

## 1999 Sample Mid-Term Exam—Answer Sheet

### 1) True, False, Uncertain

**a) TRUE.** Videos supplied by the two stores are substitutes. As the price charged by the Hollywood store in Somerville increases, some people decide to go to rent videos from the Cambridge Blockbuster instead.

**UNCERTAIN** may also be acceptable, if in addition to above, a statement is added to the effect that positive correlation is an indication that the price of one good is constraining the price of another. However, correlation does not imply causation.

**Common Problems:** There are still some people who do not realize that you only need demand substitutability or supply substitutability to be in a market, not both. Also, a number of people seem to be having trouble keeping the vocabulary straight (using the word complement instead of substitute OR saying the problem described demand substitutes rather than supply substitutes). Another common error was thinking that you needed evidence that ALL consumers in Cambridge/Somerville must be willing to switch to the other store in order to be in the same market (i.e., effectively implying that only perfect substitutes or products with perfect geographic overlap can be in the same market).

**b) FALSE.** A portion of Air France's average variable cost is the "user cost of capital". This user cost of capital depends on the opportunity cost of capital, which is equal to the interest rate times the market value of the capital. Thus, when the market value of Air France's planes increases, its average variable cost also increases.

**UNCERTAIN is an alternative answer.** The answer may differ depending on whether we are thinking about long run or short run costs. In the long-run, when capital is variable, costs would be affected, but in the very short-run, when capital is sunk or fixed, variable costs would not be affected.

**Common Problems:** Most common mistake was thinking that the user cost of capital was a fixed rather than variable cost. Most people in this category implied that Air France would need to shut down completely to get rid of their planes rather than considering that they could sell/lease just a part of their fleet. If the person made a well-stated argument that the plane costs were fixed in the short term but variable in the long term WITH an argument describing the increased UCC, we gave full credit. The other common mistake was not realizing that UCC was the key to the question.

**c) FALSE.** Without a log calculator, this problem requires that you translate a percentage change in  $W$  to a percentage change in price, and then compare the resulting price change to the additional cost of \$9.78. The percentage change in weight  $W$  is

$$(8-10)/10 = -.20 = -20\%$$

This gives an (approximate) percentage change in price of

$$-.583 * -.2 = .1166 = 11.66 \%$$

or an overall (approximate) change in price of

$$.1166 * (\$ 59.95) = \$6.99$$

This is less than the cost of \$9.78 to change the weight, so the answer is FALSE, you should not recommend this change.

**Alternative 1:** (Different Approximate Percentage Change) Suppose that you used a different base for the percentage change, namely

$$(8-10)/8 = -.25 = -25\%$$

This gives an (approximate) percentage change in price of

$$-.583 * -.25 = .1457 = 14.57 \%$$

or an overall (approximate) change in price of

$$.1457 * (\$ 59.95) = \$8.74$$

This is less than the cost of \$9.78 to change the weight, so the answer is FALSE, you should not recommend this change.

**Alternative 2:** (Exact Solution) With a log calculator, you can directly compute

$$-.583 * (\text{Log } 8 - \text{Log } 10) = .1300 = 13.00 \%$$

We solve for the new price  $P^*$  as

$$\text{Log } P^* - \text{Log } 59.95 = .1300, \text{ or } P^* = \exp(.1300 + \text{Log}(59.95)) = \$ 68.27$$

So the price change is \$ 68.27 - \$ 59.95 = \$ 8.32, which is less than the cost of \$9.78 to change the weight, so the answer is FALSE, you should not recommend this change.

### Common Problems:

Some students multiplied the absolute value of the change (-2 Lbs) by the elasticity (0.583) and compared the result with the incremental cost (e.g. \$1.166 vs. \$9.78)

Some students multiplied the % change in the weight by the elasticity, multiplied the result by 100 and then compared this value with the incremental cost (e.g., \$11.66 vs. \$9.78)

Several students said that they didn't have enough information to answer the question. Some said that they needed to know the t-stat of the different parameters in order to give a correct answer.

**d) FALSE.** If a firm with seller market power faces a positive network externality, it will set price and quantity in any period taking account of the effect of the marginal unit on future demand and marginal revenue. For example, in a two period example, the firm will operate in period 1 such that

$$MR_1 = MC_1 - \frac{\partial R_2}{\partial Q_1}$$

where the marginal change in second-period revenue with an additional unit of production in the first period is positive. If the marginal cost is very low (*e.g.*, in the provision of some internet services where it may be near zero), then may maximize profit by producing for this period in a region where  $MR_1$  is negative.

### Common Problems:

Many people misread the question and answered “uncertain”, saying that generally a monopolist will produce a certain quantity  $Q$  such that  $MR = MC$ , but that it could happen that  $MR < 0$  in a certain time period due to learning or positive network externalities. If well argued, no points were taken off.

