Overview:Time and Uncertainty

- Intertemporal Prices and Present Value
- Uncertainty
- Irreversible Investments and Option Value

Economics of Time: Some Issues

- Cash now versus cash payments in the future?
- Future payments are uncertain?
- When should we undertake a new project – now, later or never?
- How do we manage resources over time?
  - When do we end a profitable project?
  - How do we use up a non-renewable resource?

Intertemporal Prices

- Interest rate r,   Today is t = 0:

  \$1 invested today becomes \$(1 + r) at t = 1

  \$1 invested today becomes $\$(1 + r)^2$ at t = 2, etc.

- Today's price of ($1 at t = 1) is 1/(1+r)

  (I.e. $ 1/(1+r) invested today becomes $1 at t = 1)

- Today's price of ($1 at t = 2) is $1/(1+r)^2$, etc.

Present Value

- Present Value of a stream of cash flows is the value in today's prices

$$PV = C_0 + C_1/(1+r) + C_2/(1+r)^2 + \ldots + C_T/(1+r)^T$$

where C_t is the (positive or negative) cash flow at time t

- PV Criterion: Invest in projects with PV > 0

- r is "discount rate," PV often computed for many values

Example

Consider two projects, A and B

						Present Value	
	t = 0	1	2	3		r = 1%	10%
Project A	-200	50	50	120		15	-23
Project B	100	50	50	-220		-15	21
			Difference B - A			-30	45

(Timing of payments matters, with discount rate very important)

Choice under Uncertainty

- Another aspect of future cash flows is uncertainty. This is modeled via random variables with a distribution.
- How do you react to uncertainty?
  - Cover yourself; avoid big losses at all costs
  - Make decisions using average (mean) values, ignoring the randomness.
  - Take big risks, relishing in the thrill of the unknown ("*the wonder of it all*")

Risk Aversion

- Suppose you are offered a job with a financial firm, and there are two alternative compensation packages.

  A. $ 100,000 Salary
     $ 100,000 Bonus

  You expect to receive the bonus with probability .5.

  B. $ W Salary only, where W > 100,000.

- What is the smallest value of W that would cause you to take B over A?

Risk Aversion (continued)

- If your answer is

W = 150,000 = E(package A), You are *risk neutral*

W < 150,000,   You are *risk averse*.

W > 150,000,   You are *risk loving*`

- Risk Premium: what you would pay to avoid facing the risk, e.g. W = 130,000 gives risk premium of

20,000 = E(package A) - W.

Figure 5.3 Risk Aversion

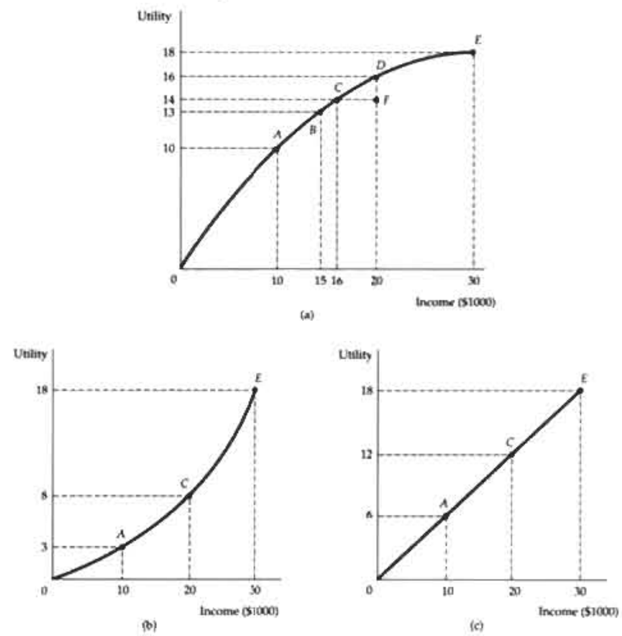
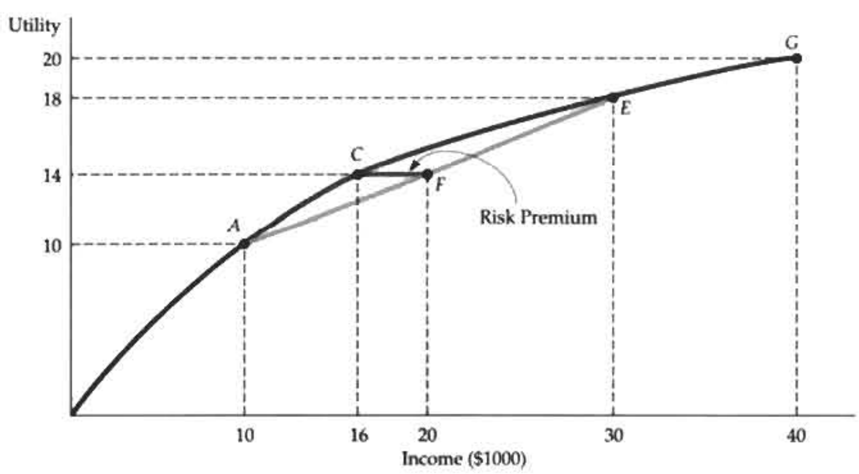


