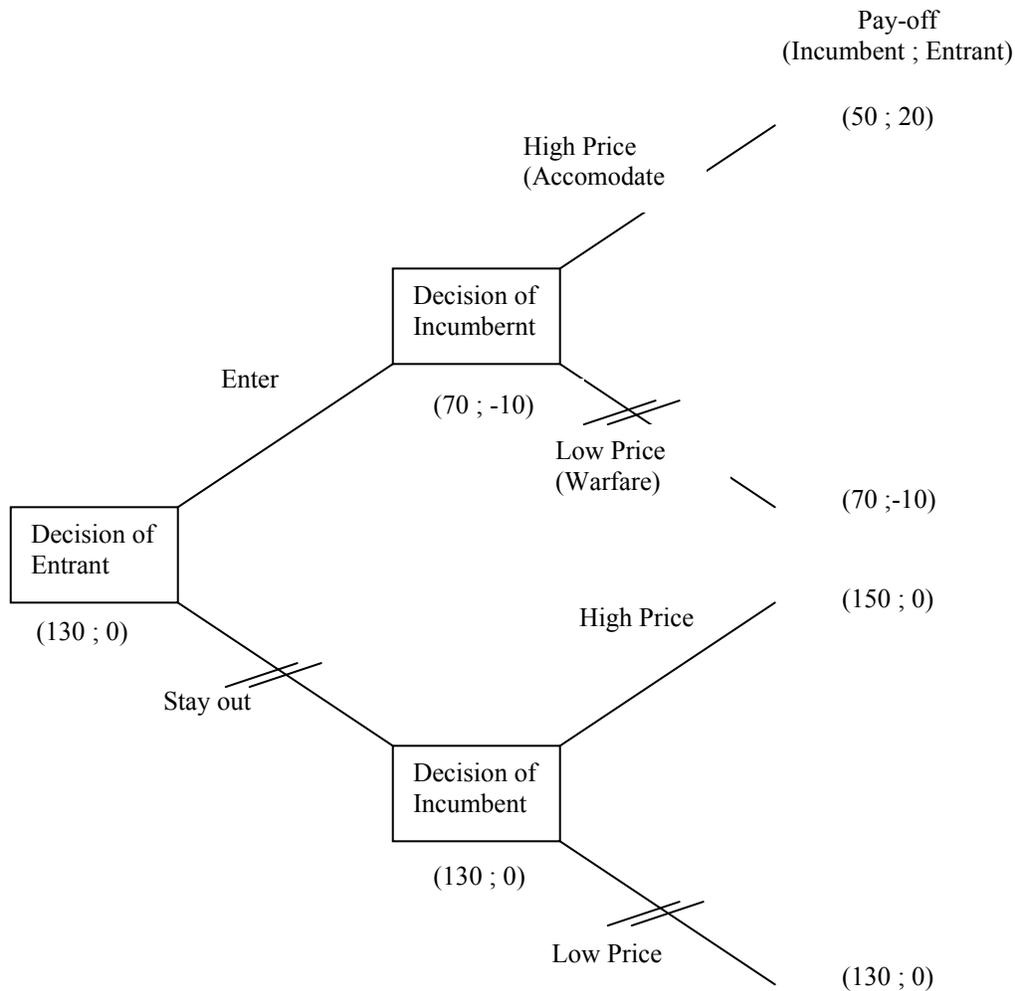
- a. *Does either player have a dominant strategy?*
- Incumbent: it will always earn higher profits by charging a high price.
 - Entrant: its choice depends on the Incumbent's strategy (ie, no dominant strategy).
- b. *Is there a Nash Equilibrium?*

We know that the Incumbent will choose to have higher prices. The profit maximizing choice for the new player would be to enter the market. This is a Nash Equilibrium because neither player can move to a better position.

c. *As the incumbent, do you like the results of this game?*

Well, you should be missing the nice profits from having high prices and entry deterrence. Can you credibly threaten to lower prices? How can you do this?

- Your challenge is to change the payoff matrix in a way that makes the threat of price competition credible.
- By making an investment in capacity expansion before the competitor's entry, you are making an irrevocable commitment that will engage in price warfare (since you have made already the costly investment in capacity; this is now a sunk cost). The commitment to expand capacity needs to be made before the pricing decision. Is there a Nash Equilibrium? How have dominant strategies changed?



d. *As the incumbent, how can you deter entry without making the costly move of expanding production capacity?*

- Establish reputation for punishing entry by fighting--short term losses build reputation leading to less of a need to accommodate entry in the future. This essentially means re-evaluating the payoffs by including future profits. This makes future announcements that you will fight more credible.
- Cut price now -- make first move. This is discussed in P&R as limit pricing.

3.2 Example of a Cartel

Let's use again the example of the T-shirt seen in the last recitation. As you recall, you are the CEO of a firm that is producing T-shirts for the Latin American C-Function (the year's "Best C-Function" and a Best of Boston Hall-of-Famer according to Boston Magazine). Assume you are operating as a duopolist. Both you and your competitor must decide now how many T-shirts to produce. (There is some lead-time involved in producing the T-shirts.) Once you have decided how many shirts to produce, you cannot revise your decision and you cannot produce a second set of shirts (i.e. you only play the "game" once). Assume that both yours and your competitor's T-shirts are undifferentiated products.

There is a market study (accessible to you and your competitor) that analyzes the demand for these T-shirts. The market demand is given by the following equation:

$$Q=100-P$$

You and your competitor both have constant marginal cost equal to

$$MC_1=MC_2=10$$

How many T-shirts should you produce, assuming the two firms will collude? How much should be produced assuming that you have found a way to maximize industry profits? Let's assume that you have agreed to split profits evenly with your competitor.

The total quantity is given by summing the production of both firms:

$$Q = Q_1 + Q_2 \quad \text{Eqn. (I)}$$

Knowing the market demand, you can calculate the industry profits, maximizing output by setting MC equal to MR.

$$\text{Market demand } Q = 100 - P \longrightarrow P = 100 - Q \quad \text{Eqn. (II)}$$

$$\text{And } TR = 100 Q - Q^2$$

$$\Rightarrow MR = dTR / dQ = d(P * Q) / dQ = 100 - 2 * Q$$

$$MR = MC$$

$$\Rightarrow 100 - 2 * Q = 10$$

$$\Rightarrow Q = 45$$

Since you agreed to split profits evenly, each firm will produce $Q_1 = Q_2 = 22.5$. At this quantity, the price and each firm's profit is equal to:

$$P = 100 - 45 = 55$$

$$\pi_1 = TR_1 - TC_1 = P \cdot Q_1 - 10 \cdot Q_1 = 55 \cdot 22.5 - 10 \cdot 22.5 = 1012.5$$

Similarly, $\pi_2 = 1012.5$

(Note: Every combination of Q_1 and Q_2 that sums up to 45 will maximize industry profits. However, at any quantity other than 22.5, the profits are no longer split evenly between the two firms.)

In the collusive outcome, there is a large incentive for the players to cheat and gain higher profits by cheating and moving away from the agreed levels of quantity.