Many people affirmed that it is possible that  $MR < MC$  due to learning or network externalities, but said that  $MR$  could never be negative. In support of this argument many said that: a) if  $MR$  is negative, this means that the firm is paying the customers to buy the product, or b) if  $MR$  is negative, this means that the firm is making negative profits. Neither is necessarily true.

Some students answered that  $MR$  will always be equal to  $MC$  as a result of the monopolist's market power. In this case  $MR$  will never be a negative value because  $MC$  cannot be negative.

## 2) Footballs

**a)**  $E Q/P = dQ/dP$ . Hence if  $Q(\text{demand}) = a + bP$  and  $Q(\text{supply}) = c + dP$ , then  $b = Ed*Q/P = -4*5/10 = -2$ ,  $d = 2*5/10 = 1$ , and  $a = Q - bP = 5 - (-2)10 = 25$ ,  $c = Q - dP = 5 - 1*10 = -5$

Thus  $Q_s = -5 + P$  and  $Q_d = 25 - 2P$ .

(Note for (b) that the intercept of the demand curve is at  $P = \$12.5$  and the intercept of the supply curve is at  $P = \$5$ )

**b)**  $Q_s = -5 + (P - \text{tax}) \Rightarrow Q_s = -8 + P$

The new equilibrium is where  $Q_s = Q_d$ :  $-8 + P = 25 - 2P \Leftrightarrow 33 = 3P$  or  $P = \$11$ . This is the price buyers receive. Suppliers receive only \$8 per unit. At this supply price,  $Q_s = Q_d = 3$  million units. Hence the tax revenue raised is  $\$3 * (3 \text{ million units}) = \$9 \text{ million}$ .

$CS(\text{old}) = \text{area of triangle above demand curve at Price of } \$10 = 0.5 * (12.5 - 10) * 5 = \$6.25 \text{ million}$

CS(new) = area of triangle above demand curve at Price of \$11 =  $0.5 \cdot (12.5 - 11) \cdot 3 =$   
\$2.25 million

Difference = cs(new) - cs(old) = -\$4 million

PS(old) = area above supply curve and below price of 10 =  $0.5 \cdot 5 \cdot (10 - 5) =$  \$12.5 million

PS(new) = area above supply curve at new supply price of \$8 =  $0.5 \cdot 3 \cdot (8 - 5) =$  \$4.5 million

Difference = PS(new) - PS(old) = - \$8 million.

### 3) Monopolist in Two Markets

(a) This is from Homework Set ..., with a revision in the numbers. With two separate geographic markets, we can serve the two demands separately, charging different prices.

For **Market #1**,  $TR_1 = P_1 Q_1$ , and the inverse demand function can be written as  $P_1 = 15 - Q_1/2$ . Substituting  $P_1$  into  $TR_1$  gives

$$TR_1 = 15Q_1 - 1/2Q_1^2$$

Taking the derivative with respect to  $Q_1$  gives us the marginal revenue for Market #1

$$MR_1 = 15 - Q_1$$

Since  $TC = 5 + 2(Q_1 + Q_2)$ , taking the derivative with respect to  $Q_1$  gives us the marginal cost for Market 1:

$$MC_1 = 2$$

We know that monopolists price such that  $MR = MC$ , so set  $MR_1 = MC_1$ :

$$15 - Q_1 = 2 \text{ or } Q_1 = 13 \text{ units.}$$

Substituting  $Q_1$  back into the inverse demand function gives the price

$$P_1 = 15 - 1/2Q_1 = \$8.5$$

We can use the same process for **Market #2** to derive  $Q_2$  and  $P_2$ .

$$Q_2 = 11 \text{ units and } P_2 = \$13.$$

The profit function is  $\pi = P_1 Q_1 + P_2 Q_2 - TC$ , and we can substitute the values of  $P_1$ ,  $Q_1$ ,  $P_2$ , and  $Q_2$  into the equation to calculate profits:

$$\pi = (13)(8.5) + (11)(13) - [5 + 2(11 + 13)] = \$200.5$$

b) With open borders there will be one larger market and price cannot vary between the two areas. First, sum the demands of the two areas:  $Q = Q_1 + Q_2$

$$Q = 54 - 3P$$

For the whole market,  $TR = PQ$ . The inverse demand function can be written as

$$P = 18 - Q/3.$$

Substituting this value for P into TR gives

$$TR = 18Q - 1/3Q^2$$

Taking the derivative with respect to Q gives us the marginal revenue for the market

$$MR = 18 - 2/3Q$$

We know that  $MC = 2$ , and that monopolists price such that  $MR = MC$ , so

$$18 - 2/3Q_1 = 2$$

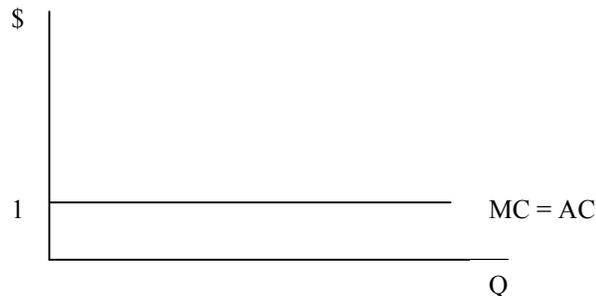
Solving for Q gives us  $Q = 24$  units, and substituting Q back into the inverse demand function gives the price,  $P = \$10$ . The profit is  $\pi = (10)(24) - [5 + 2(24)] = \$197$ .