Figure 5.4 Risk Premium



Production Technology Choice

- Choice of a technology commits a firm to a production process
  - Risks arise from uncertainty in input prices
  - Risks arise from uncertainty in quantity or output prices
- Consider choosing a 'high FC + low MC' technology over a 'low FC + high MC' technology
  - This is a bet on high quantity or high output prices, enough to cover the high FC.
  - If substantial chance of low quantity or low prices, low FC choice is safer.

Example: Production Technology

- A risk neutral firm must choose between two available technologies
  - Technology 1: FC = 400 and MC = 9 (low FC + high MC)
  - Technology 2: FC = 4,000 and MC = 4 (high FC + low MC)
- Technology installed at cost FC today (year 0) and production occurs in year 1, with r = .1.
- In year 1, quantity is either 200 with probability p and 1000 with probability 1 – p
  - We consider p = .1, .2 and .5
- Price P = 12

Example: Production Technology (1)

- We must compute PV for each technology in each possible situation.
- For instance, with Technology 1
  - Q = 200: Variable profits: (P – MC)*200 = (12-9)*200 = 600
    - Present value with r = .1

      - 400 + 600 / (1+ .1) = 145
  - Q = 1000: Variable profits:(P – MC)*1000 = (12-9)*1000 = 3000
    - Present value with r = .1

      - 400 + 3000 / (1+ .1) = 2,327
- Technology 1: Expected Present Value at p = .2 and r = .1

  EPV = .2 * 145 + .8 * 2,327 = 1,891

Example: Production Technology (2)

- Expected Present Values

	Probability p							
	of Low Quantity		0.1		0.2		0.5	
Discount Rate r								
	10%	Tech 1	2109		1891		1236	*
		Tech 2	2691	*	2109	*	364	
	25%	Tech 1	1808		1616	*	1040	*
		Tech 2	1888	*	1396		-160	

* denotes preferred choice

Irreversibility and Option Value

- Many investment decisions are irreversible
  - Once committed, costs are (at least partially) sunk
- With uncertainty, there is a value to waiting
  - There is an ‘option’ value to flexibility – postponing decision while uncertainty resolves.
- Consider pricing with
  - Season tickets
  - Rent-to-buy arrangements

Example: Irreversibility

- Two possible technologies, B and V; it is uncertain which one will become the standard
- If you develop the right technology, then profits are 100. If not, your profits are 40 (since you have to license from someone else).
- Your market research suggests that there is a 80% probability that V will be the standard.
- How much do you want to pay to keep the B option alive until uncertainty resolves?

Example: Irreversibility (2)

- If you research only one technology, then you should research V, and your expected profits are:

$$\pi = 0.8 * 100 + 0.2 * 40 = 88$$

- If you research both, then expected profits are:

$$\pi = 0.8 * 100 + 0.2 * 100 = 100$$

- Value of keeping both options open is 12. This is what you are willing to pay.

Issues for Discussion

1. ("When to cut down the tree?" problem.) Suppose I have a process that is increasing in value, when do I halt it?

2. ("When to sell the oil?" problem.) Suppose we have a non-renewable resource, how do we best use it up?

When to cut down a tree?

- We assume that process initially increases rapidly in value and then slows down.

- Essential Logic: At any moment, you can halt the process and invest the value at interest r.

Optimal to keep the process going when it's value is growing at a rate greater than r, and halt it when the growth rate drops below r.

When to sell the oil?

- Consider two time periods: t = 0 and t = 1, fixed amount of oil to sell, price taker.
- Essential Logic: Sell when you get the highest profit for each unit. If you sell in both periods, marginal unit must have same profit PV, namely

$$P_0 – MC_0 = (P_1 – MC_1)/(1+r)$$

- Note, if MC flat or rising, P must rise.
- P – MC increases at rate of interest.

Take Away Points

- Money today and money tomorrow are different things. Present value is the correct way to combine such cash flows.
- People tend to be risk-averse. This is an important consideration for e.g. incentives.
- Flexibility has value (option value) which can be priced.