#### 4) New Heart Drug

Demand:  $Q = 30 - 10P$

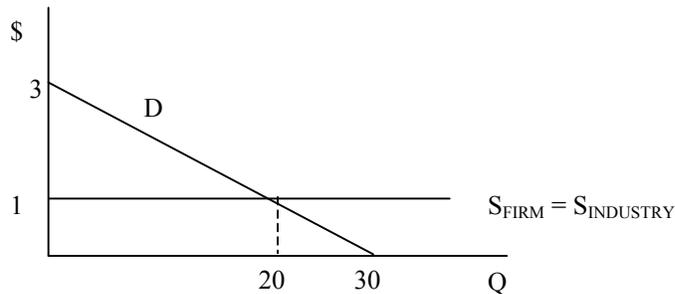
Cost:  $MC = \$1$

a) With a large number of firms, we have a perfectly competitive market. To determine the competitive market equilibrium, we need to analyze what each firm will do (derive supply curves). A generic firm's cost curves look like:



Note that since marginal cost is constant and there are no fixed costs, AC is also constant at \$1.

Thus, at market prices below \$1, the firm will not produce. At a market price of \$1, the firm would be willing to supply any amount of the product. Thus, the supply curve of the firm looks like:



The market supply curve is the horizontal sum of the firm supply curves – therefore, it is also flat at  $P=\$1$ .

Equilibrium in the market is where market supply equals market demand. At this point,  $P=\$1$ , so  $Q = 30 - 10*(1) = 20$ . Total firm profits or producer surplus at this equilibrium are exactly 0, as firms are selling at a price equal to average cost.

**b)** Profits for a monopolist (as a function of  $Q$ ) are:

$$\Pi = P(Q)*Q - C(Q)$$

where  $P(Q)$  is the inverse demand curve, in this case  $P = 3 - 0.1Q$ .

$C(Q) = Q$  because  $MC$  and  $AC$  are constant at  $\$1$ .

$$\text{Thus, } \Pi = (3 - 0.1Q)Q - Q$$

To maximize profits, set derivative with respect to  $Q$  equal to zero:

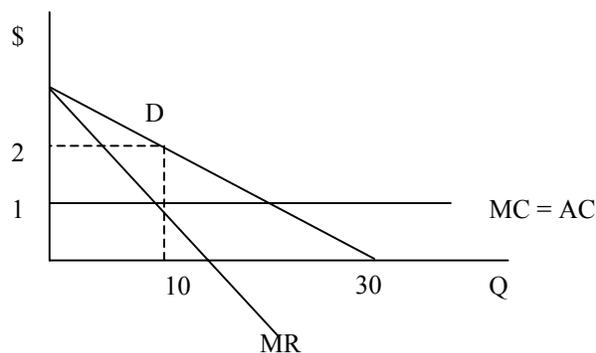
$$\begin{aligned} 3 - 0.2Q - 1 &= 0 \\ Q &= 10 \end{aligned}$$

$$\text{and } P = 3 - 0.1Q = 2$$

At this monopoly equilibrium:

$$\Pi = (3 - 0.1Q)Q - Q = (3 - 0.1*10)10 - 10 = 10$$

Graphically:



c) From part b), a monopolist would want to price at \$2. With a price ceiling below \$2, the monopolist will set price as high as it can, equal to the ceiling \$1.20. Quantity sold at that price is  $Q = 30 - 10 \cdot 1.2 = 18$ . Profits are:

$$PI = (3 - 0.1Q)Q - Q = (3 - 0.1 \cdot 18)18 - 18 = 3.6$$

**Common Problems:**

(4a) Several students calculated producer surplus to be the area under the supply curve to the left of the equilibrium quantity. This is wrong: this area actually represents the total variable cost of the suppliers. The producer surplus (or supplier profit in this case) was equal to zero, since the marginal cost curve was flat and equal to price.

(4b) Several students misunderstood producer surplus, which in this case should be supplier revenue minus cost. Also, many students had difficulty algebraically manipulating the demand equation to isolate price in terms of quantity, and calculating the marginal revenue curve.

(4c) Many students set price equal to marginal revenue as calculated in part (b), and used this equation to calculate equilibrium quantity. The correct way to do this part is to substitute the \$1.20 price into the demand curve to determine quantity, not to substitute into the marginal revenue curve. (As a logical check, notice that the supplier profit is higher when you use the demand curve than when you use the marginal revenue curve - thus, the supplier will want to choose quantity from the demand curve to maximize profit.)